RECOMMENDATIONS.

From Prof. E. A. Andrews.

About two years ago, I had the pleasure of meeting with Mr. Tracy, and of having some conversation with him on the subject of an Arithmetic which he was preparing for schools. I was particularly pleased with that part of his system which related to *canceling*, and which appeared to me to possess *great practical value*. Within a few days, Mr. Tracy has put into my hands a part of his manuscript, that I might become more minutely acquainted with his system. My time has been so far occupied with my own business, that I have been able to examine a part only of the manuscript put into my hands; but with this part I have been much gratified. It appears to me, that, when carefully and thoroughly revised and perfected, as the author designs to do, it may become a *most valuable work*, inferior to none of the Arithmetics now used in our schools. Such is my confidence in the ability of the author to complete and polish the work, that I look upon its success as quite certain.

From E. H. Burritt, Author of the Geography of the Heavens, &c.

Through the politeness of Mr. Tracy, I have been favored with a perusal of an Arithmetic, in manuscript, which he is preparing for publication. The work is intended as a universal class-book in elementary arithmetic. It is the production of a gentleman of known abilities and experience in teaching; and he has, with great care, arranged its several parts, and given the rules, and selected the examples, step by step, in that natural order, and easy method, which his own judgment and experience approved. There are some excellences in his Arithmetic, some facilities of dealing with figures, which, so far as I know, are entirely peculiar to this treatise, and which distinguish it from all others. On this ground especially, and that of its general merit, I think it a work which will commend itself to the attention of teachers.

I entirely coincide in the above opinion, having been particularly gratified with the ease and facility with which many difficult operations are performed by the new principle introduced by the author. J. P. BRACE, *Principal of Hartford Female Seminary.*

From E. C. Herrick, Esq.

I have cursorily examined the manuscript of Mr. C. Tracy's treatise on Arithmetic. The most prominent feature of the work is the introduction of a peculiar mode of stating numerous classes of problems, which are then solved by an abridged process, called canceling. This appears to me an important improvement on the books in common use, and one which renders the publication of this treatise very desirable.

From La Fayette S. Foster, Esq.

Mr. Calvin Tracy has submitted to my examination, in manuscript, an Arithmetic prepared by himself for publication. From the known ability of Mr. Tracy as an instructor, I was prepared to entertain a high opinion of any treatise designed to facilitate the acquisition of knowledge, of which he might be the author; and from the attention which I have bestowed on his Arithmetic, I have no hesitation in bearing testimony to its high meritorious character. His plan appears to me to be highly judicious, and ably and skillfully executed. The work, in my opinion, will be a valuable addition to a very important branch of education.

Extract of a Letter from J. J. Van Antwerp, Esq., Principal of Coxsackie Academy, N. Y.

Dear Sir — I have heretofore given the preference to "Adams's New Arithmetic," and have always made use of it. I consider yours equally as good, if not superior to it, as far as your system is *not* "New." Your system of canceling I regard as an important improvement, especially for those pupils who are called "good in figures," and I am not sure but it may help to arouse the stupid and dormant faculties of the *dull*. We wish to make ready and correct accountants; — your system strikes me as tending directly to such a result.

Extract of a Letter from M. N. Morris, Esq., Principal of Colchester (Conn.) Academy.

From the defective manner of treating the principle of canceling, so far as I had seen allusions to it in works published previously to yours, I had been led to think unfavorably of it, — the method having been merely alluded to, without any clear investigation of the principle itself. Instead of so doing, you have, however, first led the scholar in the plain, obvious course, to the result, and have then taught him to abbreviate the operation, by the application of well-known and obvious principles. The defective manner to which I have alluded, you have therefore avoided. In applying the principle of canceling, as laid down in your book, the student secures even a clearer [illegible] of his subject, than in the ordinary

mode of solution. I can therefore cheerfully express the opinion, that, although several valuable treatises on arithmetic have appeared within a few years, your work combines excellences which are rarely, if ever, to be found without resorting to different systems. The general perspicuity and conciseness of explanations and illustrations, the completeness of the work as a system, and the happy application of the principle of canceling, give it strong claims on the attention of those who have the care of educating the young.

From S. Smith, Esq., Principal of Boarding School, Poughkeepsie, N. Y.

I have examined a new system of Arithmetic by C. Tracy, Esq., and find it well adapted to the use of schools. The arrangement and mode of treating the subject are more definite and perspicuous, and, in fact, better than are those of any other Arithmetic with which I am acquainted.

From R. S. Howes, Principal of Academy, Troy, N. Y.

I have examined a system of Arithmetic by C. Tracy, Esq., and, as the result, am satisfied that it is superior to any other work with which I am acquainted. The method of canceling introduced must secure success to the work, being, as it is, well calculated to render those who study it quick in figures and prompt in business.

From E. Wilson, Jr., Principal of Monitorial School, Troy, N. Y.

I have recently examined a treatise on Arithmetic by C. Tracy, Esq. Both the matter and manner of the work are judicious, — the former embracing all that the habits of our men of business require ; the latter, all the advantages which other systems contain, together with the new and peculiar mode of canceling, which very much abridges the processes of solution. In these particulars, the work is an improvement upon former systems.

From J. H. Rogers, Esq., Principal of Prospect Hill High School.

Messrs. Durrie & Peck,

Gentlemen — From a hasty examination of Tracy's Arithmetic, I believe it worthy of being ranked among the best school books. The method of canceling, very fully brought into practice in this work, greatly abridges many operations, and may be mentioned as one of its most valuable features. Sincerely yours, J. H. ROGERS.

From Rev. A. Bond, Pastor of the Second Congregational Church in Norwich.

Having examined the general plan of an Arithmetic, prepared by Mr. Tracy, Principal of Norwich Academy, I can cheerfully recommend it as a system possessing, in some important particulars, a superiority over any other system with which I am acquainted. The method of *canceling*, which is carried through the work, excepting the Roots, greatly facilitates the process of arithmetical calculations, and will give it a decided advantage in the estimation of business men. The part on foreign exchanges will enhance its value with the commercial community. While its simplicity adapts it to the use of common schools, its comprehensiveness, and the ease and accuracy with which complicated problems may be solved, will be likely to secure for it a prominent place in the counting room.

From Rev. L. N. Tracy, formerly Principal of New Britain Academy.

I have spent considerable time in a careful examination of an Arithmetic prepared by Mr. C. Tracy, Principal of Norwich Academy. For my own benefit and pleasure, I have carefully examined every rule ; and though I have daily used the best Arithmetics extant, while engaged for many years in teaching, I am led to believe that there is not a text-book on arithmetic in use which presents equal excellences. Its grand feature — that which distinguishes it from every other arithmetical treatise — is *the principle of canceling*, introduced and applied throughout the work. The extent and facility of its application to all operations in which multiplication and division are both concerned, are fully and clearly illustrated. It is safe to say that two thirds, and often four fifths of the labor and time usually required for arithmetical solutions, are saved. While it contains an amount of matter equal to any other Arithmetic in use, it is still a strictly elementary work.

The following is from the Board of Visitors of the First School Society of the town of New Haven, who are by law appointed, to determine what books shall be used in the schools under their superintendence.

The undersigned, Visitors of the First School Society in the town of New Haven, having, by means of a committee of our body, made examination of the "New System of Arithmetic," prepared by Mr. C. Tracy, Principal of Norwich Academy, and considering the work to contain important improvements on the treatises in common use, do hereby direct the said book to be adopted in the Schools of this Society.

R. S. HINMAN,	R. H. OSBORN,	*School Visitors.*
ALLING BROWN,	GEORGE F. SMITH,	
EDWARD C. HERRICK,	WYLLIS PECK,	

New Haven, *April* 29, 1840.

A

NEW SYSTEM

OF

ARITHMETIC;

IN WHICH IS EXPLAINED AND APPLIED TO PRACTICAL PURPOSES, IN ADDITION TO THE ORDINARY RULES OF OPERATION,

THE PRINCIPLE OF CANCELING,

BEING AN

ABBREVIATED MODE

OF

ARITHMETICAL SOLUTION.

DESIGNED FOR SCHOOLS AND ACADEMIES.

alvin

By C. TRACY, A. M.
PRINCIPAL OF NORWICH ACADEMY.

FOURTH EDITION.

NEW YORK:
MARK H. NEWMAN,
199 BROADWAY.

1845.

STEREOTYPED AT THE
BOSTON TYPE AND STEREOTYPE FOUNDRY.

INTRODUCTION.

It will readily be conceded, that all efforts in behalf of the general diffusion of useful knowledge, are in themselves commendable. There is, however, and probably ever will be, a difference of opinion relative to the extent to which books of any particular description, and treating upon the same general topic, may be multiplied, and the interests of education uniformly advanced thereby.

This difference of opinion exists especially in relation to *books designed for the use of common schools and academies*, and which treat upon the more common subjects of study. The multiplication of books of this description, to the extent realized at the present day, is regarded by many as injurious to the general good. That its tendency is to increase, *in some small degree*, the expense of education — at least in some parts of the country — will not be denied. But, before sentence of final condemnation is pronounced, it always becomes those, who sit as umpires, to take as extended views of the subject before them, as the nature of the case will admit.

The question now presented is, *how far the general good is advanced by the multiplication of school books.*

To answer this, let it be supposed that only a single work in each department of science studied in common schools, had ever been presented to the public, and that each work were such as it should be. Books of this description would obviously find ample circulation, sufficient, perhaps, to satisfy both authors and publishers, without embracing *one half of the ground to be occupied.* The consequence would be, that the more recently settled parts of our country would be but poorly supplied with the means of education, for at least some considerable period of time. But, as it now is, with such a multiplicity of school books constantly emanating from the press, a spirit of rivalry is created, — a desire is excited on the part of both authors and publishers to give to their several works a more extended circulation than can be obtained without exploring the whole ground. As a natural consequence, the inhabitant of the less favored portion of our land is scarcely settled in his log cabin, before books of every description necessary for the education of his sons and daughters, are presented him, as it were, at his own door. His attention is thus directed to a subject second in importance to none of a temporal nature; and one which, when duly presented, will be likely to be regarded, and to receive a consideration, which otherwise might be long neglected.

The truth of our supposition, that any one set of school books is such, in all respects, as is required, may, however, very reasonably be doubted. Many of them are unquestionably of a high order, and probably owe some degree of their merit to the fact, that other minds have been, are, and yet will be, traversing the same ground which their authors trod, and are preparing other works to supersede them, if possible, in the estimation of the public.

The effect of the multiplication of school books is, therefore, *to render the means of education as perfect as the nature of their subjects will allow, and to convey these means, thus perfected, to every part of our entire country.*

From the preceding considerations, the author is inclined to regard the multiplication of school books as favorable to the cause of general education. It therefore only remains to point out some of the more important

features of the following work, before introducing it to the ordeal of public opinion. That it is worthy of public attention and patronage, belongs not to him to decide. It certainly will be found to possess some *peculiarities*, which are of course regarded by him as *improvements*. Whether they are indeed such, remains for others to determine.

A peculiar feature of the following pages, and one which distinguishes this work from every other on the same subject, is the "System of Canceling," which, in connection with the ordinary mode of solution, is introduced throughout, and applied to such arithmetical problems as embrace in their operation both multiplication and division. This is regarded by the author, and by many others acquainted with his system, *as a decided improvement upon all Arithmetics heretofore presented to the public.*

The following are some of the advantages of the new system: —

1st. The statement required, or, rather, recommended, for canceling, *is itself a complete analysis of the sum proposed.* Suppose, for illustration, that 12 yards of cloth cost \$48, and that it is required to find the value of 15 yards of the same.

We analyze the preceding sum, either by first finding the value of one yard of the cloth, viz., $\$48 \div 12\text{yd.} = \$4$, and then multiplying that price by the number of yards, as, $\$4 \times 15\text{yd.} = \$60$, *Ans.*; or by finding the ratio of the number of yards of which the price is given, and of those of which the price is required, and then multiplying that ratio by the given cost. This ratio is $\frac{15}{12} = \frac{5}{4}$; and $\frac{5}{4} \times 48 = 60$, the number of dollars required, and the same as above.

The same stated for canceling: —

$$\frac{48.\ 15}{12}$$

(See rule for canceling, in Single Proportion.) By the above statement, \$48 is to be increased by the ratio 12 : 15; or $\frac{15}{12} = \frac{5}{4}$. It is, however, obvious that $\frac{1}{12}$ of 48, multiplied by 15, is the same as $\frac{1}{12}$ of 15 multiplied by 48. The above sum is therefore canceled and solved thus: —

$$\frac{\overset{4}{\cancel{48}}.\ 15}{\cancel{12}}, \text{ and } 15 \times 4 = 60, \text{ the dollars required.}$$

The application of the canceling principle is, however, more completely illustrated by the solution of sums in which the ratio of one of the given quantities to a required quantity of the same kind, is traced through several simple ratios. The following sum may serve as an illustration: —

If 3 men, in 16 days, of 9 hours each, build a wall 20 feet long, 9 feet high, and 4 feet thick, in how many days, of 8 hours each, will 12 men build a wall 200 feet long, 8 feet high, and 6 feet thick?

It is obvious that the ratio of the given days to the required number of days, is compounded of the ratios, 8 hours : 9 hours; 12 men : 3 men; 20 feet in length : 200 feet in length; 6 feet in height : 8 feet in height; and 4 feet in thickness : 6 feet in thickness. Or, these ratios may be fractionally expressed, thus; $\frac{9}{8}$, $\frac{3}{12}$, $\frac{200}{20}$, $\frac{8}{6}$, and $\frac{6}{4}$.

Now, the given days of 9 hours each are changed to days of 8 hours each by the following statement: —

$$\frac{16.\ 9}{8} = 18 \text{ days.}$$

The time, in days of 8 hours each, required for the 12 men to complete the work of 3 men, is obtained by uniting the second of the preceding ratios to the above statement, thus:

$$\frac{16.\ 9.\ 3}{8.\ 12} = 4\tfrac{1}{2} \text{ days.}$$

By introducing into the same statement, the third of the preceding ratios, we obtain the requisite time for completing the 200 feet of wall, allowing the height and thickness of each wall to be the same; thus,

$$\frac{16.\ 9.\ \ 3.\ 200}{8.\ 12.\ \ 20} = 45 \text{ days.}$$

If the fourth ratio be united, we obtain the time required, allowing each wall to be of the same thickness; thus,

$$\frac{16.\ 9.\ \ 3.\ 200.\ 8}{8.\ 12.\ \ 20.\ 6} = 60 \text{ days.}$$

Lastly, if the fifth and last ratio be introduced, the number of days required by all the conditions of the question is obtained, viz.:

$$\frac{16.\ 9.\ \ 3.\ 200.\ 8.\ 6}{8.\ 12.\ \ 20.\ 6.\ 4} = \text{the required days.}$$

The last statement canceled;

$$\frac{\cancel{16}.\ 9.\ \ \cancel{3}.\ \overset{10.}{\cancel{200}}\ \overset{2}{8}.\ \cancel{6}}{8.\ \underset{2}{\cancel{12}}.\ \ \cancel{20}.\ \cancel{6}.\ \cancel{4}}, \text{ and } 10 \times 9 = 90 \text{ days, } Ans.$$

2dly. A second advantage to be derived from the canceling system, is the facility afforded by it for reducing several operations to a single statement. The following example will afford an illustration: —

Bought 5 cwt. of sugar, at 6 d., New York currency, per pound, after making a deduction of 8 lb. on every 112 lb. for tare, &c. How much shall I receive for the whole quantity, if I sell it at an advance of 20 per cent. on the purchase price?

The ordinary mode of solving this sum would be as follows: — Having reduced the 5 cwt. to pounds, we make the statement, 112 : 104 : : 560 : *Ans.*; viz., 520 lb. This 520 lb. is the net weight of the sugar; therefore, $520 \times 6 = 3120$, the pence the sugar cost. We next reduce the pence to dollars by dividing by 12 and 8, the currency being that of New York, and obtain, as the cost of the sugar, $32.50. On this sum I wish to make an advance of 20 per cent., for which the following is the statement: —

100 : 120 : : 32.50 : *Ans.*, which is 39 dollars.

By canceling, these several operations are reduced to one; thus,

$$\frac{5.\ 4.\ 28.\ 104.\ \ 6.\ \ \ 120}{112.\ 12.\ 8.\ 100}. \text{ The same canceled; } \frac{\cancel{5}.\ \cancel{4}.\ \cancel{28}.\ \overset{13.}{\cancel{104}}.\ \overset{3.}{\cancel{6}}.\ \overset{6}{\cancel{120}}}{\cancel{112}.\ \cancel{12}.\ \underset{2.}{\cancel{8}}.\ \underset{20}{\cancel{100}}};$$

and $13 \times 3 = 39$, the dollars required, and the same as before.

3dly. A great advantage of the canceling system over all others, arises from the *expedition* it affords in arithmetical solutions. Instead of multiplying and dividing by all the numbers which the nature of the sum proposed would naturally require, the multipliers and divisors are made to cancel each other; that is, equal factors are rejected from both. Hence, they are all made to exert their appropriate influence in procuring the answer, while the labor of multiplying and dividing is avoided. The statement of each sum for canceling is a fractional answer of the same; and it is obvious that the value of fractions is not affected by rejecting equal factors from their numerators and denominators.

The processes of reduction, which occur very frequently in common Arithmetics, are mostly avoided by this system. Suppose it be required to find how many pounds sterling 5 hogsheads of wine would cost, at 10 d. per pint. By the canceling system, it is necessary only to write

down the numbers required to effect the reduction, and the question is then solved by canceling those numbers as far as practicable; thus,

$$\frac{5.\ 63.\ 4.\ \ 2.\ 10}{12.\ 20}.$$

The numbers above the line are obviously those which, when multiplied together, will give the answer in pence. The numbers below the line are those required to reduce pence to pounds sterling. The above sum canceled:

$$\frac{5.\ \overset{21}{\cancel{63}}.\ \cancel{4}.\ \ \cancel{2}.\ \cancel{10}}{\cancel{12}.\ \underset{3}{\cancel{20}}};\ \text{then, } 21 \times 5 = 105\ \pounds.,\ \textit{Ans.}$$

That the above method of solving arithmetical problems is easily comprehended and applied by the scholar, has been fully tested by the author. The experience of nine or ten years entirely devoted to the business of instruction, leaves him no room to doubt on this point. Being, however, fully aware that his Arithmetic might fall into the hands of some, who would not at once comprehend and apply the principle of canceling, he has introduced the ordinary rules of solution, in connection with those of canceling, and has endeavored to render both modes plain and familiar, by frequent and clear illustrations.

The constant aim of the teacher should be to prepare his pupils for the *active duties* of life; and, in the department of arithmetic, this is accomplished only when the scholar has acquired correctness and *expedition* in effecting his solutions.

To make good arithmeticians, it is first necessary to acquire a correct and extensive comprehension of the simple or fundamental rules of arithmetic. When this is done, their application will be obvious. The danger, therefore, is not that the scholar will spend too much time on what is usually regarded as the more simple part of arithmetic, but that he will leave it too soon.

In the use of this treatise, the author would recommend, that, when the pupil shall have passed the simple rules, and commenced those operations to which canceling may be applied, he be required to solve each problem both by the ordinary rule and by the rule for canceling. More practice will thus be secured, and, consequently, greater expedition acquired.

In the illustrations which are given in connection with the different rules, it has been the design of the author fully to acquaint the scholar with the nature of the subject presented, without carrying his explanations so far as to take the work which properly belongs to the scholar, out of his hands. No important acquisition can be made without corresponding effort. This fact seems to have been overlooked in the preparation of some Arithmetics now in use, and special effort made to render every thing as *easy as possible* for the scholar; that is, to enable him to effect the solutions with very little mental labor. The intellectual powers are, however, developed and strengthened only by being brought into *vigorous exercise.* "In arithmetic, the young beginner should find just enough assistance to encourage and stimulate him to effort. That is not the best system which enables the learner to advance from rule to rule with the least amount of study; but that which, while it helps him over some difficulties, leaves him examples enough to task his powers to the utmost." (Dr. Humphrey's Thoughts on Education.)

With these introductory remarks, the following work is commended to the candor of an enlightened public.

THE AUTHOR.

NORWICH ACADEMY, *May*, 1840.

TABLE OF CONTENTS.

APPENDIX.

ARITHMETIC.

§ 1. ARITHMETIC explains the properties and relation of numbers, and makes known their practical application.

There are six fundamental operations, with which the scholar must become perfectly familiar, before he can advance successfully, viz. *Notation*, *Numeration*, *Addition*, *Subtraction*, *Multiplication*, and *Division*. These operations are called *fundamental*, because all others are founded upon, or are wrought by, the application of one or more of them. They therefore require to be first clearly understood.

NOTATION.

§ 2. Notation is the art of expressing numbers by numerical characters. The characters employed to express numbers are, 1, 2, 3, 4, 5, 6, 7, 8, 9, 0, and are called *figures*. Each of these figures has its own *specific*, and also its *local* value, as will be learned from Numeration. Besides these characters, there are others used to express operations.

1st. The sign of *addition*, viz. +, (or *plus*, more;) requiring the numbers between which it is placed to be added: 3 + 2 are 5; that is, 3 added to 2 are 5; usually read, 3 plus 2 are 5.

2d. The sign of *subtraction*, viz. —, (or *minus*;) showing that the number following is to be taken from that which

precedes it; thus, $4 - 2$ is 2; that is, 2 taken from 4, 2 remains.

3d. The sign of *multiplication*, viz. $\times$; requiring the number placed before it to be multiplied by that which follows; thus, 3×4 is 12; that is, 3 multiplied by 4 is 12.

4th. The sign of *division*, viz. $\div$; requiring the number preceding it to be divided by that which follows; thus, $8 \div 2$ is 4; that is, 8 divided by 2 is 4.

In the use of each of the preceding signs, the figure preceding the sign is to be operated upon by that which follows it.

5th. The signs of *proportion*, viz. : :: : ; showing that the numbers including and between these dots, are proportionals; thus, $2 : 4 :: 6 : 12$; that is, 2 bears the same relation to 4 as 6 to 12. The numbers are thus read: 2 is to 4 as 6 is to 12.

6th. The sign of *equality*, viz. $=$; expressing the equality of the numbers between which it is placed; or, that the numbers on the right equal those on the left; thus, $9 + 7 = 20 - 4$.

7th. The *vinculum*, or horizontal line drawn over several numbers, and showing that they are all subjected to the same operation; as, $18 - \overline{9 + 2} = 7$; that is, the sum of 9 and 2 being taken from 18, 7 remains.

8th. The characters $\sqrt{}$, $\sqrt[3]{}$, $\sqrt[4]{}$, $\sqrt[5]{}$, &c., require some root of the number before which they stand to be extracted. The figure placed over the sign always shows what root is required. When the character is used without any figure, it then indicates the square root.

By the use of these characters, any arithmetical operation may be indicated. If it be required to add 9 to 16, from the amount to subtract 5, to divide the remainder by 4, and to multiply the quotient by 6, the operation would be thus expressed: $\overline{9 + 16 - 5} \div 4 \times 6 = 30$.

QUESTIONS.—What does Arithmetic explain? What application does it make of numbers? How many are the fundamental operations of Arithmetic? What are they? Why are these called fundamental operations? What is *Notation*? What are the characters used to express numbers called? What twofold value has each figure? For what purposes are other characters used? What is the sign of addition?—and for what is it used? The sign of subtraction?—what does it require? The sign of multiplication?—what does it require? The sign of division?—what does it require? The signs of proportion?—what do they show? The sign of equality?—what does it show? What is the vinculum? What is the character used to express the extraction of roots called? *Ans.* The radical sign. What does the figure placed over the radical sign show?

NUMERATION.

§ 3. The scholar has seen, under Notation, the characters used to express the first nine numbers, viz. that to express one whole object or thing, 1 is used; to express two whole things, 2 is employed; and for three whole things, 3 is taken, &c.; so that each character has its own *specific* value; and this it always expresses when it stands alone. But each figure has also a *local* value; that is, a value depending on the *place* it occupies; thus, the value of 3 differs in each of the following numbers, viz., 003, 030, and 300. In the first number, its value is three units, or ones; in the second number, it is three tens, or thirty; and in the last, it is three hundreds. It will therefore be readily perceived, that the position of a figure materially affects its value. Numeration enables us to determine this *local* value of any figure, and consequently to ascertain the *total value of any number of figures.*

Let it therefore be remembered, that units always occupy the *first place* on the right hand; tens, the *second place;* hundreds, the *third place,* &c.; also, that any figure is increased in a *tenfold* ratio by having a single figure placed on the right of it; thus, 6 alone is 6 units; but if another figure be placed on the right of this, its value is ten times as great as before; thus, in 63, the 6 is six tens, equal to 60, and the 3 is three units. This value is increased a *hundred fold* by having two figures placed on the right; thus, 600; and a *thousand fold* by having three figures on the right of it; thus, 6000. In the first of the last two examples, the value of 6 is six hundred, and in the second it is six thousand.

Hence the scholar will see the necessity of terms by which to designate this local value of figures, and will also readily see the appropriateness of those used, viz.: —Units, Tens, Hundreds, Thousands, Tens of Thousands, Hundreds of Thousands, Millions, Tens of Millions, Hundreds of Millions, &c. These nine terms are sufficient to express any number in common practice. The higher denominations are Billions, Tens of Billions, Hundreds of Billions; Trillions, Tens of Trillions, Hundreds of Trillions; Quadrillions, Tens of Quadrillions, Hundreds of Quadrillions; Quintillions, Tens of ——, Hundreds of ——; Sextillions, Tens of ——, Hundreds of ——; Septillions, Tens of ——, Hundreds of ——; Octillions, Tens of ——, Hundreds of ——; Nonillions, Tens of ——, Hundreds of ——, &c.

Six units are written,	6
Six tens, or sixty units,	60
Six hundreds, or sixty tens,	600
Six thousand, or sixty hundred,	6,000
Sixty thousand,	60,000
Six hundred thousand,	600,000
Six millions,	6,000,000
Sixty millions,	60,000,000
Six hundred millions,	600,000,000
By uniting the preceding numbers, we obtain	666,666,666

This last expression is obviously a union of all the preceding numbers, each figure retaining its relative position and value.

§ 4. It will be observed that, as the first three figures, reckoning from the right, are units, tens, and hundreds, so every succeeding three are appropriated to the units, tens, and hundreds of the succeeding higher denominations. The following table will serve as an illustration: —

of Octillions.	of Septillions.	of Sextillions.	of Quintillions.	of Quadrillions.	of Trillions.	of Billions.	of Millions.	of Thousands.	of Units.
369,	342,	900,	976,	368,	265,	371,	502,	634,	436.
Hundreds tens units	Hundreds tens units	Hundreds tens units	Hundreds tens units	Hundreds tens units	Hundreds tens units	Hundreds tens units	Hundreds tens units	Hundreds tens units	Hundreds tens units

This table will enable the scholar to see at a glance, that the names and value of figures are entirely dependent on their location. If they be counted from the right hand towards the left, the first figure in any line of figures is units; the second is tens; the third, hundreds; the fourth, thousands; the fifth, tens of thousands, &c.; and whatever station or place any figure may occupy, its value becomes ten times as great by being moved one degree farther to the left.

§ 5. It will be observed, on examining either the preceding or following table, that *three figures* are appropriated to *each denomination;* and that, under each denomination, the words *units*, *tens*, and *hundreds*, are invariably employed to express the *relation these figures sustain to each other.*

To make the reading of figures easy, especially if the numbers are large, they are separated by the comma into periods of *three figures each.* A familiar acquaintance with the *order of the periods*, and the *relative* value of the figures composing them, is therefore requisite to insure uniform accuracy in assigning them their *true* value.

NUMERATION TABLE.

Period 6th			Period 5th			Period 4th			Period 3d.			Period 2d.			Period 1st.		
Hundreds of Quadrillions.	Tens of Quadrillions.	Quadrillions.	Hundreds of Trillions.	Tens of Trillions.	Trillions.	Hundreds of Billions.	Tens of Billions.	Billions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.
3	4	5,	6	7	8,	9	1	2,	4	5	9,	7	1	6,	2	0	4
·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	3	5	6
·	·	·	·	·	·	·	·	·	·	·	·	·	2	6,	7	8	5
·	·	·	·	·	·	·	·	·	3	7	0,	0	0	0,	6	2	7
·	·	·	·	·	5,	4	2	0,	4	1	5,	6	0	0,	0	0	1
·	·	·	5	7	9,	1	3	5,	2	4	6,	8	0	1,	3	5	7
·	·	4,	6	7	8,	2	0	0,	0	0	3,	6	0	0,	4	2	0
·	5	9,	2	0	0,	4	6	8,	1	3	9,	6	4	2,	1	0	0
9	9	9,	8	8	8,	7	7	7,	6	6	6,	5	5	5,	4	4	4
·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	·	8	2
·	·	·	·	·	·	·	·	·	·	·	·	·	·	9,	6	5	4
·	·	·	·	·	·	·	·	·	·	4	0,	0	0	0,	0	0	8
·	·	·	·	·	·	·	·	6,	0	4	0,	3	0	0,	0	0	2
·	·	·	·	·	·	3	8	6,	5	9	0,	4	6	8,	5	9	6
·	·	·	6	8	4,	6	8	3,	0	9	5,	8	6	4,	9	5	6

The scholar should be thoroughly exercised on the preceding table, or a similar one written on his slate.

QUESTIONS.—What are the characters used to express the first nine numbers? Give an example. When does each figure express its own specific value? What other value has each figure? On what does the local value of a figure depend? Give an illustration. What, therefore, does Numeration teach us to do? What does it enable us to do? In what ratio is the value of any figure increased by having a single figure placed on the right? In what ratio is it increased by having two figures on the right of it? In what, by having three placed on its right? In what ratio do numbers continue to increase from the right to the left? *Ans.* In a tenfold ratio. What are the terms by which the local value of figures is expressed? To what are each three successive figures in any number appropriated? *Ans.* To the units, tens, and hundreds of each denomination. In tracing the figures from the right to the left, what is the first figure called? The second? The third? The fourth? &c. What effect is produced on the value of any figure by moving it one place to the left?

§ 6. Enumerate the following numbers, viz.:—6; 27; 467; 568; 4269; 13786; 27599; 367595; 1729567; 67596422; 586; 379872689; 278063; 596402606; 295; 336003; 300300303; 505050505; 467327986427; 585950876; 688001; 100000000; 99999999999; 6393742913; 4678.

The scholar should be taught to read these figures accurately; for example, suppose he be required to enumerate the last number, viz., 4678; let him commence and repeat thus: eight units, seven tens, six hundreds, and four thousands; and then unite them, thus: four thousand six hundred and seventy-eight. Let him also be required to give the value of any figure as it may vary by being written at different points under any line of figures.

§ 7. After the scholar has become familiar with the preceding exercise, he may write the following numbers on his slate in figures, taking care to express each number accurately: — 1. Thirty-five. 2. Three hundred and seventy-five. 3. Three hundred and five. 4. Seven thousand six hundred and thirty-five. 5. Seven thousand and thirty-five. 6. Seventy-five thousand four hundred and sixteen. 7. Seventy-five thousand and sixteen. 8. Seventy-five thousand and six. 9. Seventy-five thousand. 10. Three hundred and thirty-three thousand three hundred and thirty-three. 11. Three hundred thousand and three. 12. Three hundred thousand three hundred and three. 13. Five millions and five. Six millions and seventy-five. One hundred and sixty millions. Forty-seven millions, one hundred and five thousand and sixty. 14. One hundred millions, one hundred and one. 15. One hundred and seven millions, one hundred and seven thousand, one hundred and seven. 16. Two billions, three hundred and three millions, five hundred and five thousand and six. 17. Seven hundred and seven trillions, six hundred and seventy-two billions, nine millions, three hundred and five thousand, six hundred and nine.

§ 8. There is yet another method of expressing numbers, viz., the Roman method; in which the letters of the alphabet are used, as may be seen from the following table: —

ROMAN TABLE.

I	One.	LX	Sixty.
II	Two.	LXX	Seventy.
III	Three.	LXXX	Eighty.
IV	Four.	XC	Ninety.
V	Five.	C	One hundred.
VI	Six.	CC	Two hundred.
VII	Seven.	CCC	Three hundred.
VIII	Eight.	CCCC	Four hundred.
IX	Nine.	D	Five hundred.
X	Ten.	DC	Six hundred.
XX	Twenty.	DCC	Seven hundred.
XXX	Thirty.	DCCC	Eight hundred.
XL	Forty.	DCCCC	Nine hundred.
L	Fifty.	M	One thousand.

ADDITION OF SIMPLE NUMBERS.

§ **9.** Addition is an operation by which two or more numbers are united in one. The number obtained is called the *sum*, or *amount*, and always contains as many units, tens, hundreds, &c., as the numbers united. When the numbers so united increase in value from the right hand to the left, in the constant ratio of 10, (see § 3,) the operation is called *Simple Addition.*

In reciting the following, and other tables, the teacher should be careful that the scholar, as he repeats his answers, perform the necessary mental operation; — he must teach the pupil to *think.*

ADDITION TABLE.

§ **10.** The signs plus and minus, &c., are introduced into the following tables, that the scholar may early learn the use of them. The sign + implies *addition*, and = signifies *equality*.

2 plus 1 equals 3	3 + 1 = 4	4 + 1 = 5	5 + 1 = 6
2 + 2 = 4	3 + 2 = 5	4 + 2 = 6	5 + 2 = 7
2 + 3 = 5	3 + 3 = 6	4 + 3 = 7	5 + 3 = 8
2 + 4 = 6	3 + 4 = 7	4 + 4 = 8	5 + 4 = 9
2 + 5 = 7	3 + 5 = 8	4 + 5 = 9	5 + 5 = 10
2 + 6 = 8	3 + 6 = 9	4 + 6 = 10	5 + 6 = 11
2 + 7 = 9	3 + 7 = 10	4 + 7 = 11	5 + 7 = 12
2 + 8 = 10	3 + 8 = 11	4 + 8 = 12	5 + 8 = 13
2 + 9 = 11	3 + 9 = 12	4 + 9 = 13	5 + 9 = 14
2 + 10 = 12	3 + 10 = 13	4 + 10 = 14	5 + 10 = 15
2 + 11 = 13	3 + 11 = 14	4 + 11 = 15	5 + 11 = 16
2 + 12 = 14	3 + 12 = 15	4 + 12 = 16	5 + 12 = 17
6 + 1 = 7	7 + 1 = 8	8 + 1 = 9	9 + 1 = 10
6 + 2 = 8	7 + 2 = 9	8 + 2 = 10	9 + 2 = 11
6 + 3 = 9	7 + 3 = 10	8 + 3 = 11	9 + 3 = 12
6 + 4 = 10	7 + 4 = 11	8 + 4 = 12	9 + 4 = 13
6 + 5 = 11	7 + 5 = 12	8 + 5 = 13	9 + 5 = 14
6 + 6 = 12	7 + 6 = 13	8 + 6 = 14	9 + 6 = 15
6 + 7 = 13	7 + 7 = 14	8 + 7 = 15	9 + 7 = 16
6 + 8 = 14	7 + 8 = 15	8 + 8 = 16	9 + 8 = 17
6 + 9 = 15	7 + 9 = 16	8 + 9 = 17	9 + 9 = 18
6 + 10 = 16	7 + 10 = 17	8 + 10 = 18	9 + 10 = 19
6 + 11 = 17	7 + 11 = 18	8 + 11 = 19	9 + 11 = 20
6 + 12 = 18	7 + 12 = 19	8 + 12 = 20	9 + 12 = 21

$10+1=11$	$11+1=12$	$12+1=13$
$10+2=12$	$11+2=13$	$12+2=14$
$10+3=13$	$11+3=14$	$12+3=15$
$10+4=14$	$11+4=15$	$12+4=16$
$10+5=15$	$11+5=16$	$12+5=17$
$10+6=16$	$11+6=17$	$12+6=18$
$10+7=17$	$11+7=18$	$12+7=19$
$10+8=18$	$11+8=19$	$12+8=20$
$10+9=19$	$11+9=20$	$12+9=21$
$10+10=20$	$11+10=21$	$12+10=22$
$10+11=21$	$11+11=22$	$12+11=23$
$10+12=22$	$11+12=23$	$12+12=24$

§ **11.** The first consideration to which the scholar's attention should here be directed, is, that *like things* only can be added to or subtracted from each other. It would be absurd to attempt to add together books and chairs, to see how many books, or how many chairs, the whole would make; the number of each would evidently remain unaffected. If, however, we add books to books, we obtain a number greater than either of the original numbers, that is, just equal to them both. Neither can we add units to tens; for the amount would be neither units nor tens; but units must be added to units, tens to tens, and hundreds to hundreds, and so on. But ten units make one ten, ten tens make one hundred, and ten hundreds make one thousand, &c.; that is, simple numbers increase and decrease in a tenfold ratio. Hence, 10 in the column of units is equal to 1 in the column of tens; and 10 in the column of tens is equal to 1 in the column of hundreds. If, then, in adding up the column of units, the whole should amount to just 10, it is obvious that nothing is lost, if these ten units are converted into 1 ten, and the 1 ten added to the column of tens; for 10 units = 1 ten. The same is true of other denominations. Therefore, the following general rule will be found applicable to simple addition:—

§ **12.** **Rule.**—1st. *Write down the numbers, placing units under units, tens under tens, &c.*

2d. *Draw a line underneath, and commence at the right hand, and add together all the figures in the first column.*

3d. *If the sum be less than 10, set it down at the foot of that column; if it be 10, or more than 10, it will consist of two figures at least; set down the right-hand one as before, and add the left-hand one to the next column, it being in all cases so many tens, when compared with the figures added.*

4th. *Continue to perform the same operation with the remaining columns, observing only to write down the whole amount of the left-hand column.*

5th. *To detect any error that may have been committed, commence again, and add each column downwards; if the same numbers are obtained by each operation, the work is probably right.*

Now, to apply this rule, let us add together the four following numbers, viz., 1234, 2345, 6420, and 5796. The rule says, write these numbers with units under units, tens under tens, &c., thus:—

1 2 3 4
2 3 4 5
6 4 2 0
5 7 9 6
1 5 7 9 5

Now, to add these numbers, I commence at the right hand, and first add together the unit figures, viz., 6, 0, 5, 4; the sum I find to be 15 units, equal to 1 ten and 5 units; I write down the 5 units, but add the 1 ten to the next column; thus, 1 added to 9 is 10, and 2 are 12, and 4 are 16, and 3 are 19; as before, I write down the 9, and add the 1 to the next column; this being added, gives the amount 17; the 7 is written down, and the 1 again carried to the next and last column; and here the amount is 15, which being the last column, the whole number is written down. This operation gives the amount of the four numbers, 15795. The scholar will readily comprehend the nature of Simple Addition, viz., that it consists in uniting two or more numbers of the same denomination, so as to find their amount.

The scholar will carefully examine the following sums, which are added, to see if he obtains the same result.

2.
8 4 7 3 9 6
3 6 4 8 6 9
4 8 2 4 3 6
6 2 2 4 3 9
2 3 1 7 1 4 0

In this second example, in the unit column, there are 3 to carry, because there are 3 tens, or 30. For the same reason there are 2 to carry in all the remaining columns but one.

We are frequently required to *prove* an operation, to ascertain its correctness. In Addition, the proof is thus obtained: After adding the several numbers, as already directed, cut off the *upper* one by a horizontal line, and find the amount of those standing *below* it; then add this amount to the upper number, and, if the work be correct, the *amount last obtained* will equal the first amount, or sum total.

We will now apply our proof to Example 2.

8 4 7 3 9 6	= the upper number, cut off after the whole have been added, and the sum total obtained.
3 6 4 8 6 9	
4 8 2 4 3 6	
6 2 2 4 3 9	
2 3 1 7 1 4 0	= amount, or sum total.
1 4 6 9 7 4 4	= amount after the upper number was cut off.
2 3 1 7 1 4 0	= amount of the preceding amount and number cut off above. This last amount equals the first; we therefore conclude that the work is correctly performed.

The principle applied in proving addition is, that any whole or integral object is equal to the sum of all its parts. For example, if an apple be divided into any number of parts, — say four, — the *whole* apple equals the *four parts*. The first amount, or sum total, of the above sum, may therefore be regarded as the whole object; and the four given numbers, which, when united, produce it, as the several parts into which it is divided. The second amount, or 1469744, is the amount of only three of the parts, and is consequently the whole, wanting one of the given parts; hence, if the part wanting be added to this, the whole, or 2317140, is obtained.

The scholar should be required to prove his sums.

3.	4.
4 5 6 7 8 9 4 6 0	1 2 3 4 5 6 7 8 9
6 8 0 2 4 6 8 0 2	9 8 7 6 5 4 3 2 1
1 3 5 7 9 1 3 5 7	1 2 3 4 5 6 7 8 9
4 2 3 6 5 0 8 2 5	9 8 7 6 5 4 3 2 1
3 7 1 5 7 4 6 2 8	1 2 3 4 5 6 7 8 9
8 5 6 4 5 6 3 3 3	9 8 7 6 5 4 3 2 1
2 9 2 4 5 0 9 4 0 5	3 3 3 3 3 3 3 3 3 0

5.	6.	7.
2 4 6 8 0 2 4	9 7 9 6 3 6 5	1 7 9 8 6 7 0
1 3 5 7 9 1 2	5 6 3 6 9 7 9	6 1 2 4 3 5 6
4 2 0 8 6 4 2	8 2 4 6 0 2 4	7 2 1 2 3 4 5
2 1 3 5 7 9 1	5 9 6 3 8 1 6	8 4 6 3 7 3 8
3 3 5 5 7 7 9	4 6 7 4 3 6 4	5 7 3 9 1 6 8
8 8 6 6 4 4 8	5 7 9 6 3 7 8	9 1 5 6 4 2 3
2 2 3 9 2 5 9 6	4 0 1 1 3 9 2 6	

8.	9.	10.
9176435	43457096	70819634
5683214	61728349	64753278
2345678	28394563	86542365
0123456	56831234	23456789

11.	12.	13.
81828384	39724638	45768903
18283848	97236452	18924683
99999999	15707934	73579246
77777777	65972109	97576887

14. Add 479, 3845, 4294, 6765, and 291, together. *Ans.* 15674.

15. Add 30003, 5005, 606, 6, 22, 956, 7856, 45652, and 1, together. *Ans.* 90107.

16. What is the amount of 17, 467, 5678, 62195, 156379, 6804596? *Ans.* 7029332.

17. What is the amount of 9427642, 527219, 32875, 2649, 839, 66, and 9? *Ans.* 9991299.

18. What is the amount of 46735, 6456, 6734, 88, 444? *Ans.* 60457.

19. What is the amount of 8888, 7777, 6666, 5555, 4444, 3333, 2222, 1111? *Ans.* 39996.

20. A farmer sold his wheat for 320 dollars, his corn for 275 dollars, his oats for 78 dollars, his barley for 162 dollars, and one horse for 132 dollars. What was the amount of his sales? *Ans.* $967.

21. A merchant owned four vessels, which were worth, the first, $4800; the second, $5200; the third, $6000; and the fourth, $6800: he had also goods on board one of these vessels worth $2700, besides $3500 deposited in the bank. What was the amount of his property? *Ans.* $29000.

22. Three farmers have each 562 acres of land; how many have they all? *Ans.* 1686 acres.

23. A farmer fattened and killed an ox for market; the hind quarters weighed, the one 182 pounds, and the other 177; each of the fore quarters weighed 163 pounds, the hide 116 pounds, and the tallow 120 pounds. What was the weight of the ox? *Ans.* 921 pounds.

24. In a certain town there are five schools, containing the following numbers of scholars, viz., 72, 28, 65, 84, and 91. How many children are attending these five schools? *Ans.* 340.

25. A steamboat performed, in one week, four trips from Hartford to New York; on the first trip, she took, for passengers, $378, for freight, $175; on the second trip, for passengers, $402, for freight, $278; on the third, for passengers, $263, for freight, $147; on her fourth trip, for passengers and freight, $500. What did her bills amount to for the week? *Ans.* $2143.

26. A carpenter contracted for the building of five dwellings; for the first he was to receive $1800; for the second, $2100; for the third, $2221; for the fourth, $2850; and for the fifth, $3172. To what did his contracts amount? *Ans.* $12143.

27. In 1834, A traveled 1320 miles; in 1835, he traveled 1162 miles; in 1836, 2100 miles; in 1837, 1400 miles; in 1838, 1992 miles. How many miles did he travel from 1834 to 1838, inclusive? *Ans.* 7974.

28. Bought of my neighbor four loads of hay; the first weighed 1600 lb.; the second, 2100 lb.; the third, 1999 lb.; and the fourth, 1709 lb. What was the whole weight? *Ans.* 7408 pounds.

29. A wholesale dealer in grain has, in one bin, 242 bushels of wheat; in another he has 2356 bushels of rye; in a third, 1556 bushels of oats; in a fourth, 876 bushels of barley. How many bushels of grain has he of all kinds? *Ans.* 5030.

30. Suppose I am indebted to A, $2560; to B, $27; to C, $169; to D, $3470; and to E, $17; how much do I owe in all? *Ans.* $6243.

31. A man, having three sons and two daughters, gave to each of his sons $379, and to each of his daughters $199. How much money did he give them all? *Ans.* $1535.

32. A gentleman, being asked how old he was, said he was married when he was 29 years of age, — that he lived with his wife 8 years before the birth of their son, who was now 27 years of age. What was the father's age? *Ans.* 64 years.

33. A man bought a horse for $75, a chaise for $150, and a harness for $45; he then sold his horse for $150, his chaise for $125, and his harness for $30. What did he pay for the whole, and what did he receive for the whole? *Ans.* Paid, $270; received, $305.

34. Add together three hundred and seventy-five thousand and sixty-five, nine hundred thousand and three, one million six hundred thousand seven hundred and ninety-nine. *Ans.* 2875867.

35. Add, also, ninety-nine millions, seven hundred and fifty-five millions, six hundred and thirty-three. *Ans.* 854000633.

36. There are 5784 apples in one pile, 538 in another, 84

in a third, and seven hundred and seventy-nine in a fourth. How many are there in all? *Ans.* 7235.

37. A man collected the following sums of money, viz., \$327, \$832, \$29, \$567, and \$1396. How many dollars did he collect? *Ans.* \$3151.

38. If a man travel 24 miles on Monday, 28 on Tuesday, 30 on Wednesday, 34 on Thursday, 36 on Friday, and 40 on Saturday, how many miles will he travel during the week? *Ans.* 192 miles.

39. If a locomotive engine run 346 miles on Monday, 288 on Tuesday, 400 on Wednesday, 175 on Thursday, 89 on Friday, and 179 on Saturday, how many miles does it run during the week? *Ans.* 1477 miles.

40. Suppose there are four numbers, the second of which is 256, the third 388, the fourth 577, and the first as much as the second and fourth together. What is the sum of the four numbers? *Ans.* 2054.

41. A butcher killed a cow, one of whose fore quarters weighed 128 lb., and the other 112 lb.; one of the hind quarters weighed 136 lb., and the other 130 lb.; the tallow weighed 72 lb., and the hide 96 lb. What was the weight of the whole cow? *Ans.* 674 lb.

42. Three men commencing trade together, the first advanced \$2850, the second \$1722, and the third \$3428. How much did they all advance? *Ans.* \$8000.

QUESTIONS.—What is Addition? What is the number obtained called? What things only can be added to, or subtracted from, each other? In what does Simple Addition consist? Can we add units to tens? To what must units, &c., be added? For what number do we carry in Simple Addition? Why for 10? *Ans.* Because numbers increase in a tenfold ratio. 10 units equal how many 10's? What is the rule for Addition? How do you write down the numbers? Where do you commence to add? If the sum of the figures added be less than 10, what is to be done? What, if it be 10, or more than 10? What is observed respecting the last, or left-hand column? How may errors in Addition be detected?

SIMPLE SUBTRACTION.

§ **13.** This rule is directly the reverse of the preceding. While we are there taught to unite several numbers into one, we are here taught the operation by which one number is taken from another. A familiar acquaintance with the following table should be the first object of the scholar.

The sign — implies *subtraction.*

SUBTRACTION TABLE.

1—1= 0	2—2= 0	3—3= 0	4—4= 0
2—1= 1	3—2= 1	4—3= 1	5—4= 1
3—1= 2	4—2= 2	5—3= 2	6—4= 2
4—1= 3	5—2= 3	6—3= 3	7—4= 3
5—1= 4	6—2= 4	7—3= 4	8—4= 4
6—1= 5	7—2= 5	8—3= 5	9—4= 5
7—1= 6	8—2= 6	9—3= 6	10—4= 6
8—1= 7	9—2= 7	10—3= 7	11—4= 7
9—1= 8	10—2= 8	11—3= 8	12—4= 8
10—1= 9	11—2= 9	12—3= 9	13—4= 9
11—1=10	12—2=10	13—3=10	14—4=10
12—1=11	13—2=11	14—3=11	15—4=11
13—1=12	14—2=12	15—3=12	16—4=12
5—5= 0	6—6= 0	7—7= 0	8—8= 0
6—5= 1	7—6= 1	8—7= 1	9—8= 1
7—5= 2	8—6= 2	9—7= 2	10—8= 2
8—5= 3	9—6= 3	10—7= 3	11—8= 3
9—5= 4	10—6= 4	11—7= 4	12—8= 4
10—5= 5	11—6= 5	12—7= 5	13—8= 5
11—5= 6	12—6= 6	13—7= 6	14—8= 6
12—5= 7	13—6= 7	14—7= 7	15—8= 7
13—5= 8	14—6= 8	15—7= 8	16—8= 8
14—5= 9	15—6= 9	16—7= 9	17—8= 9
15—5=10	16—6=10	17—7=10	18—8=10
16—5=11	17—6=11	18—7=11	19—8=11
17—5=12	18—6=12	19—7=12	20—8=12
9—9= 0	10—10= 0	11—11= 0	12—12= 0
10—9= 1	11—10= 1	12—11= 1	13—12= 1
11—9= 2	12—10= 2	13—11= 2	14—12= 2
12—9= 3	13—10= 3	14—11= 3	15—12= 3
13—9= 4	14—10= 4	15—11= 4	16—12= 4
14—9= 5	15—10= 5	16—11= 5	17—12= 5
15—9= 6	16—10= 6	17—11= 6	18—12= 6
16—9= 7	17—10= 7	18—11= 7	19—12= 7
17—9= 8	18—10= 8	19—11= 8	20—12= 8
18—9= 9	19—10= 9	20—11= 9	21—12= 9
19—9=10	20—10=10	21—11=10	22—12=10
20—9=11	21—10=11	22—11=11	23—12=11
21—9=12	22—10=12	23—11=12	24—12=12

When the scholar has become familiar with the preceding table, he will begin to practice with his slate and pencil. It will already have been observed, that only two numbers are em-

ployed in a single operation of subtraction. The larger of these two numbers is called the *minuend,* and the smaller, the *subtrahend.* The object of the rule is to find the difference between the two; that is, to find how much will remain of the larger after the smaller is taken from it. The number obtained by the operation is called the *remainder.*

The scholar may be guided by the following rule:

§14. **Rule.**—1st. *Write the less of the two numbers under the greater, with units under units and tens under tens, &c., and draw a line beneath them.*

2d. *Commence with the right-hand figure of the lower line or subtrahend, and take it from the figure which stands directly above it, if practicable. Do the same with the remaining figures in the subtrahend, if practicable, and the operation will be completed.*

3d. *But, whenever this cannot be done, that is, when the lower figure is the larger,* 10 *should be added to the upper figure, and the lower one taken from the sum.*

4th. *Whenever* 10 *is added to an upper figure,* 1 *must be carried or added to the next lower figure; that is,* 1 *is to be carried whenever* 10 *is borrowed.*

5th. *To prove the work, add the remainder to the subtrahend, and, if the work be right, the amount will correspond with the minuend.*

The scholar will easily comprehend the nature of this rule, unless he should find difficulty in understanding why, when we borrow 10, we are required to carry only 1. He must however remember, that, by the addition of this 10 to the upper figure, he has increased its value 10 units, 10 tens, 10 hundreds, or 10 thousands, according to the place the figure occupies. If he add it to the units, the *value* of the addition is 1 ten; if to the tens, the *value* is 1 hundred, because 10 units make 1 ten, and 10 tens, 1 hundred, &c. Now, by the rule, if 10 be borrowed, 1 must be carried to the next *lower* figure; by which operation 1 more will be taken from the figure in the minuend; and this 1 more, which is thus removed, is just equal in value to the 10 that was added, for it is taken from a figure one degree farther to the left. But this subject will be more clearly comprehended, when illustrated by example. Take the following sum:

From	6 3 5 4 2 8
Take	3 8 2 5 1 6
Rem.	2 5 2 9 1 2

In this example, it is evident that, if 6 be taken from 8, 2 will remain; and if the 1 ten be taken from 2 tens, 1 ten will remain. But how is 5 in the place of hundreds to be taken from the 4 above it? Evidently by the third section of the rule; that is, 10 is added to the 4 in the minuend, by which addition it will become 14 hundred, from which if 5 hundred be taken, 9 hundred will remain, which is the third figure in the remainder. But by this operation the minuend has been increased 10 hundred; if, therefore, I add 1 to the 2 thousand, it will become 3 thousand, and consequently, when subtract-

ed from the figure 5 above it, will take 1 thousand more from the minuend, so that only 2 will remain. If, therefore, 10 hundred was in one instance added to the minuend, in the other, 1 thousand, its equal, has been taken from it. The same reasoning is applicable to the 8; 10 is added to the 3, which increases it to 13; the 8 is taken from the 13, and 5 remains. There is then 1 to carry to the 3, which, thus increased, is taken from the 6, and 2 remains. The whole remainder, therefore, is 252912.

§ **15.** Subtraction is proved by adding the *remainder* to the *subtrahend.* If the operation be accurately performed, their sum will equal the *minuend.*

THE ABOVE OPERATION PROVED.

$$\begin{array}{ll} \text{Min.} & 635428 \\ \text{Subtr.} & \underline{382516} \\ \text{Rem.} & 252912 \end{array} \Big\} \text{Added.}$$

Amount of the subtr. and rem. = 6 3 5 4 2 8 = Min.

The principle involved in the above proof, is the same as that illustrated in Addition. The operation of subtraction separates the minuend into two parts, viz, the subtrahend and remainder. Since, therefore, the whole equals the sum of all its parts, the *sum* of these two parts must equal the minuend.

ADDITIONAL ILLUSTRATION.

The true value of the above minuend, viz., 635428, may be expressed in the following manner, viz.: —

hund. of thou.	tens of thou.	thousands.	hundreds.	tens.	units.
5	13	4	14	2	8

We here have taken 1 from the 6 hundred thousand, which equals 10 in the next right-hand column; to which if the given 3 tens of thousands be added, we obtain 13 tens of thousands, as here written. 1 is also removed from the 5 thousands, (consequently leaving only 4 in the place of thousands,) and added to the 4 in the place of hundreds; by which operation 14 hundred is obtained.

The sum may therefore be thus written, viz.: —

$$\begin{array}{rrrrrrl} 5 & 13 & 4 & 14 & 2 & 8 & = \text{Min.} \\ 3 & 8 & 2 & 5 & 1 & 6 & = \text{Subtr.} \\ \hline 2 & 5 & 2 & 9 & 1 & 2 & = \text{Rem.} \end{array}$$

Here, then, each figure in the subtrahend has directly above it a number larger than itself, from which it may be subtracted, and the true remainder obtained.

Such is the nature of the operation when, in subtracting, the lower figure is the larger. In the ordinary mode of operating, the 10 borrowed is not, it is true, taken *directly from the next minuend figure,* as here represented; the same effect is, however, produced by carrying 1 to the next lower figure, as the rule directs.

The scholar should be required to prove his work.

2.	3.	4.
From 66683	From 894673	From 987654321
Take 25966	Take 768596	Take 123456789
40717	126077	864197532

5.	
Min.	1 0 0 0 0 0 0 0 0 0
Subtr.	9 9 9 9 9 9 9 9 9

6.	
Min.	1 0 7 5 6 0 8 7 5 6
Subtr.	6 9 8 4 5 3 8 7 4

7.	
Min.	9 6 3 4 6 5 7 9 4 2
Subtr.	8 3 2 6 5 4 7 2 8 6

8.	
Min.	3 0 0 0 0 0 0 0
Subtr.	2 9 9 9 9 9 9 9

9.	
Min.	1 0 0 0 0 0 0 0 0 0
Subtr.	9

10.	
Min.	9 0 8 0 7 0 6 0 5 0 4
Subtr.	8 0 9 0 6 0 7 0 4 0 0

11.	
Min.	9 6 5 8 4 3 1 2 5
Subtr.	4 2 8 6 4 2 0 8 6

12.	
Min.	5 6 7 8 9 3 5 7 9 1 3
Subtr.	4 1 3 5 7 2 4 3 6 4 8

13.
8 1 2 3 4 5 6 7 8 9 4 6
4 8 8 7 6 5 4 3 2 1 0 9

14.	
Min.	7 5 3 9 7 5 3 1 6 8
Subtr.	3 6 4 0 8 5 4 2 7 9

15.
1 2 3 4 5 6 7 8 9 0 1 2
9 2 5 6 7 8 4 5 0 2 3

16.
5 6 7 8 5 3 5 3 4 7 9 6 3
4 7 5 6 2 4 6 1 2 5 7 8 7

17. From 84762 take 49819. *Ans.* 34943.
18. From 66666 take 59999. *Ans.* 6667.
19. From 100000 take 99999. *Ans.* 1.
20. From 86429 take 19561. *Ans.* 66868.
21. From 1146942 take 915641. *Ans.* 231301.
22. From 877777 take 788888. *Ans.* 88889.

APPLICATION.

§ **16.** 23. A was born in 1679. How old was he in 1777? *Ans.* 98 years.

24. From 1600000 take 900000, and from the remainder take 699999, and how much will remain? *Ans.* 1.

25. A man has two flocks of sheep; in the one there are 693, and in the other 499. What is the difference in these flocks? *Ans.* 194.

26. A man has in his possession property to the amount of $15728, and he owes $7869. How much will remain in his hands when his debts are paid? *Ans.* $7859.

27. America was discovered in 1492. How long will it have been discovered in 1846? *Ans.* 354 years.

28. A man, being asked how old he was when his eldest son was born, said that his own age was 79 years, and his son's 42 years. What was his age at the birth of his son? *Ans.* 37 years.

29. The amount of A's debts was $2356, the amount of his property $5672. How much had he left after his debts were paid? *Ans.* $3316.

30. A merchant bought a quantity of cloth for $572, and sold it for $526. Did he gain or lose? and how much?
Ans. Lost $46.

31. To what number must I add 576 to make the amount 1726? *Ans.* 1150.

32. Bought cotton to the value of $572896, and sold the same for $600027. How much did I gain? *Ans.* $27131.

33. If the sum of two numbers be 2793, and one of those numbers 1892, what is the other? *Ans.* 901.

34. A merchant bought 742 yards of cloth, and sold all but 7 yards. How much did he sell? *Ans.* 735 yards.

35. A man paid $1182 for a house, and sold the same for $1069. How much did he lose? *Ans.* $113.

36. A farmer purchased a farm, for which, including the buildings, he paid $6782; the buildings were worth $2896. What was the value of the land? *Ans.* $3886.

37. A person owed a merchant $999, and paid him all but $179. How much did he pay him? *Ans.* $820.

Sums, requiring in their Solution the Application of both Addition and Subtraction.

§ **17.** 38. I have in my possession two notes against my neighbor B., one for $560, and the other for $70. Now, suppose that he pays me $320 in cash, and $260 in goods, how much will he then owe me? *Ans.* $50.

39. There are $1000 in four different purses: in the first there are $96; in the second, $310; in the third, $205. How many are there in the fourth? *Ans.* $389.

40. Four men agreed to contribute for a benevolent object, as follows: the first, $34; the second, $50; the third, $100; and the fourth, $150. Three of them having paid, the sum amounted to $234. Which subscription was unpaid?
Ans. The third.

41. A had 172 yards of cloth, of which he sold 57 to B, and 42 to C. How many yards were left? *Ans.* 73.

42. A man, having $3986, paid one debt of $1997, and another of $1089. How many dollars had he left? *Ans.* $900.

43. A man at his death left an estate of $9876. In his will he gave to each of his three sons $1800; to his daughter,

$1500; and the remaining part he left to his wife. What was the wife's portion? *Ans.* $2976.

44. A person, commencing business, found that he had $790 in money; in goods, $1260; he held also three notes of $150 each. After trading six years, he retired from business, and found that his property amounted to $6000. How much had he gained by trading? *Ans.* $3500.

45. Bought four chests of tea, weighing 72, 79, 83, and 87 pounds. From these, I sold to one man 46, to another 95, and to a third 113 pounds. How much tea had I remaining? *Ans.* 67 pounds.

46. A man owed for his farm, $2100; for house furniture, $156; for a horse, $96; for a yoke of oxen, $120; for a flock of sheep, $86. In one year he sold from his farm grain to the value of $462, butter and cheese to the value of $156, stock to the amount of $320. How much did he owe at the end of the year? *Ans.* $1620.

47. A man received $7000 as a legacy; he was previously worth $8560; he then commenced traveling, and in 7 years he spent $9873. How much was he then worth? *Ans.* $5687.

60−19=41+21=62 ans

48. A man, being asked how old he was, replied that he married at 21 years of age, and that in 19 years more he should have been married 60 years. How old was he? *Ans.* 62 years.

49. Bought 1000 pounds of coffee; from this quantity I sold at one time 376 pounds, and at another 512 pounds. How much had I remaining? *Ans.* 112 pounds.

50. A man bought two hogsheads of melasses, the one containing 65 gallons, and the other, 69 gallons; from the two he sold 112 gallons. How much had he left? *Ans.* 22 gallons.

QUESTIONS.—How does this rule compare with Addition? What are we taught in Subtraction? How many numbers are employed in this rule? What are they called? What is the object of the rule? What name is given to what is left after the operation? What is the rule for subtraction? What is to be done when the lower figure is the larger? When is 1 to be carried? Do you ever have more than 1 to carry in subtraction? *Ans.* We do not. How do you prove the work? How will you show that carrying 1, as directed, is equivalent to the 10 borrowed?

SIMPLE MULTIPLICATION.

§ 18. The rule to which the scholar's attention will now be directed, is one by which a number is produced from two given numbers, which shall contain either of these given numbers as many times as there are units in the other; or, it is the

repeating of one number as many times as there are units in the other. For example, let 8 and 4 be the numbers; that is, let 8 be repeated 4 times. The result of these four repetitions of 8 is obviously 32. But 32 contains 8 four times, or 4 eight times; or, in other words, 32 contains either number as many times as there are units in the other. The scholar must first learn the following table. The sign × implies *multiplication.*

MULTIPLICATION TABLE.

1 × 1 = 1	2 × 1 = 2	3 × 1 = 3	4 × 1 = 4
1 × 2 = 2	2 × 2 = 4	3 × 2 = 6	4 × 2 = 8
1 × 3 = 3	2 × 3 = 6	3 × 3 = 9	4 × 3 = 12
1 × 4 = 4	2 × 4 = 8	3 × 4 = 12	4 × 4 = 16
1 × 5 = 5	2 × 5 = 10	3 × 5 = 15	4 × 5 = 20
1 × 6 = 6	2 × 6 = 12	3 × 6 = 18	4 × 6 = 24
1 × 7 = 7	2 × 7 = 14	3 × 7 = 21	4 × 7 = 28
1 × 8 = 8	2 × 8 = 16	3 × 8 = 24	4 × 8 = 32
1 × 9 = 9	2 × 9 = 18	3 × 9 = 27	4 × 9 = 36
1 × 10 = 10	2 × 10 = 20	3 × 10 = 30	4 × 10 = 40
1 × 11 = 11	2 × 11 = 22	3 × 11 = 33	4 × 11 = 44
1 × 12 = 12	2 × 12 = 24	3 × 12 = 36	4 × 12 = 48
5 × 1 = 5	6 × 1 = 6	7 × 1 = 7	8 × 1 = 8
5 × 2 = 10	6 × 2 = 12	7 × 2 = 14	8 × 2 = 16
5 × 3 = 15	6 × 3 = 18	7 × 3 = 21	8 × 3 = 24
5 × 4 = 20	6 × 4 = 24	7 × 4 = 28	8 × 4 = 32
5 × 5 = 25	6 × 5 = 30	7 × 5 = 35	8 × 5 = 40
5 × 6 = 30	6 × 6 = 36	7 × 6 = 42	8 × 6 = 48
5 × 7 = 35	6 × 7 = 42	7 × 7 = 49	8 × 7 = 56
5 × 8 = 40	6 × 8 = 48	7 × 8 = 56	8 × 8 = 64
5 × 9 = 45	6 × 9 = 54	7 × 9 = 63	8 × 9 = 72
5 × 10 = 50	6 × 10 = 60	7 × 10 = 70	8 × 10 = 80
5 × 11 = 55	6 × 11 = 66	7 × 11 = 77	8 × 11 = 88
5 × 12 = 60	6 × 12 = 72	7 × 12 = 84	8 × 12 = 96
9 × 1 = 9	10 × 1 = 10	11 × 1 = 11	12 × 1 = 12
9 × 2 = 18	10 × 2 = 20	11 × 2 = 22	12 × 2 = 24
9 × 3 = 27	10 × 3 = 30	11 × 3 = 33	12 × 3 = 36
9 × 4 = 36	10 × 4 = 40	11 × 4 = 44	12 × 4 = 48
9 × 5 = 45	10 × 5 = 50	11 × 5 = 55	12 × 5 = 60
9 × 6 = 54	10 × 6 = 60	11 × 6 = 66	12 × 6 = 72
9 × 7 = 63	10 × 7 = 70	11 × 7 = 77	12 × 7 = 84
9 × 8 = 72	10 × 8 = 80	11 × 8 = 88	12 × 8 = 96
9 × 9 = 81	10 × 9 = 90	11 × 9 = 99	12 × 9 = 108
9 × 10 = 90	10 × 10 = 100	11 × 10 = 110	12 × 10 = 120
9 × 11 = 99	10 × 11 = 110	11 × 11 = 121	12 × 11 = 132
9 × 12 = 108	10 × 12 = 120	11 × 12 = 132	12 × 12 = 144

§ **19.** It has already been said, that two numbers are required for the operation of multiplication, viz., the *multiplicand*, or number to be multiplied or repeated and the *multiplier*, or number showing how many times the multiplicand is to be taken or repeated. The multiplier and multiplicand, when spoken of together, are called *factors*. The ***number*** obtained by the operation is called the *product*. A short illustration will show this rule to be an abbreviation of addition. Suppose it be required to multiply 5 by 4. If the scholar turn to his table, he will find the product of these two numbers is 20. The same result is obtained if four 5's be added together; thus, $5+5+5+5=20$.

It will, then, be perceived that if one of the two numbers which are to be multiplied together, be written down as many times as there are units in the other, and these several numbers be then added, we obtain the same result as when these two numbers are multiplied together. Multiplication is therefore a short method of performing addition.

In multiplying, it is usual to make the smaller of the two given numbers the multiplier. This, however, is not necessary, but merely convenient; for the product of the two factors, 4 and 5, equals $5+5+5+5=20$; that is, 5 taken four times equals 20; or, $4+4+4+4+4=20$; that is, 4 taken five times equals 20.

CASE I.

§ 20. WHEN THE MULTIPLIER DOES NOT EXCEED 12.

Rule.—*Commence at the right hand, and multiply each figure in the multiplicand by the multiplier, carrying and setting down as in the preceding rules.*

Ex. 1. Multiply 452 by 3.

OPERATION.

```
       4 5 2
           3
       -----
Prod. 1 3 5 6

       4 5 2
       4 5 2
       4 5 2
       -----
      1 3 5 6
```

I first say, 3 times 2 are 6, which, being less than 10, I set down; next, 3 times 5 are 15, which is 5 units and 1 ten; the units I set down, and carry the ten; thus, 3 times 4 are 12, and 1 to carry is 13. I thus find the whole product to be 1356. This number must consequently contain the multiplicand 3 times, and may be obtained by adding together three 452's, as in the margin.

The same result is therefore obtained by multiplication as by addition, but more expeditiously.

The operation is proved by ***dividing the product by the multiplier***, which, if the work be correct, will give the *multiplicand*. The scholar is not, however, supposed yet to understand division, and will not be required to prove his work till more advanced.

2. Multiply 6432 by 4.

OPERATION.

```
  6 4 3 2
        4
  -------
2 5 7 2 8 = Prod.
```

3. Multiply 123456 by 6. *Prod.* 740736.
4. Multiply 234567 by 8. *Prod.* 1876536.

5. Multiply 345678 by 10. *Prod.* 3456780.
6. Multiply 456789 by 12 *Prod.* 5481468.
7. Multiply 729468 by 5. *Prod.* 3647340.
8. Multiply 295105538 by 7 *Prod.* 2065738766.
9. Multiply 4285637 by 9. *Prod.* 38570733.
10. Multiply 462838 by 11. *Prod.* 5091218.
11. Multiply 99887766 by 9. *Prod.* 898989894.
12. Multiply 765987879 by 7 *Prod.* 5361915153.
13. Multiply 9864579 by 8. *Prod.* 78916632.
14. Multiply 7799886655 by 12. *Prod.* 93598639860.

APPLICATION.

15. Bought 72 yards of cloth for 3 dollars per yard. What did it cost? *Ans.* $216.

16. Sold 137 sheep, at 5 dollars per head. How much did I receive? *Ans.* $685.

17. Employed 196 men one week, for $8 per week. How much did I pay them all? *Ans.* $1568.

18. How many miles will a man travel in 297 days, if he travel 12 miles a day? *Ans.* 3564.

19. If I take 11 steps in one minute, how many steps shall I take in 2 hours 56 minutes, or 176 minutes? *Ans.* 1936 steps.

20. If a horse trot 7 miles in one hour, how far will he trot in 76 hours? *Ans.* 532 miles.

CASE II.

§ 21. When the Multiplier is a Composite Number.

Note. — A composite number is one which can be produced by the multiplication of two or more small numbers; thus, 6 and 3 are the component parts of 18, because $6 \times 3 = 18$.

Rule. — *Multiply first by one of the component parts of the multiplier, and this product by the other component part; the last product will be the one sought.*

Ex. 1. Multiply 4568 by 24. The component parts of 24 are 6 and 4; therefore,

$$\begin{array}{r} 4568 \\ 6 \\ \hline 27408 = 6 \text{ times the multiplicand.} \\ 4 \\ \hline 109632 = 24 \text{ times the multiplicand.} \end{array}$$

Ex. 2. Multiply 459684 by 36. $36 = 4 \times 9$; therefore,

$$
\begin{array}{r}
459684 \\
9 \\
\hline
4137156 = 9 \text{ times the multiplicand.} \\
4 \\
\hline
16548624 = 36 \text{ times the multiplicand.}
\end{array}
$$

It is immaterial in what order the component parts are taken.

3. Multiply 5634236 by 18. *Prod.* 101417148.
4. Multiply 4327648 by 27. *Prod.* 116846496.
5. Multiply 7295678 by 36. *Prod.* 262644408.
6. Multiply 4639546 by 48. *Prod.* 222698208.
7. Multiply 3695475 by 42. *Prod.* 155209950.
8. Multiply 54639578 by 60. *Prod.* 3278374680.
9. Multiply 578016937 by 96. *Prod.* 55489625952.
10. Multiply 7987561З by 63. *Prod.* 5032165509.

APPLICATION.

11. Bought 75 tons of hay, at $15 per ton. What did the whole cost? *Ans.* $1125.
12. How many hours are there in 76 days? *Ans.* 1824.
13. How many minutes are there in 49 hours? *Ans.* 2940.
14. How many days are there in 21 years? *Ans.* 7665.
15. What cost 172 acres of land, at $36 per acre? *Ans.* $6192.
16. What will 876 pounds of coffee cost, at 28 cents per pound? *Ans.* 24528 cents.

CASE III.

§ 22. When the Multiplier exceeds 12, and is not a Composite Number.

It is highly important that the scholar should obtain accurate views of the value of the product arising from the multiplication of any two numbers. There will be no difficulty in this respect, when it is required to multiply unit figures only, or when a unit figure only is given as a multiplier; for then the product of each figure in the multiplicand will be of the same denomination as the figure itself. But when the two factors consist each of several figures, so that tens are to be multiplied by tens, and hundreds by hundreds, the scholar will not so readily comprehend the nature of the operation. He must, however, remember, that, when his multiplying figure is *tens*, it will raise the value of the product of each figure in the multiplicand *one degree*; when it is hundreds, it will raise the value of each *two degrees*; and when thousands, *three degrees*, &c. Let the scholar carefully notice what is here stated. If tens in the multiplier be multiplied into *units* in the multiplicand, the product is *tens*; if

into *tens*, the product is *hundreds*. If hundreds in the multiplier be multiplied into units in the multiplicand, the product is *hundreds;* if into tens, the product is *thousands;* and if into hundreds, the product is *tens of thousands, &c.* This explanation will enable the scholar to understand the following rule, relative to writing down the several products. He will readily perceive it to be nothing more or less than writing units under units, and tens under tens, &c.

Rule.—*Multiply each figure in the multiplicand by each figure in the multiplier separately, commencing with the right-hand figure of each, and set down the first figure of each product directly under the multiplying figure. After each figure in the multiplier has been taken, add together the several products; the amount will be the required product.*

Ex. 1. Multiply 342635 by 125.

OPERATION.

```
  342635
     125
--------
 1713175 = Product of 5 units.
 685270  = Product of 2 tens removed 1 place to the left.
342635   = Product of 1 hundred, 2 places to the left.
--------
42829375 = Product of 125.
```

Ex. 2. Multiply 167498 by 231.

OPERATION.

```
  167498
     231
--------
  167498 = Product of 1 unit.
 502494  = Prod. of 3 tens removed 1 place to the left.
334996   = Prod. of 2 hundreds, 2 places to the left.
--------
38692038 = Prod. of 231.
```

3. Multiply 36598674 by 432. *Prod.* 15810627168.
4. Multiply 46354897816 by 56843. *Prod.* 2634951456554888.
5. Multiply 378199886432 by 42395. *Prod.* 16033784185284640.
6. Multiply 85698436946 by 46743. *Prod.* 4005802038166878.
7. Multiply 6739542 by 346. *Prod.* 2331881532.
8. Multiply 72926495 by 4567. *Prod.* 333055302665.
9. Multiply 89764267 by 999. *Prod.* 89674502733.
10. Multiply 46371674 by 49684. *Prod.* 2303930251016.
11. Multiply 8429638 by 7294. *Prod.* 61485779572.
12. Multiply 7364951 by 888. *Prod.* 6540076488.

13. There are 69 pieces of cloth, containing each 112 yards. How many yards are there in all? *Ans.* 7728.

14. Suppose a man travel by steam 21900 miles in a year, how far will he travel in 67 years? *Ans.* 1467300 miles.

15. In a volume of 675 pages, each page containing 156 lines, and each line 136 letters, how many letters?
Ans. 14320800.

16. How many hills are there in a field of corn, containing 149 rows, with 96 hills in a row? *Ans.* 14304.

17. On the preceding supposition, how many ears of corn are there in the field, allowing the average to be 9 to a hill? and how many kernels of corn, allowing 300 to an ear?
Ans. 128736 ears of corn, and 38620800 kernels.

CASE IV.

§ **23.** WHEN THERE ARE CIPHERS ON THE RIGHT HAND OF THE MULTIPLIER, OR MULTIPLICAND, OR BOTH.

Rule. — *Omit the ciphers, and multiply by the significant figures only, and annex to the right hand of the product as many ciphers as were omitted.*

Ex. 1. Multiply 2100 by 70.

PERFORMED.

```
  2100
    70
------
147000
```

I multiply the 21 by the 7 only, and then annex three ciphers to 147, the product.

2. Multiply 47600 by 150. *Prod.* 7140000.
3. Multiply 9560 by 1200. *Prod.* 11472000.
4. Multiply 462000 by 190. *Prod.* 87780000.
5. Multiply 760 by 1000. *Prod.* 760000.

CASE V.

§ **24.** WHEN THE MULTIPLIER IS ANY NUMBER BETWEEN 11 AND 19, INCLUSIVE.

Rule. — *Multiply by the right-hand figure only, and place the product under the multiplicand one place to the right; then add it to the multiplicand. The sum will be the true product.*

Ex. 1. Multiply 468 by 15.

PERFORMED.

468×5
$\underline{\ \ 2340} =$ Product of 468 multiplied by 5.
$7020 =$ Product of 468 multiplied by 15.

The reason of this rule is plain. Were we to multiply in the ordinary mode, the only difference would be, that the number 2340 would stand above 468, instead of below it.

Ex. 2. Multiply 37464 by 17.

PERFORMED.

$$\begin{array}{r} 37464\times 7 \\ \underline{262248} \\ 636888 \end{array} = \text{Product.}$$

3. Multiply 65328 by 13.	*Prod.* 849264.
4. Multiply 23456789 by 14.	*Prod.* 328395046.
5. Multiply 65432 by 15.	*Prod.* 981480.
6. Multiply 123456 by 16.	*Prod.* 1975296.
7. Multiply 437426 by 17.	*Prod.* 7436242.
8. Multiply 653842 by 18.	*Prod.* 11769156.
9. Multiply 603040 by 19.	*Prod.* 11457760.
10. Multiply 999999 by 11.	*Prod.* 10999989.

CASE VI.

§ 25. WHEN THE MULTIPLIER IS EITHER 21, 31, 41, 51, 61, 71, 81, OR 91.

Rule.—*Multiply by the left-hand figure only, and place the product under the multiplicand one place to the left.*

Ex. 1. Multiply 634982 by 21.

PERFORMED.

$$\begin{array}{rl} 634982\times 2 & \\ \underline{1269964\,\,} & = \text{Product of 2 tens 1 place to the left.} \\ 13334622 & = \text{Product of 21.} \end{array}$$

2. Multiply 9382716 by 31.	*Prod.* 290864196.
3. Multiply 1234567 by 41.	*Prod.* 50617247.
4. Multiply 4364369 by 51.	*Prod.* 222582819.
5. Multiply 6937845 by 61.	*Prod.* 423208545.
6. Multiply 364812 by 71.	*Prod.* 25901652.
7. Multiply 482436 by 81.	*Prod.* 39077316.
8. Multiply 2468 by 91.	*Prod.* 224588.

NOTE.—When, in either of the two preceding cases, ciphers intervene between the figures of the multiplier, the same mode of operation may be adopted, if care be taken to give each figure its true place.

Ex. 1. Multiply 6456 by 105.

PERFORMED.

6 4 5 6×5
3 2 2 8 0 = Product of the 5 placed two degrees to the right.
6 7 7 8 8 0 = Product of 105.

2. Multiply 37562 by 601.

PERFORMED.

3 7 5 6 2×6
2 2 5 3 7 2 = Product of 6 hundred placed two degrees to the left.
2 2 5 7 4 7 6 2 = Product of 601.

3. Multiply 695378 by 5001. *Prod.* 3477585378.
4. Multiply 2579678 by 1007. *Prod.* 2597735746.

APPLICATION.

1. Bought 52 horses, at $75 each. What did they cost? *Ans.* $3900.

2. What cost 84 tons of hay at $15 per ton? *Ans.* $1260.

3. If a man can travel 43 miles in one day, how far can he travel in 60 days? *Ans.* 2580 miles.

4. There are 144 square inches in one square foot. How many square inches are there in 67 square feet? *Ans.* 9648.

5. If there be 18 panes of glass in one window, how many are there in a house which has 56 such windows? *Ans.* 1008.

6. Bought 342 bales of linen, each containing 56 pieces of 25 yards each. How many yards did I buy? *Ans.* 478800.

7. There is an orchard consisting of 126 rows of trees, and in each row there are 109 trees. How many apples are there in the orchard, allowing an average of 1007 on a tree? *Ans.* 13830138.

8. A certain state contains 50 counties; each county, 35 towns; each town, 300 houses, and each house, 8 persons. What is the population of the state? *Ans.* 4200000.

QUESTIONS. — What is the nature of Multiplication? How many numbers are employed in the operation? What are they called, and what is peculiar to each? What is the number obtained called? Of what rule is Multiplication an abbreviation? Illustrate. What are the multiplicand and multiplier called when spoken of together? What is Case I.? What is the rule? Case II.? The rule? What is a composite number? What are the component parts of a number? What is Case III.? What is the value of each figure in the product when you multiply by a unit figure only? Units multiplied by units give what? Units by tens? Units by hundreds? When the multiplying figure is tens, what effect will it have on the value of the product of each figure in the multiplicand? and what will be the effect if the multiplying figure be hundreds? Give further illustration of the value of the product figures. What is the rule for Case III.? What is Case IV.? The rule? Case V.? The rule? Case VI.? The rule?

SIMPLE DIVISION.

§ **26.** We now come to the reverse of the preceding rule. There we had two factors given to find their product. Here we have given the product, or what corresponds to it, and one of the factors, and are required to obtain the other factor.

Multiplication, as was shown, could be performed by repeated *additions*; Division may be performed by repeated *subtractions.* Suppose it be required to ascertain how many times 4 is contained in 12. It may be done by taking 4 from 12, till nothing remains, or till a number less than 4 remains. Thus,

$$\begin{array}{l} 12 \\ \underline{\ \ 4} \\ \ \ 8 = 12 - 4. \\ \underline{\ \ 4} \\ \ \ 4 = 12 - \overline{4 + 4}. \\ \underline{\ \ 4} \\ \ \ 0 = 12 - \overline{4 + 4 + 4}. \end{array}$$

The operation shows three 4's may be taken from 12. 4 is therefore contained in 12 three times. This is, however, a slow mode of operation. A more expeditious one must be sought; and, preparatory for it, the scholar is required to learn the following table.

The sign $\div$ implies *division.*

DIVISION TABLE.

$2 \div 2 = 1$	$3 \div 3 = 1$	$4 \div 4 = 1$	$5 \div 5 = 1$
$4 \div 2 = 2$	$6 \div 3 = 2$	$8 \div 4 = 2$	$10 \div 5 = 2$
$6 \div 2 = 3$	$9 \div 3 = 3$	$12 \div 4 = 3$	$15 \div 5 = 3$
$8 \div 2 = 4$	$12 \div 3 = 4$	$16 \div 4 = 4$	$20 \div 5 = 4$
$10 \div 2 = 5$	$15 \div 3 = 5$	$20 \div 4 = 5$	$25 \div 5 = 5$
$12 \div 2 = 6$	$18 \div 3 = 6$	$24 \div 4 = 6$	$30 \div 5 = 6$
$14 \div 2 = 7$	$21 \div 3 = 7$	$28 \div 4 = 7$	$35 \div 5 = 7$
$16 \div 2 = 8$	$24 \div 3 = 8$	$32 \div 4 = 8$	$40 \div 5 = 8$
$18 \div 2 = 9$	$27 \div 3 = 9$	$36 \div 4 = 9$	$45 \div 5 = 9$
$20 \div 2 = 10$	$30 \div 3 = 10$	$40 \div 4 = 10$	$50 \div 5 = 10$
$22 \div 2 = 11$	$33 \div 3 = 11$	$44 \div 4 = 11$	$55 \div 5 = 11$
$24 \div 2 = 12$	$36 \div 3 = 12$	$48 \div 4 = 12$	$60 \div 5 = 12$
$6 \div 6 = 1$	$7 \div 7 = 1$	$8 \div 8 = 1$	$9 \div 9 = 1$
$12 \div 6 = 2$	$14 \div 7 = 2$	$16 \div 8 = 2$	$18 \div 9 = 2$
$18 \div 6 = 3$	$21 \div 7 = 3$	$24 \div 8 = 3$	$27 \div 9 = 3$
$24 \div 6 = 4$	$28 \div 7 = 4$	$32 \div 8 = 4$	$36 \div 9 = 4$
$30 \div 6 = 5$	$35 \div 7 = 5$	$40 \div 8 = 5$	$45 \div 9 = 5$
$36 \div 6 = 6$	$42 \div 7 = 6$	$48 \div 8 = 6$	$54 \div 9 = 6$
$42 \div 6 = 7$	$49 \div 7 = 7$	$56 \div 8 = 7$	$63 \div 9 = 7$
$48 \div 6 = 8$	$56 \div 7 = 8$	$64 \div 8 = 8$	$72 \div 9 = 8$
$54 \div 6 = 9$	$63 \div 7 = 9$	$72 \div 8 = 9$	$81 \div 9 = 9$
$60 \div 6 = 10$	$70 \div 7 = 10$	$80 \div 8 = 10$	$90 \div 9 = 10$
$66 \div 6 = 11$	$77 \div 7 = 11$	$88 \div 8 = 11$	$99 \div 9 = 11$
$72 \div 6 = 12$	$84 \div 7 = 12$	$96 \div 8 = 12$	$108 \div 9 = 12$

$10 \div 10 = 1$	$11 \div 11 = 1$	$12 \div 12 = 1$
$20 \div 10 = 2$	$22 \div 11 = 2$	$24 \div 12 = 2$
$30 \div 10 = 3$	$33 \div 11 = 3$	$36 \div 12 = 3$
$40 \div 10 = 4$	$44 \div 11 = 4$	$48 \div 12 = 4$
$50 \div 10 = 5$	$55 \div 11 = 5$	$60 \div 12 = 5$
$60 \div 10 = 6$	$66 \div 11 = 6$	$72 \div 12 = 6$
$70 \div 10 = 7$	$77 \div 11 = 7$	$84 \div 12 = 7$
$80 \div 10 = 8$	$88 \div 11 = 8$	$96 \div 12 = 8$
$90 \div 10 = 9$	$99 \div 11 = 9$	$108 \div 12 = 9$
$100 \div 10 = 10$	$110 \div 11 = 10$	$120 \div 12 = 10$
$110 \div 10 = 11$	$121 \div 11 = 11$	$132 \div 12 = 11$
$120 \div 10 = 12$	$132 \div 11 = 12$	$144 \div 12 = 12$

§ **27.** The scholar must now apply what he has learned from the preceding table to division on a more extensive scale. He will have noticed that, for each operation, *two numbers* are given; viz., a number to be divided, which is called the *dividend;* and a number by which to divide, called the *divisor.* The number obtained is called the *quotient;* a word which signifies *how many,* because this number always determines how many times the divisor is contained in the dividend. After the operation is performed, there is frequently a number left. This is called the ***remainder,*** and is always less than the divisor. When the division is performed, if there be no remainder, the quotient multiplied by the divisor will always produce the dividend; and if there be a remainder, the dividend will be produced by multiplying as before, and adding the remainder to the product. Hence division is proved by multiplication. The scholar will readily perceive that these two rules are the reverse of each other.

The operations in division will be illustrated under two general heads; viz., Short Division, and Long Division.

I. SHORT DIVISION.

§ **28.** When the divisor does not exceed 12, the process is abbreviated by keeping the computation in the mind, and writing down only the quotient figures.

Rule.—1st. *Write down the dividend, and place the divisor on the left, with a curve line drawn between them.*

2d. *Take as many figures on the left of the dividend, as will contain the divisor once or more, and write the figure expressing the number of times, directly under those divided.*

3d. *If, in dividing, there be a remainder, imagine the next figure in the dividend to be placed on the right hand of it. This will form a new number, which may be divided as before. Continue the same process till all the figures of the dividend have been disposed of, and the number obtained will be the quotient required.*

4th. *If, in taking any figure of the dividend, the number produced be not sufficient to contain the divisor once, a cipher must be placed in the quotient, and an additional figure of the dividend taken.*

Ex. 1. Divide 496 by 4.

PERFORMED.

$$4\overline{)4\,9\,6}$$
$$1\,2\,4 = \text{Quotient.}$$

I first say, 4 is in 4 once, and place the 1 as directed by the rule. I next say, 4 is in 9 twice, and 1 remains; I write down the 2 as directed, and on the right hand of the 1 I place the 6, and thus obtain 16. Lastly, I say, 4 is in 16 four times. This operation gives 124 as the number of times which 496 contains 4. Now, the scholar will readily perceive, that this is reversing a process of multiplication. If he multiply 124 by 4, he will obtain 16 units, 8 tens, and 4 hundreds; and these are precisely the numbers divided. But the 16 units are equal to 1 ten and 6 units; the whole is, therefore, equal to 4 hundreds, 9 tens, and 6 units; or to 496.

Division is therefore proved by multiplication.

Ex. 2. Divide 1512 by 7.

PERFORMED.

$$7\overline{)1\,5\,1\,2}$$
$$2\,1\,6 = \text{Quotient.}$$

The numbers here, as they are severally divided, are 15, 11, and 42. If 216 be multiplied by 7, it will produce the same numbers.

3. Divide 5463 by 3. *Quo.* 1821.
4. Divide 1256 by 2. *Quo.* 628.
5. Divide 63548 by 4. *Quo.* 15887.
6. Divide 256788 by 8. *Quo.* 32098, and 4 remains.
7. Divide 65342167 by 4 and by 5, and add the quotients. *Ans.* 29403974, and 5 remains.
8. Divide 735649 by 5 and by 7, and add the quotients. *Ans.* 252221, and 9 remains.
9. Divide 456789 by 6 and by 8, and add the quotients. *Ans.* 133229, and 8 remains.
10. Divide 68890 by 7 and by 9, and add the quotients. *Ans.* 17495, and 7 remains.
11. Divide 78901 by 8 and by 10, and add the quotients. *Ans.* 17752, and 6 remains.
12. Divide 89012 by 9 and by 11, and add the quotients. *Ans.* 17982, and 2 remains.
13. Divide 90123456 by 10 and 12, and add the quotients. *Ans.* 16522633, and 6 remains.
14. Nine persons drew a prize of $198. What was each one's share? *Ans.* $22.
15. Paid $750 for 30 cows. What was the average price? *Ans.* $25.
16. A person dying leaves an estate of $4500 to 9 children. What will be each one's share? *Ans.* $500.

17. A man traveled 224 miles in 7 days. What was his daily progress? *Ans.* 32 miles.

18. If 12 ounces make a pound of silver, how many pounds are there in 2040 ounces? *Ans.* 170.

19. How many times may 12 be subtracted from 1416? *Ans.* 118.

20. Four persons boarded at a public house till the bill of their board was $184. What was the average bill? *Ans.* $46.

§ **29.** When the divisor is more than 12, and is a composite number, the same mode of operation can be adopted.

Rule 2d.—*Divide first by one of the component parts, and the quotient arising from this division, by the other.*

The only difficulty which will here present itself, will be to ascertain the true remainder. The scholar needs only to remember, that, if a remainder occur after the *first* division only, that is the true remainder. If after the *second* division only, the true remainder is obtained by *multiplying this remainder* by the *first* divisor. If there be a remainder after each division, the true remainder is found by multiplying the *last* remainder by the *first* divisor, and adding the *first* remainder.

Ex. 1. Divide 864 by 18. The component parts of 18 are 6 and 3; therefore,

6) 8 6 4

3) 1 4 4 = Quotient of 864 divided by 6.

4 8 = Quotient of 144 divided by 3. This is the true quotient of $864 \div 18$.

Ex. 2. Divide 793 by 18. The component parts are 6 and 3.

6) 7 9 3

3) 1 3 2 and 1 remains.

4 4 and nothing remains. Therefore, (see remark under Rule 2d,) 1 is the whole remainder.

Ex. 3. Divide 792 by 16. Component parts of 16 are 4 and 4.

4) 7 9 2

4) 1 9 8 and nothing remains.

4 9 and 2 remains; therefore, (see remark as above,) $4 \times 2 = 8$, the true remainder.

Ex. 4. Divide 162641 by 72. The component parts are 9 and 8; therefore,

9) 1 6 2 6 4 1

8) 1 8 0 7 1 and 2 remainder.

2 2 5 8 and 7 remainder. Therefore, $7 \times 9 + 2 =$ the true remainder, viz., 65.

5. Divide 2592 by 63.	*Quo.* 41, and 9 remains.
6. Divide 7776 by 108.	*Quo.* 72.
7. Divide 6750 by 15.	*Quo.* 450.
8. Divide 437639 by 42.	*Quo.* 10419, and 41 remains.
9. Divide 738246 by 27.	*Quo.* 27342, and 12 remains.
10. Divide 60400 by 25.	*Quo.* 2416.
11. Divide 45678 by 16.	*Quo.* 2854, and 14 remains.
12. Divide 4688 by 48.	*Quo.* 97, and 32 remains.
13. Divide 347628 by 84.	*Quo.* 4138, and 36 remains.

II. LONG DIVISION.

§ 30. When the Divisor exceeds 12, and is not a Composite Number.

Rule.—1st. *Write down the dividend, and, drawing a curve line on the right and left of it, place the divisor on the left.*

2d *Find how many times the divisor is contained in the fewest figures that will contain it, taken from the left of the dividend; and place the figure expressing the number of times on the right of the dividend, as the first quotient figure.*

3d. *Multiply the divisor by this quotient figure, and place the product under the figures divided.*

4th. *Subtract, and to the remainder bring down the next figure of the dividend.*

5th. *Divide again, and place the result as the second figure in the quotient.*

6th. *Continue the process of multiplying, subtracting, bringing down, &c., till all the figures have been divided.*

7th. *If, after all the figures have been divided, there be still a remainder, place it as the numerator, and the divisor as the denominator of a fraction, on the right hand of the quotient.*

Note 1.—Whenever a figure has been placed on the right of the remainder, and the number produced will not contain the divisor, a cipher must be placed in the quotient.

Note 2.—If the remainder, after subtracting, be greater than the divisor, the quotient figure is too small. If the number obtained by multiplying the divisor by the quotient figure, be greater than the number divided, that quotient figure is too large.

Ex. 1. Divide 15341 by 29.

PERFORMED.

```
29)15341(529
   145
   ---
    84
    58
   ---
    261
    261
    ---
    000
```

EXPLANATION. — I, in the first place, notice that at least three figures are required to contain the divisor, and that in this number, 153, the divisor is contained 5 times. 5 is then my first quotient figure. I then proceed to multiply and subtract as the rule directs, and obtain a remainder of 8. To this, I bring down the next figure of the dividend, and obtain 85. I proceed to divide, multiply, and subtract, as before, and obtain a quotient figure 2, and a remainder of 26. Again I bring down, divide, multiply, &c., and obtain the quotient figure, 9, and no remainder.

NOTE 3. — It will be observed, that each figure in the quotient is obtained by *four successive operations*, and that these operations uniformly succeed each other in the same order. In the first place, we are required to *divide;* in the second, to *multiply* the divisor by the quotient figure; in the third, to *subtract* the product of this multiplication from the figures divided; and, lastly, to the remainder thus obtained, to *bring down* another figure of the dividend.

Ex. 2. Divide 6283459 by 29.

PERFORMED.

```
29)6283459(216.671 = Quotient.
   58
   --
    48
    29
    ---
    193
    174
    ---
     194
     174
     ---
      205
      203
      ---
        29
        29
        --
        00
```

3. Divide 7461300 by 95. *Quo.* 78540.
4. Divide 1893312 by 2076. *Quo.* 912.
5. Divide 455678 by 78. *Quo.* 5842, and 2 rem.
6. Divide 6495685 by 85. *Quo.* 76419, and 70 rem.
7. Divide 9424789962 by 978. *Quo.* 9636799, rem. 540.

8. Divide 2686211248 by 296. *Quo.* 9075038.
9. Divide 84764367 by 431. *Quo.* 196669, rem. 28.
10. Divide 4683579 by 234. *Quo.* 20015, rem. 69.
11. Divide 3579864 by 135. *Quo.* 26517, rem. 69.
12. Divide 1748 by 18. *Quo.* 97, and 2 rem.

NOTE 4.—When the divisor is 10, 100, 1000, 10000, &c., point off as many figures from the right of the dividend, as there are ciphers in the divisor; the figures on the *left* of the point will be the quotient, and those on the *right* the remainder.

Ex. 1. Divide 19375468 by 10000. *Quo.* 1937, rem. 5468.
2. Divide 99885566 by 100000. *Quo.* 998, rem. 85566.
3. Divide 47429 by 10. *Quo.* 4742, rem. 9.
4. Divide 463581 by 100. *Quo.* 4635, rem. 81.
5. Divide 618293 by 1000. *Quo.* 618, rem. 293.

NOTE 5.—When the divisor consists of a number of figures with ciphers annexed, the ciphers may be cut off from the divisor, and an equal number of figures from the right of the dividend, and the remainder of the dividend divided by the significant figures of the divisor. After the division, the figures cut off from the dividend are to be placed at the right of the remainder.

Ex. 1. Divide 36418235700 by 98700.
Quo. 368979, rem. 8400.
2. Divide 11579112 by 890000. *Quo.* 13, rem. 9112.
3. Divide 8317642500 by 814600. *Quo.* 10210, rem. 576500.

APPLICATION.

1. If 246 men incur an expense of $175152, what is each man's share? *Ans.* $712.

2. A gentleman left an estate of $65468 to 6 sons. What was each one's share? *Ans.* 10911\frac{2}{6}$.

3. Twelve men own a bridge, from which they annually receive $2352 in toll. What is each man's share? *Ans.* $196.

4. Suppose 7776 peach-trees to be planted in 108 rows; how many trees are there in a row? *Ans.* 72.

5. If light comes from the sun to the earth in 8 minutes, how far does it travel in one minute, the distance being 95000000 miles? *Ans.* 11875000 miles.

6. If a man travel 9125 miles in a year, what is his average daily progress? *Ans.* 25 miles.

7. If a horse run 288 miles in 36 hours, how far does he run in one hour? *Ans.* 8 miles.

8. In 437850 yards of cloth, how many rolls of 75 yards each? *Ans.* 5838.

APPLICATION OF THE PRECEDING RULES.

1. A farmer sold 3 yoke of oxen, at $96 each; 12 cows, at $24 each; 83 sheep, at $3 a head; 239 bushels of wheat, at $2 per bushel; and distributed the avails equally among his 7 sons. What was each one's share? *Ans.* 186\frac{1}{7}$.

2. A man, to whom was intrusted the settlement of an estate, found that the whole value of the estate was, $95688. There were five claims against the estate, viz., one of $8672; another, of $3421; a third, of $10637; a fourth, of $356; and a fifth, of $1673. After the payment of these several claims, the balance was to be divided equally among 9 heirs. What was the share of each? *Ans.* $7881.

3. A farmer had 16 calves, worth $5 per head; 45 sheep, worth $3 per head; and 75 bushels of grain, worth $2 per bushel. He gave the whole for a horse, worth $136; a carriage, worth $195; and a harness, worth $63. Did he gain, or lose? and how much? *Ans.* Gained $29.

4. A merchant received by boat, 9696 bushels of salt, and hired it carted 16 miles, at a dollar a load of 24 bushels. How much did the cartage cost? *Ans.* $404.

5. When the dividend is 290864196, and the quotient 9382716, what is the divisor? *Ans.* 31.

6. A man purchased a farm, for which he paid $18000. He sold 60 acres, for $50 an acre; and then the remainder stood him at $75 per acre. How much land did he purchase? *Ans.* 260 acres.

QUESTIONS.—How does Division differ from Multiplication? How may multiplication be performed? How may division be performed? How many numbers are given in division? What are they, and what are they called? What is the number obtained by the operation called? What does it signify? What is the remainder? How does it always compare with the divisor? How is division proved? What is Short Division? What is the rule for it? What is the rule when the divisor is more than 12, and a composite number? How is the true remainder obtained, when we divide by the component parts of the divisor? What is Long Division? What is the rule? How do you divide by 10, 100, 1000, &c.? When the divisor consists of a figure greater than 1, with ciphers annexed, how do you divide?

FEDERAL MONEY.

§ **31.** Federal Money is the currency of the United States. Its denominations are mills, cents, dimes, dollars, and eagles, increasing, like simple numbers, in a tenfold ratio, as represented in the following table:—

TABLE OF FEDERAL MONEY.

10 mills (*marked* m.)	make	1 cent,	*marked*	c. *or* ct.
10 cents	"	1 dime,	"	d.
10 dimes	"	1 dollar,	"	\$ *or* dol.
10 dollars	"	1 eagle,	"	E.

The coins of the United States are of three kinds, viz., gold, silver, and copper coins.

The gold coins are,

The eagle,	= \$10.
Half-eagle,	= \$5.
Quarter-eagle,	= \$2½.

The silver coins are,

The dollar,	= 100 cts.
Half-dollar,	= 50 cts.
Quarter-dollar,	= 25 cts.
Dime,	= 10 cts.
Half-dime,	= 5 cts.

The 6¼ cent and 12½ cent pieces, &c., are not American coins.

The copper coins are, the cent and the half-cent. Half-cents are seldom used. The gold and silver coins are not composed of pure metal, but are alloys, that is, compounds of these metals with the baser metals. The purity of a metal is expressed by the word *carat;* which word is used to express a twenty-fourth part of a given quantity. If, for example, a quantity of gold be said to be 18 carats fine, the meaning is, that 18 equal portions of the whole are gold, and 6 equal portions are of a less valuable metal.

By recurring to the preceding table, it will be seen that the denominations of Federal Money may be added, subtracted, multiplied, and divided by the same rules as the simple numbers, as *they increase* in the *same ratio*. The scholar should, however, remember that the *dollar* is the *unit money;* and that *dimes*, *cents*, and *mills*, are *decimals;* or tenths, hundredths, and thousandths of the dollar or unit dénomination.

It is, therefore, always important to know which of a number of figures is the dollar or unit figure. This is shown by the decimal point (.) or period, which always stands between the dollars and dimes; thus, 63.78, read, sixty-three dollars and seventy-eight cents.

The first figure on the left of the point, is dollars, and the second, eagles. They are, however, called dollars indiscriminately; as, in the above number, the 6 is eagles, and the 3, dollars; but usually read, 63 dollars. Likewise, the 78 is usually read, 78 cents, instead of 7 dimes and 8 cents. Both methods express the same value.

To reduce the higher denominations to the lower, it is

necessary to bear in mind, 1st. That cents are converted into mills by annexing one cipher; thus, 8 cents = 80 mills. 2d. That dollars may be changed into cents by annexing two ciphers; thus, 3 dollars = 300 cents; and into mills by annexing three ciphers; thus, $3 = 3000 mills. 3d. The reverse operation will convert mills into cents, and cents into dollars.

Ex. 1. How many mills in 47 cents? *Ans.* 470.
2. How many mills in 69 cents? *Ans.* 690.
3. How many mills in 156 cents? *Ans.* 1560.
4. In 78 dollars, how many cents? *Ans.* 7800.
How many mills? *Ans.* 78000.
5. In $637, how many cents? *Ans.* 63700.
How many mills? *Ans.* 637000.
6. In 450 mills, how many cents? *Ans.* 45.
7. In 470 mills, how many cents? *Ans.* 47.
8. In 6700 mills, how many cents? *Ans.* 670.
9. In 6700 mills, how many dollars? *Ans.* 67.

10. Change $6 into cents. 11. Change 42 cents into mills. 12. Change $95 into mills. 13. Change 460 mills into cents. 14. 28000 mills into dollars. 15. 439 mills into cents. 16. 9876 mills into dollars.

17. Reduce the following sums to mills, and find their amount, viz., $21, $31.64, $45.01, $0.90, and $109.

Ans. 207550 mills.

TABLE OF FEDERAL MONEY.

Hund. of eagles, or thou. of dolls.	Tens of eagles, or hund. of dolls.	Eagles, or tens of dolls.	Dollars.		Dimes.	Cents.	Mills.	Tenths of mills.	Hundredths of mills.	Thousandths of mills.	Tenths of thou. of mills.
.	.	.	.		.	.	.	.	.	.	5
.	.	.	.		.	.	.	.	.	6	5
.	.	.	.		.	.	.	.	7	6	5
.	.	.	.		.	.	.	4	5	6	7
.	.	.	.		.	.	6	3	0	8	2
.	.	.	.		.	4	0	5	8	2	6
.	.	.	.		5	6	0	3	2	4	5
.	.	.	5	.	4	3	2	0	7	1	6
.	.	8	6	.	2	7	4	6	0	3	4
.	9	0	3	.	4	5	6	1	2	3	6
2	6	4	5	.	7	9	2	6	1	4	3

ADDITION OF FEDERAL MONEY.

§ 32. **Rule.** — *Set the numbers one under another, so that dollars shall stand under dollars, dimes under dimes, cents under cents, and mills under mills. Then add up the several columns as in Simple Addition, and place the decimal point, in the amount, directly under those in the numbers added.*

If the above rule be followed in writing down the several numbers, the separating points will stand directly under each other.

Ex. 1. What is the sum of 136 dollars 21 cents, 75 dollars 13 cents, 7 dollars 78 cents, 66 dollars 19 cents, and 196 dollars 72 cents? *Ans.* $482.03.

PERFORMED.

```
 136.21
  75.13
   7.78
  66.19
 196.72
 ------
$482.03 Amount.
```

2. Add together $432.73, $297.38, $172.66, and $333.62. *Amount*, $1236.39.

3. What is the sum of $1.55, $0.72, $340.89, $0.01, $1460.99? *Ans.* $1804.16.

4. What is the sum of $72.01, $1, $0.01, $0.10, $40.70, $560.88? *Ans.* $674.70.

5. What is the sum of $101.01, $20.15, $42.89, $79.81, $41.41, $51.51, $38.41? *Ans.* $375.19.

6. What is the sum of $16.64, $20.84, $462.573, $29.922, $56.32, $84.48? *Ans.* $670.775.

7. A farmer bought a cow, for $23.75; a yoke of oxen, for $96.78; a horse, for $69.83; and a pig, for $1.625. What did they all cost him? *Ans.* $191.985.

8. A grocer paid for a box of cheese, $37.21; for candles, $8.32; for a cask of wine, $7.38; for a box of raisins, $3.625. What was the whole cost? *Ans.* $56.535.

9. Bought 5 gallons of melasses, for $1.80; 3 pounds of tea, for $1.125; 3 yards of broadcloth, for $9.82; 11 yards of cotton cloth, for $1.83; 6 yards of linen, for $3.82. What was the amount of my bill? *Ans.* $18.395.

10. Paid for building my house, $2169.72; for my barn, $972.87; for my out-houses, $1272.69; for digging my well, $56.38. What is the amount of my expenses? *Ans.* $4471.66.

11. My expenses for a journey were as follows: for stage

fare, $66.89; for meals, $18.50; for lodging, $6.58; for carrying baggage, $2.57; for brushing boots, $1.36; for washing, $2.66. What is the amount? *Ans.* $98.56.

12. My hat cost me $4.75; my coat, $18.96; my pantaloons, $9.74; my vest, $5.82; and my boots, $6.54. What did the whole cost? *Ans.* $45.81.

SUBTRACTION OF FEDERAL MONEY.

§ **33.** **Rule.** — *Place the numbers as in Simple Subtraction, and, after subtracting, place the separating point as directed in Addition of Federal Money.*

Ex. 1. From $463.42 take $399.99. *Rem.* $63.43.
2. From $179.364 take $88.449. *Rem.* $90.915.
3. From $125 take $9.09. *Rem.* $115.91.
4. From $642.99 take $99.99. *Rem.* $543.
5. From $127.01 take $41.10. *Rem.* $85.91.
6. From $200 take $0.90. *Rem.* $199.10.
7. From $2 take $0.05. *Rem.* $1.95.
8. From $99 take $0.99. *Rem.* $98.01.

9. I have $473, and my brother twice as much, wanting 90 cents. How much has my brother? *Ans.* $945.10.

10. Having in my possession $1600, I paid $516.95 for a span of fine horses; $156.55 for a carriage; $221.19 for a gold watch, and spent $450.71 in traveling. How much had I left? *Ans.* $254.60.

11. If a man has an income of $4500, and spends $2461.85, how much does he lay up? *Ans.* $2038.15.

12. If one man lays up $339.86 in a year, and another $299.99, how much does one lay up more than the other? *Ans.* $39.87.

13. Bought a farm for $3946, and sold a part for $1426.82. What did the remaining part cost me? *Ans.* $2519.18.

MULTIPLICATION OF FEDERAL MONEY.

§ **34.** **Rule.** — *Multiply as in simple numbers. The product will always be in the lowest denomination given, which may be reduced to dollars, cents, and mills, by the preceding rules.*

Ex. 1. What will 36 yards of cloth cost, at $4.50 per yard? *Ans.* $162.00.

PERFORMED.

```
   4.50
     36
  -----
  27.00
 135.0
 ------
 162.00
```

The given price being 450 cents, the whole price is 16200 cents, which equals $162.00.

2. What will 29 pairs of shoes cost, at $1.50 per pair? *Ans.* $43.50.

3. What will 35 pounds of beef cost, at 8 cents per pound? *Ans.* $2.80.

4. Bought 280 reams of paper, at $2.35 per ream. What was the whole cost? *Ans.* $658.00.

5. What cost 600 pounds of lard, at 15 cents per pound? *Ans.* $90.00.

6. Bought 15 tons of hay, at $16.42 per ton. What was the whole cost? *Ans.* $246.30.

7. What cost 349 acres of land, at $15.49 per acre? *Ans.* $5406.01.

8. Bought 18 yoke of oxen, for $72.50 per yoke. What was the whole cost? *Ans.* $1305.00.

9. Bought 32 pounds of butter, at 20 cents per pound; 45 pounds of loaf sugar, at 18 cents per pound; 56 pounds of coffee, at 15 cents per pound; 26 pounds of tea, at $1.75 per pound; 21 cwt. of raisins, at $6.75 per cwt.; 42 barrels of flour, at $7.50 per barrel; and 29 pairs of boots, at $4.50 per pair. What did the whole cost me? *Ans.* $655.65.

DIVISION OF FEDERAL MONEY.

§ **25.** Division of Federal Money is employed whenever the cost of a number of articles, as yards, pounds, &c., is given, and the price of one required.

Rule.—*Divide the cost by the number of articles, and point off as many figures from the quotient, for cents and mills, as there are in the given sum. If dollars only be given, and ciphers are added to complete the division, these ciphers must be regarded as cents and mills.*

Ex. 1. If 9 pounds of butter cost $2.25, what is the value of one pound? *Ans.* $0.25.

2. Sold 69 bushels of wheat, for $625. What was the price per bushel? *Ans.* $9.057.

3. Paid $75.00 for 500 lb. of butter. What was the price per pound? *Ans.* $0.15.

4. Paid $311.70 for 15 tons of hay. What was the price per ton? *Ans.* $20.78.

5. Paid $658 for 280 reams of paper. What did I pay per ream? *Ans.* $2.35.

6. Paid $505.44 for 144 lb. of tea. What was the price of one pound? *Ans.* $3.51.

7. Paid $375 for 50 firkins of butter. What was the price per firkin? *Ans.* $7.50.

8. Paid $43.79 for 29 pairs of boots. What was the price per pair? *Ans.* $1.51.

9. Paid $2.80 for 35 lb. of beef. What was the price per pound? *Ans.* .08.

APPLICATION OF THE PRECEDING RULES.

1. A man dying, left an estate of $12000, which was divided equally among 7 children, after his wife had received her third. What was the portion of the wife, and what did each child receive? *Ans.* $4000, wife's portion;
$1142.857, each child's portion.

2. A man, settling with his grocer, finds himself charged with 15 lb. of tea, at 75 cts. per pound; 42 pounds of brown sugar, at 11 cts. a pound; 3 barrels of flour, at $7.50 per barrel; 8 gallons of lamp oil, at $1.25 per gallon; and 45 pounds of ham, at 15 cts. per pound. He is also credited $11.62. How much does he owe his grocer? *Ans.* $43.50.

3. A man sells a horse, for $84; 5 cows, for $25 each; and agrees to take 80 sheep in pay. How much do the sheep cost him per head? *Ans.* $2.61¼.

4. A person agrees to furnish a grocer with 56 bushels of rye, at 50 cts. a bushel, and to take his pay in coffee, at 15 cts. a pound. How many pounds of coffee will he receive?
Ans. 186⅔.

5. If I pay $21311 for 844 acres of land, what do I pay per acre? *Ans.* $25.25.

6. Bought 350 yards of cloth, at $3.50 per yard. Of this I sold 200, at $5.00 per yard. How much money did I pay out? how much did I receive? how many yards had I left? and how much did it cost me per yard? *Ans.* I paid out $1225; I received $1000; 150 yards were left; and what remained cost me $1.50 per yard.

7. A man sold his farm for $8456, and his stock for $1560; at the same time, he had on deposit in the bank, $872.97. He then purchased a house in the city, for which he paid $3845;

he also purchased a horse and carriage for $392.53, and paid up his old debts to the amount of $1787. How much money had he left? *Ans.* $4864.44.

8. Suppose the man in the preceding sum to have laid out the balance of his money in wagons, at $72 each; and that he took these wagons to the south, and sold them for $97 each. How many wagons would he have purchased, and how much gained by the transaction, allowing his expenses to have been $297.83? *Ans.* He would have purchased 67 wagons, and had $40.44 left; and he would have gained $1377.17.

9. On the first of January, a spendthrift was in possession of $3860.90: after 30 days, he had only $1680 remaining. How much had he spent during the whole time? and how much daily? *Ans.* Whole sum, $2180.90; daily, $72.696.

10. A man bought 20 pounds of coffee, for 15 cents a pound; and 18 pounds of sugar, at 12 cts. a pound. He paid 96 cents in cash, and the balance in butter at 20 cents a pound. How much butter did it take? *Ans.* 21 pounds.

11. How much tea, worth 56 cents a pound, must be given for 16 sacks of salt, worth $2.87 per sack? *Ans.* 82 pounds.

MERCHANTS' BILLS, &c.

1.

S. Dean

Bought of J. James,

12 lb. of Tea, at 75 cts. per pound, ...	$ 9.00
28 lb. of Sugar, at 11 cts. "	3.08
17 lb. of Lard, at 15 cts. "	2.55
9 lb. of Butter, at 21 cts. "	1.89

Required the amount of the bill. *Ans.* $16.52

2.

Samuel Bancroft

Bought of Stephen Sawtell,

9 yd. of Broadcloth, at $4.50 per yard, ...	$ 40.50
6 yd. of Cotton Cloth, at 13 cts. "	0.78
8 sticks of Twist, at 7 cts. a stick, ...	0.56
9 Buttons, at 56 cts. per dozen, ...	0.42
½ yd. of Velvet, at $3 per yd., ...	1.50

The amount is required. *Ans.* $43.76

3.

S. Downer

Bought of H. Mann,

16 pairs of Shoes, at $1.50 per pair, ... $ 24 00
21 yd. of Sheeting, at 17 cts. per yard, ... 3.57

Amt. $27.57

Received payment,

H. Mann.

4.

G. Gordon, Esq.

Bought of S. Sanborn,

1 pair of Boots, at $4.50, ... $ 4.50
3 pairs of Kid Slippers, at 75 cts. per pair, ... 2.25
8 pairs of Brogans, at $1.25 per pair, ... 10.00
6 pairs of Slippers, at 58 cts. per pair, ... 3.48

Amt. $20.23

Received payment,

S. Sanborn.

Norwich, *July* 30, 1841.

5.

J. Hazzletine, Esq.

Bought of Samuel Richards,

1 qr. of a hundred weight of Sugar, at $11 per cwt., $ 2.75
9 Eggs, at 24 cts. per dozen, ... 0.18
7 bushels of Oats, at 25 cts. per bushel, ... 1.75
56 lb. of Rice, at $4 per 100 pounds, ... 2.24
5 gal. of Melasses, at 42 cts. per gal., ... 2.10

Amt. $9.02

Received payment,

S. Richards.

Hartford, *Aug.* 3, 1841.

6.

Thos. Thornton, Esq.

Bought of Jas. Eaton,

36 lb. of Coffee, at 14 cts. per pound, ... $ [illegible]
75 lb. of Sugar, at $9 per 100 pounds, ... 6.75
42 lb. of Butter, at 22 cts. per pound, ... 9.24

Amt. $21.03

Received payment,

Jas. Eaton.

Hartford, *Sept.* 8, 1841.

7.

Geo. Jameson, Esq.

Bought of Robt. Read,

130 bags of Coffee, each 96 lb., at 12 cts. per pound, $1497,60
72 chests of Tea, at $54 per chest, ... 3888,00
32 bbl. Flour, at $7.50 per barrel, ... 240 00
18 casks of Oil, each 84 gal., at $1.12½ per gallon, . 1701,00
27 boxes of Raisins, at $2.25 per box, ... 60,75

Amt. $7387.35

Received payment,

Robt. Read.

New Haven, *Oct.* 8, 1841.

8.

James Grosvenor, Esq.

Bought of Jacob Andrews,

5 bales of Sheeting, each 56 yds., at 15 cts. per yard, $42,00
72 yd. of Broadcloth, at $3.75 per yard, ... 270,00
67 yd. of Nankeen, at 27 cts. per yard, ... 18 09
133 yd. of Ribbon, at 8 cts. per yard, ... 10,64
8 pairs of Socks, at 36 cts. per pair, ... 2,88
12 Cravats, at $1.16 each, ... 13,92
15 yd. of Irish Linen, at 64 cts. per yard, ... 9,60

Amt. $367.13

Received payment,

Jacob Andrews.

Boston, *Sept.* 6, 1841.

9.

Joseph Dunham, Esq.

Bought of Sam'l Osborn,

18 lb. of Tea, at 56 cts. per pound, ... $
27 lb. of Coffee, at 14½ cts. " . . . .
48 lb. of Sugar, at 11½ cts. " . . . .
21 lb. of Raisins, at 15 cts. " . . . .
31 lb. of Butter, at 21 cts. " . . . .
19 lb. of Lard, at 13 cts. " . . . .

Amt. $31.645

Received payment,

Sam'l Osborn.

New London, *Aug.* 8, 1841.

10

Joseph Bond, Esq.

Bought of Benj. Sawtell,

3 boxes of Sugar, each 108 lb., at 12 cts. per pound, . .	$ 38, 88
3 hhd. of Melasses, each 63 gal., at 36 cts. per gallon,	68, 04
5 chests of Tea, each 96 lb., at 84 cts. per pound, . .	403, 20
6 casks of Rice, each 280 lb., at 5 cts. per pound, . .	84 00
Amt.	$594.12

Received payment,

Benj. Sawtell.

NORWICH, *Dec.* 21, 1841.

11.

N. Newman, Esq.

Bought of Levi Barrows,

360 yd. of Cotton Cloth, at 14½ cts. per yard, ...	$ 52, 20
320 yd. of Broadcloth, at $2.87 per yard, ...	918, 40
970 yd. of Ribbon, at 3 cts. per yard, ...	29, 10
16 rolls of Tape, each 21 yd., at 3 cts. per yard, . .	10, 08
Amt.	$1009.78

Received payment,

Levi Barrows.

NEW HAVEN, *Oct.* 18, 1841.

QUESTIONS.—What is Federal Money? What are its denominations? How do they increase in value? Repeat the table. What are the coins of the United States? What are the gold coins? What are their values? What are the silver coins? What are their values? Are the four-pence-half-penny and the nine-penny pieces American coin? What are the copper coins? Are the gold and silver coins pure metal? What is their composition? By what word is the purity of a metal expressed? What does that word express? If a quantity of gold be said to be 18 carats fine, what is meant? Suppose it be said to be 22 carats fine, what is meant? How may the denominations of Federal Money be added? Which is the unit money? What are dimes, cents, and mills? How is the dollar or unit figure always shown? How does the period always stand? What is the first figure on the left of the point? The second? How are they usually read? How are the figures on the right of the point read? How are cents converted into mills? How are dollars converted into cents? What is the rule for the addition of Federal Money? How is the decimal point placed? What is the rule for the subtraction of Federal Money? How is the point placed? What is the rule for the multiplication of Federal Money? How many figures are cut off from the right of the product? When is division of Federal Money employed? What is the rule? How many figures do you point off in the quotient? If dollars only are given, and ciphers are added, how are they to be regarded?

CANCELING.

§ **36.** Let 12 be multiplied by 4, and the product divided by 4.

$12 \times 4 = 48$, and $48 \div 4 = 12$; that is, the multiplier and divisor being the same, the value of 12, the number operated upon, is not affected; the multiplier and divisor may therefore be rejected. Let 12 be multiplied by 4, and the product divided by 2. The multiplier is twice as large as the divisor; therefore $12 \times 2 = 24$, the number required, for $12 \times 4 = 48$, and $48 \div 2 = 24$.

Again, let 16 be multiplied by 9, and the product divided by 3. The multiplier, 9, is three times as large as the divisor, 3; therefore $16 \times 3 = 48$, *Ans.*; for $16 \times 9 = 144$, and $144 \div 3 = 48$.

Hence we see, that, in all arithmetical operations, it is important not only to know *how* a question may be solved, but how it may be done *most expeditiously.*

The object of this rule is to acquaint the scholar with a principle by which *peculiar expedition* is attained in the solution of such sums as involve in their operation both *multiplication* and *division.* This principle is founded on the following facts:

First. The value of any quotient depends on the *ratio,* or *relative size of the divisor and dividend;* that is, if the dividend be three times as large as the divisor, the value of the quotient is 3; and if it be four times as large, the value is 4, &c.

Second. If two or more numbers are to be multiplied together, and their product divided by any other number, the true result is obtained by first dividing one of these numbers by the dividing number, and then multiplying the quotient by the remaining number or numbers. Thus, if it be required to multiply 8 by 4, and to divide the product by 2, first divide 8 by 2, and multiply the quotient by 4; thus, $8 \div 2 = 4$, and $4 \times 4 = 16$.

The advantage of this process will be more obvious if we take large numbers. Suppose we wish to multiply 288 by 16, and to divide the product by 144. The usual process would be thus:

```
 288            144)4608(32, Quotient.
  16                432
----                ---
1728                288
288                 288
----                ---
4608 = Product.     000
```

But by first dividing, the operation is much abbreviated; thus:

```
144)288(2
    288
    ---
    000, and 16 × 2 = 32.
```

By the usual method, 37 figures are required; by the other, only 18. There is still another advantage. The scholar can see at a glance that 144 is contained in 288 twice; and that twice 16 is 32; so that an operation which is long and protracted, is often reduced nearly or quite to a mental operation.

Third. (When any large number is to be divided by the product of two or more smaller numbers, it may be divided by each number separately.) This needs no explanation; it is the same as dividing by the component parts of any number, instead of the number itself.

Fourth. (When the operation is of such a nature as to require the *product* of *several numbers* to be *divided* by the *product* of several other numbers, these numbers may be divided before multiplication, and their quotients used instead of the numbers themselves.) For illustration, suppose the product of 36 and 42 is to be divided by the product of 6 and 7. The usual mode of operation would be as follows, viz.:

```
   42           7 × 6 = 42, divisor; therefore,
   36                   42)1512(36, the required
  ---                      126         Quotient.
  252                      ---
 126                       252
 ----                      252
 1512, Dividend.           ---
```

But, by the preceding fourth principle, $36 \div 6 = 6$, and $42 \div 7 = 6$, and $6 \times 6 = 36$, *Ans.* In this example, the divisors are, as it were, expunged or lost, since they divide without remainder. This will not, however, always be the case. It will frequently be necessary to assume some number, which will divide some two given numbers, without remainder, agreeably to a rule soon to be given.

But, for further illustration, suppose it be required to multiply the numbers 36, 12, 27, and 72 together, and to divide the product successively by 24, 18, and 48. Now, it is evidently desirable to arrange these numbers so that they may be conveniently compared with each other. We will adopt the following mode:—We will place the numbers whose product is to form a dividend, above a horizontal line; and those whose product is to form a divisor, below the same line, thus:

$$\frac{36.\ 12.\ 27.\ 72 = \text{Dividend.}}{24.\ 18.\ 48 = \text{Divisor.}}$$

Now, by the fourth and last principle laid down, I can divide 36 in the dividend, and 18 in the divisor, by 18, without a remainder, and obtain 2 in the dividend and 1 in the divisor; thus, $\frac{2.\ 12.\ 27.\ 72}{24.\ 1.\ 48}$. I can also divide 72 in the dividend, and 24 in the divisor, by 24, and obtain 3 in the dividend and 1 in the divisor, (it will be remembered that the divisors stand below the line,) thus, $\frac{2.\ 12.\ 27.\ 3}{1.\ 1.\ 48}$. Again, I can divide 12 in the dividend, and 48 in the divisor, by 12, and obtain 1 in the dividend and 4 in the divisor; thus, $\frac{2.\ 1.\ 27.\ 3}{1.\ 1.\ 4}$. Again, I can divide 2 in the dividend, and 4 in the divisor, by 2, and obtain 1 in the dividend and 2 in the divisor; thus, $\frac{1.\ 1.\ 27.\ 3}{1.\ 1.\ 2}$. It is now evident that the division can be carried no farther without remainder. The next step, therefore, is to divide the product of the numbers remaining above the line by the product of those below it. The product of those above the line is $27 \times 3 = 81$; and of those below the line, 2; therefore, $81 \div 2 = 40\frac{1}{2}$, the number required. The same result would have been obtained by multiplying the numbers above the line, and dividing their product by the product of those below it, previous to canceling. In the above example, as the numbers have been canceled, they have been omitted, and a new statement made. This is by no means necessary. One statement is sufficient.

It will be noticed that, in every instance, division is effected without a remainder. Such must always be the case.

The following rule will be found a competent guide for the scholar in all operations of canceling.

§ **37. Rule.**—1st. *Place all the numbers whose product is to form a dividend, above a horizontal line; and all those whose product is to form a divisor, below the same line.*

2d. *Notice whether there are ciphers both above and below the line; if so, erase an equal number from each side.*

3d. *Notice whether the same number stands both above and below the line; if so, erase them both.*

4th. *Notice again if any number on either side of the line will divide any number on the opposite side, without remainder; if so, divide and erase the two numbers, retaining the quotient figure only on the side of the larger number.*

5th. *See if any two numbers, one on each side, can be divided by any assumed number, without a remainder; if so, divide them by that number, and retain only their quotients. Proceed in the same manner as far as practicable; then,*

6th. *Multiply all the numbers remaining above the line for a dividend, and those remaining below for a divisor.*

7th. *Divide, and the quotient will be the number required.*

Note 1.—The canceled numbers in the following sums are erased; thus, $\cancel{3}$.

Ex. 2. Multiply 9 by 4, and divide the product by 3. Statement: $\frac{9.\ 4.}{3.}$. By art. 4th of the preceding rule, we reduce this statement to $\frac{\overset{3.}{\cancel{9}}.\ 4.}{\cancel{3}.}$. No number is now left below the line, and 3 and 4 only are left above it. Then, $3 \times 4 = 12$, *Ans.* (See following note.)

Note 2.—Whenever the numbers below the line are all canceled, and one number only remains above it, that number is the answer required. If more numbers than one remain above the line, and none below, as in the above example, their product is the answer. If the numbers above the line be all canceled, and one only remain below, the answer will be a fraction, (an expression for something less than a unit,) and will be represented by placing the figure 1 above a short horizontal line, and that number below it; or, if more numbers than one remain below the line, their product will form the lower term of the fraction. If one or more numbers remain both above and below the line, the answer will always be a fraction, whenever the product of those above the line is less than the product of those below.

3. Multiply 21 by 6, and divide their product by 7. Statement: $\frac{21.\ 6.}{7.}$. Canceled: $\frac{\overset{3.}{\cancel{21}}.\ 6.}{\cancel{7}.}$, and $6 \times 3 = 18$, *Ans.* (See Rule, art. 4th, also Note 2.)

4. Multiply 16 by 9, and divide the product by 12. Statement: $\frac{16.\ 9.}{12.}$. Canceled: $\frac{\overset{4.}{\cancel{16}}.\ 9.}{\underset{3.}{\cancel{12}}.}$. (See Rule, art. 5th.)—Again, $\frac{\overset{4.}{\cancel{16}}.\ \overset{3.}{\cancel{9}}.}{\underset{\cancel{3}.}{\cancel{12}}.}$, (see art. 4th,) and $4 \times 3 = 12$, *Ans.* (Note 2.)

5. Multiply 36 by 27, and divide the product by 9. *Ans.* 108.

6. Multiply 42 by 5, and divide the product by 6. *Ans.* 35.

7. Multiply 45 by 3, and divide the product by the product of 5 and 9. Statement: $\frac{45.\ 3.}{5.\ 9.}$.

Note 3.—Whenever the product of two or more numbers on one side of the line equals any number on the opposite side, these numbers may all be erased. Hence, $\frac{\cancel{45}.\ 3.}{\cancel{5}.\ \cancel{9}.}$; 3 is therefore the *Ans*

8. Multiply 63 by 24, and divide their product by the product of 18 and 12.

Statement : $\frac{63.\ 24.}{18.\ 12.}$ (See art. 4th and 5th, Rule.) *Ans.* 7.

9. Multiply 27 by 15, and divide their product by the product of 9 and 5. *Ans.* 9.

10. Multiply 100 by 63, and divide their product by the product of 30 and 10. *Ans.* 21.

11. Divide the product of 8, 22, and 15, by the product of 11 and 3. *Ans.* 80.

12. Divide the product of 14, 28, and 42, by the product of 21 and 7. *Ans.* 112.

13. Divide the product of 21, 15, and 39, by the product of 3, 5, and 13. *Ans.* 63.

14. Divide the product of 72, 6, and 10, by the product of 12, 18, and 2. *Ans.* 10.

15. Divide the product of 81, 42, and 56, by the prodnct of 27, 7, and 8. *Ans.* 126.

16. Divide the product of 99, 45, 12, and 6, by the product of 11, 9, and 36. Statement: $\frac{99.\ 45.\ 12.\ 6.}{11.\ \ 9.\ \ 36.}$ *Ans.* 90.

17. Divide the product of 8, 16, 24, and 32, by 4, 8, and 48. *Ans.* 64.

18. Divide the product of 10, 15, 20, and 25, by 5, 10, 15, and 20. *Ans.* 5.

19. Divide the product of 100, 16, 24, and 36, by 60, 10, and 16. *Ans.* 144.

20. Divide the product of 96, 18, 110, 5, and 42, by 50, 27, 11, and 28. *Ans.* 96.

Whenever the numbers to be canceled are large, the scholar will find much aid from the following Additional Rules : —

1. All even numbers are divisible by 2, without remainders.

2. To determine whether a number is divisible by 3, cast out the 3's from the number to be divided; if there be no remainder, the number is divisible by 3.

NOTE. — The 3's are cast out in the following manner : — Commence on the left, and *add* together the figures composing the number, and in every instance reject the 3's whenever the amount obtained is more than that number. As often as 3 is rejected, add the next figure to the remainder. For example, let it be required to find whether the number 12705 be divisible by 3. We commence, and add 1 to 2, and, as we obtain 3 as the result, we reject it. We then take 7, and from it reject two 3's, and have 1 remainder. We next add 5, the only remaining *integral* figure, and obtain 6 as the result, which contains two 3's, and nothing remains. We hence know that 12705 is divisible by 3, as is proved by performing the operation; thus, $12705 \div 3 = 4235$.

3. Any number is divisible by 4, without remainder, whenever the *two right-hand* figures are divisible by that number. Thus, we know that 1071124 is divisible by 4, without remainder, because 24, the two right-hand figures, is so divisible.

4. All numbers of which the right-hand figure is either 5 or 0, are divisible by 5.

5. No number is divisible by 6, unless it be an *even number*, and contain an even number of 3's. (See Rule, 2d.)

6. For 7, no convenient rule is known.

7. To determine whether a number is divisible by 8, divide the two right-hand figures by 4, and compare the result with the figure immediately preceding the last two; if both are even numbers, or both odd numbers, the whole number is divisible by 8; otherwise, not.

8. To determine whether a number is divisible by 9, cast out the 9's, as directed to cast out the 3's.

9. No number is divisible by 10, unless its right-hand figure be a cipher.

10. If a number be divided into periods of two figures each, and the sum of these periods be obtained, the sum is divisible by 11, if the number itself can be divided by that number. Every number consisting of 3 digits, the middle one of which is equal to the sum of the other two, is divisible by 11, and the quotient is obtained by rejecting the middle figure. Thus, 363 is divisible by 11, and 33 is the quotient.

Every number consisting of 3 digits, and divisible by 11, possesses the following properties:—

The *first* figure, added to the number expressed by the *two last*, is divisible by 11.

The *last* figure, subtracted from the number expressed by the *two first*, is divisible by 11.

The sum of the two extreme digits, diminished by the middle one, is always divisible by 11, being always 11 or 0.

The quotient is always easily known. Rub out the middle figure, and, if it be not equal to the sum of the other two figures, diminish the *first* by 1; and the first figure thus diminished, placed before the last figure, will be the true quotient.

The following additional sums may now be solved by canceling:—

21. Divide the product of 99, 49, 15, 20, 33, 13, and 16, by 77, 9, 45, 4, 10, and 8. *Ans.* 1001.

22. Divide the product of 164, 88, 4, 28, 9, and 9, by 328, 44, 21, 12, and 6. *Ans.* 6.

23. Divide the product of 363, 116, and 42, by 242, 84, and 29. *Ans.* 3.

24. Divide the product of 156, 484, and 19, by 78, 209, and 22. *Ans.* 4.

25. Divide the product of 105, 117, and 51, by 17, 27, and 65. *Ans.* 21.

26. If 6 horses eat 18 bushels of oats in one month, how many bushels will 36 horses eat in the same time?

Since 6 horses eat 18 bushels, 1 horse will eat one sixth part of 18 bushels, viz., 3 bushels; and 36 horses will eat 36 times 3 bushels, which is 108 bushels. We must both multiply and divide, in solving this sum; hence the principle of canceling may be applied. As we must divide by the 6 horses, we will place that number below the line, and the other two numbers above it, thus, $\frac{18.\ 36.}{6.}$ Now, if this statement be canceled, we obtain $\frac{18.\ \overset{6.}{\cancel{36}}.}{\cancel{6}.}$; and $18 \times 6 = 108$, *Ans.*

We will not, however, anticipate, but will leave the practical application of the principle of canceling to a future section.

QUESTIONS.—What is the object of the rule for Canceling? What sums may be solved by canceling? What is the first fact mentioned on which this principle is founded? Give the illustration. What is the second of these facts? Give the illustration. What is another advantage? What is the third fact? What is the fourth? Give the illustration. What is the first step of the rule for canceling? What is the second?—the third?—the fourth?—the fifth?—the sixth?—the last? How are the canceled numbers marked? What is the answer required when all the numbers below the line are canceled, and only one number is left above the line? What is the answer when more than one number remains above the line? What, when the numbers above the line are all canceled, and one or more numbers remain below the line? What numbers are divisible by 2? How can you determine whether a number is divisible by 3? How do you cast out the 3's from any number? When is a number divisible by 4? What numbers are divisible by 5?—by 6? How can you determine whether a number is divisible by 8 or not?—by 9? What numbers are divisible by 10? How can you determine when a number is divisible by 11? When a number consists of 3 digits, (that is, of 3 figures,) the middle one of which is equal to the sum of the other two, what is its quotient when divided by 11? What is the first of the three properties belonging to every number consisting of 3 digits and divisible by 11? What is the second?—the third? How can you determine the quotient of any number divisible by 11, when divided by that number?

COMPOUND NUMBERS.

§ **38.** We have thus far been operating with numbers of the same denomination, and increasing in the constant ratio of 10.

There is, however, another class of numbers, composed of *several* denominations, increasing in no *uniform* ratio, and requiring to be separately denoted or expressed. These are called Compound Numbers. Under this head are included all those denominations employed to express measures of any definite kind; such as length, breadth, solidity, weight, time, money, capacity, &c.

§ **39.** The following are the tables of these denominations. They require to be made very familiar, as they show how many units of each lower denomination are equal to a unit of the next higher denomination.

I. ENGLISH MONEY.

The denominations of English Money are pounds, shillings, pence, and farthings.

TABLE.

4 farthings (*marked* qr.). .	*make*	1 penny, ...	*marked*	d.
12 pence	"	1 shilling, ...	"	s.
20 shillings	"	1 pound, ...	"	£.

II. TROY WEIGHT.

By this weight the precious metals, such as gold and silver, also jewels and precious stones, are weighed. The following are the denominations:

TABLE.

24 grains (gr.)	*make*	1 pennyweight,	*marked*	pwt.
20 pennyweights	"	1 ounce, ...	"	oz.
12 ounces	"	1 pound, ...	"	lb.

III. AVOIRDUPOIS WEIGHT.

By this weight all coarse materials, such as hay and grain, and also the baser metals, such as copper, are weighed. 1 oz. Av. = 437½ gr. Troy.

TABLE.

16 drams (dr.)	*make*	1 ounce, ...	*marked*	oz.
16 ounces	"	1 pound, ...	"	lb.
28 pounds	"	1 quarter, ...	"	qr.
4 quarters	"	1 hun. weight, . .	"	cwt.
20 hundred	"	1 ton, ...	"	T.

IV. APOTHECARIES' WEIGHT.

This weight is used by apothecaries and physicians in mixing and preparing medicines.

TABLE.

20 grains (gr.)	*make* 1 scruple,	*marked*	℈.
3 scruples	" 1 dram,	"	ʒ.
8 drams	" 1 ounce,	"	℥.
12 ounces	" 1 pound,	"	℔.

V. CLOTH MEASURE.

This is the measure used for measuring all kinds of cloth.

TABLE.

$2\frac{1}{4}$ inches (in.)	*make* 1 nail,	*marked*	na.
4 nails	" 1 quarter yard,	"	qr.
4 quarters	" 1 yard,	"	yd.
3 quarters	" 1 ell Flemish,	"	e. Fl.
5 quarters	" 1 ell English,	"	e. E.
6 quarters	" 1 ell French,	"	e. Fr.

VI. WINE MEASURE.

This measure is used for measuring all liquors, with the exception of milk, beer, and ale. The gallon contains 231 cubic inches.

TABLE.

4 gills (gi.)	*make* 1 pint,	*marked*	pt.
2 pints	" 1 quart,	"	qt.
4 quarts	" 1 gallon,	"	gal.
$31\frac{1}{2}$ gallons	" 1 barrel,	"	bar.
63 gallons	" 1 hogshead,	"	hhd.
2 hogsheads	" 1 pipe,	"	P.
2 pipes	" 1 tun,	"	T.

VII. ALE OR BEER MEASURE.

Ale, beer, and milk are measured by this measure. The gallon contains 282 cubic inches.

TABLE.

2 pints (pt.)	*make* 1 quart,	*marked*	qt.
4 quarts	" 1 gallon,	"	gal.
36 gallons	" 1 barrel,	"	bar.
54 gallons	" 1 hogshead,	"	hhd.

VIII. DRY MEASURE.

This measure is used for measuring all kinds of grain, fruit, salt, coal, &c.

TABLE.

4 gills (gi.)	*make* 1 pint,	*marked*	pt.
2 pints	" 1 quart,	"	qt.
8 quarts	" 1 peck,	"	pk.
4 pecks	" 1 bushel,	"	bu.
36 bushels	" 1 chaldron,	"	ch.

IX. LONG MEASURE.

The following denominations and numbers are used for measuring distance.

TABLE.

3	barley-corns (b. c.)	*make*	1 inch,	*marked*	in.
12	inches	"	1 foot,	"	ft.
3	feet	"	1 yard,	"	yd.
$5\frac{1}{2}$	yards, or $16\frac{1}{2}$ feet	"	1 rod,	"	rd.
40	rods	"	1 furlong,	"	fur.
8	furlongs	"	1 mile,	"	m.
3	miles	"	1 league,	"	L.

60 geographic, or $69\frac{1}{2}$ statute miles, make 1 degree on the earth's surface. 360 degrees make the earth's circumference.

X. LAND OR SQUARE MEASURE.

TABLE.

144	square inches (sq.in.)	*make*	1 square foot,	*marked*	sq. ft.
9	square feet	"	1 square yard,	"	sq. yd.
$30\frac{1}{4}$	square yards	"	1 square rod,	"	sq. rd.
40	square rods	"	1 square rood,	"	R.
4	square roods	"	1 acre,	"	A.
640	square acres	"	1 square mile.		

Land is usually measured by Gunter's chain, which is 4 rods, or 66 feet, in length. The whole chain is divided into 100 equal parts, called *links*. The link is therefore $\frac{1}{25}$ part of the rod, and is $7\frac{92}{100}$ inches in length. 80 chains, or 320 rods, make 1 mile in length. 1 square chain makes 16 square rods, and 10 square chains make 1 acre.

XI. SOLID MEASURE.

This measure is employed in measuring substances which have three dimensions, viz., length, breadth, and thickness. Timber, stone, &c., are among these substances.

TABLE.

1728	solid inches	*make*	1 solid foot,	*marked*	s. ft.
27	solid feet	"	1 solid yard,	"	s. yd.
40	feet of round, or 50 feet of hewn timber,	"	1 ton,	"	T.
128	solid feet	"	1 cord,	"	C.

A pile of wood, 8 feet long, 4 feet wide, and 4 feet high, contains just one cord, since $8 \times 4 \times 4 = 128$.

XII. CIRCULAR MOTION.

TABLE.

60 seconds (″)	*make*	1 minute,	*marked*	′.
60 minutes (′)	"	1 degree,	"	°.
30 degrees	"	1 sign,	"	S.
12 signs, or 360°	"	1 circle,	"	C.

XIII. TIME.

TABLE.

60 seconds (sec.)	*make*	1 minute,	*marked*	m.
60 minutes	"	1 hour,	"	h.
24 hours	"	1 day,	"	d.
7 days	"	1 week,	"	w.
4 weeks	"	1 month,	"	m.
13 months, 1 day, and 6 hrs., or 365 days and 6 hours,	"	1 common year,	"	yr.

TABLE OF PARTICULARS.

12 particular things	*make*	1 dozen, *marked* doz.
12 dozen	"	1 gross.
12 gross	"	1 great gross.
20 things	"	1 score.
24 sheets	"	1 quire of paper.
20 quires	"	1 ream.
112 pounds	"	1 quintal of fish.

REDUCTION OF COMPOUND NUMBERS.

§ **40.** The scholar is now requested to turn back to the table of English Money, and from it answer the following questions: — How many farthings make a penny? How many make 2 pennies? How many make 4? How many make 6? How many make 7? How many make 8? How many 9? How many 10? How many 11? How many pence make 1 shilling? How many make 2 shillings? How many make 3? 4? 5? 6? 7? 8? 9? 10? 11? 12? How many shillings make 1 pound? How many make 2? 3? 4? 5? &c. Which is worth most, 1 penny or 4 farthings? 2 pence or 8 farthings? Which is worth most, 1 shilling or 12 pence? 2 shillings or 24 pence? 1 pound or 20 shillings? 2 pounds or 40 shillings? If these expressions are equal in value, in what do they differ? *Ans.* They are different expressions for the same value. In what, then, does Reduction consist? *Ans.* In changing numbers from one denomination to another without altering the value. Reduce 6 pence to farthings. By what do you multiply the 6? *Ans.* By 4. And why? Because 4 farthings make one penny; and, consequently, there must be four times

as many farthings as pence. Reduce 4 shillings to pence. How do you reduce shillings to pence? Reduce 3 pounds to shillings. How do you reduce pounds to shillings? In these last examples, were high denominations brought into low, or low into high? *Ans.* High denominations were brought into low. How was it effected? *Ans.* By multiplication. Reduce 12 farthings to pence. By what do you divide? *Ans.* By 4. Why? Because 4 farthings make one penny. Reduce 36 pence to shillings. By what do you divide? Reduce 40 shillings to pounds. By what do you divide? What change is here made in the denomination? *Ans.* Low denominations are brought into high. How, then, are low denominations brought into high? *Ans.* By Division. After a careful examination of the preceding questions and remarks, the scholar will readily perceive the appropriateness of the following definition:—

Reduction is an operation by which a number expressing the value of a quantity in one denomination is changed to another number, expressing the same value in a different denomination.

But it has already been shown, that high denominations are brought into low by Multiplication, and that low denominations are brought into high by Division. The scholar, therefore, needs only the rules by which to guide his operation.

§ 41. When a higher Denomination is to be reduced to a lower.

Rule 1st.—*Multiply the higher denomination by that number which it takes of the* LOWER *denomination to make* ONE *of the higher, remembering to add to the product whatever may be given of this lower denomination. If it be required to reduce the quantity still lower, multiply the number already obtained by the number required to reduce it to the next lower denomination, adding to the product the given number of that denomination, if any. Continue the same operation till you come to the required denomination.*

§ 42. When a lower Denomination is to be brought to a higher.

Rule 2d.—*Divide the lower denomination by that number which is required of that denomination to make one of the next higher. The quotient obtained will be of the higher denomination; and, if there be any remainder, it will be of the same denomination as the number divided. Divide the quotient again, (if it be not already reduced to as high a denomination as prac-*

ticable,) by the same general principle; and continue so to do, till you have reached the highest denomination of which the given quantity is susceptible.)

Ex. 1. Reduce 6£. 17 s. 9 d. 3 qr. to farthings.

PERFORMED.

	6. 17. 9. 3.
Multiply by	20
	120 = shillings in 6 pounds.
	17 = given shillings added.
	137 = whole number of shillings.
Multiply by	12
	1644 = pence in 137 shillings.
	9 = given pence added.
	1653 = whole number of pence.
Multiply by	4
	6612 = farthings in 1653 pence.
	3 = given farthings added.
	6615 = whole number of farthings.

Each pound is of the value of 20 shillings; therefore, 6 £. = 120 s., and 6 £. and 17 s. = 137 s. Each shilling is of the same value as 12 pence; therefore, 137 s. = 1644 d., and 1644 d. + 9 d. = 1653 d. Each penny = 4 farthings; therefore, 1653 d. = 6612 qr., and 6612 qr. + 3 qr. = 6615 qr.

The scholar will notice that each denomination below the pounds has been added by a separate process. This is not necessary; it may be added mentally, as in the following example:

2. Reduce 18£. 13 s. 11 d. 2 qr. to farthings.

PERFORMED.

18£. 13s. 11d. 2qr.
20

373 = shillings in 18£. 13 s.; the 13 s. being added mentally.
12

4487 = pence in 373 s. 11 d.; the 11 d. being also added mentally.
4

17950 = farthings in 4487 d. 2 qr.; the 2 qr. added as before.

In the above example, to the product of 18 multiplied by 20, I add 13; that is, I add 3 to the units, and 1 to the tens. And, when I multiply by 12, I say, 12 times 3 shillings are 36 pence; and the 11 given pence added, make 47 pence. I proceed in the same manner in reducing pence to farthings.

The preceding examples will serve to illustrate Rule 1st. An illustration or two will also be given of Rule 2d, that is, of bringing low denominations into high.

3. Reduce 17950 farthings to pounds, shillings, pence, and farthings.

PERFORMED.

4) 17950

12) 4487 — 2 qr. = 4487 d. 2 qr. obtained by first division.

20) 373 — 11 d. = 373 s. 11 d. obtained by dividing by 12.

18 — 13 s. = 18 pounds, obtained by dividing by 20, and 13 s. remain.

Since 4 farthings make 1 penny, it is evident that there are as many pence in 17950 qr. as there are 4's contained in it. The same reasoning may be applied to the other divisors.

It will be observed that, in this last example, we have reversed what was done in the second example. We there had the same *value* given, which we have here, but were required to change it from a higher to a lower denomination, instead of from a lower to a higher, as in the last example.

4. Reduce 44447 farthings to pence, shillings, and pounds.

PERFORMED.

4) 44447

12) 11111 — 3 qr. remain.

20) 925 — 11 pence remain.

46 — 5 shillings remain.

The *Ans.*, therefore, is 46 £. 5 s. 11 d. 3 qr.

5. Reduce 22685 qr. to pounds, &c.

PERFORMED.

4) 22685

12) 5671 — 1 qr. remains.

20) 472 — 7 pence remain.

23 — 12 s. remain.

The *Ans.* is 23 £. 12 s. 7 d. 1 qr.

6. Reduce 7195 pence to pounds, &c.

The scholar will observe that the given number is already pence.

PERFORMED.

12) 7195

20) 599 — 7 d. remain.

29 — 19 s. remain.

The *Ans.*, then, is 29 £. 19 s. 7 d.

NOTE.—The scholar must first consider whether the quantity given is to be brought from a higher denomination to a lower, or from a lower denomination to a higher. When this is determined, let him apply the corresponding rule.

APPLICATION OF TABLE I. — ENGLISH MONEY.

7. Reduce 23 £. 12 s. 7 d. 1 qr. to farthings. *Ans.* 22685 qr

N. B. 23 £. 12 s. 7 d. 1 qr. = 22685 qr. So, in all operations of mere reduction, the quantity given equals in value the quantity obtained.

8. Reduce 71 £. 13 s. 2 d. 3 qr. to farthings. *Ans.* 68795 qr.

9. Reduce 299924 qr. to pounds, shillings, &c. *Ans.* 312 £. 8 s. 5 d.

10. Reduce 68795 qr. to pounds, shillings, &c. *Ans.* 71 £. 13 s. 2 d. 3 qr.

11. Reduce 46 £. 5 s. 11 d. 3 qr. to farthings. *Ans.* 44447.

12. Reduce 29 £. 19 s. 7 d. to pence and farthings. *Ans.* 7195 d.; 28780 qr.

13. Reduce 5974681369 qr. to pounds, &c. *Ans.* 6223626 £. 8 s. 6 d. 1 qr.

14. Reduce 7195 pence to pounds, &c. *Ans.* 29 £. 19 s. 7 d.

15. Reduce 40320 half-pence to pounds. *Ans.* 84 £.

16. Reduce 125 £. 19 s. 11 d. 3 qr. to farthings. *Ans.* 120959 qr.

17. Reduce 475 dollars, at 6 shillings each, to pence. *Ans.* 34200 pence.

18. Reduce 312 £. 8 s. 5 d. to half-pence. *Ans.* 149962 half-pence.

19. Reduce 121 pistoles, at 22 shillings each, to pence and farthings. *Ans.* 31944 d.; 127776 qr.

20. Reduce 34200 pence to dollars, at 6 shillings each. *Ans.* 475 dollars.

21. Reduce 359548 qr. to pistoles, at 22 shillings each. *Ans.* 340 pistoles, 10 s. 7 d.

22. Reduce 740 dollars, at 8 shillings each, to pence. *Ans.* 71040 pence.

23. Reduce 79 £. to pence and half-pence. *Ans.* 18960 d.; 37920 half-pence

APPLICATION OF TABLE II. — TROY WEIGHT.

Ex. 1. Reduce 23 lb. 9 oz. 6 pwt. 22 gr. to grains.

PERFORMED.

```
     23.  9.  6.  22.
     12 oz. = 1 lb.
     ---
    285 = ounces in 23 lb. 9 oz.
     20 pwt. = 1 oz.
    ----
   5706 = pwt. in 285 oz. and 6 pwt.
     24 gr. = 1 pwt.
   -----
  22846
 11412
 ------
Ans. 136966 = grains in 5706 pwt. and 22 gr.
```

2. Reduce 8578 grains to pounds, &c.

PERFORMED.

```
24)8578
   20)357 — 10 gr. remain.
      12)17 — 17 pwt. remain.
          1 — 5 oz. remain.
```

Ans. 1 lb. 5 oz. 17 pwt. 10 gr.

3. Reduce 62 lb. 7 oz. 14 pwt. 18 gr. to grains. *Ans.* 360834 gr.

4. Reduce 360834 grains to pounds. *Ans.* 62 lb. 7 oz. 14 pwt. 18 gr.

5. Reduce 1 lb. 11 oz. 19 pwt. 23 gr. to grains. *Ans.* 11519 gr.

6. Reduce 11519 grains to pounds, &c. *Ans.* 1 lb. 11 oz. 19 pwt. 23 gr.

APPLICATION OF TABLE III. — AVOIRDUPOIS WEIGHT.

Ex. 1. Reduce 4 cwt. 3 qr. 26 lb. 10 oz. 12 dr. to drams.

PERFORMED.

```
     4. 3. 26. 10. 12.
     4   Multiply by 4, because
    --     4 quarters = 1 cwt.
    19
    28   Multiply by 28, because
   ---     28 pounds = 1 qr.
   178
   38
   ---
   558
    16   Multiply by 16, because
  ----     16 oz. = 1 lb.
  3358
  558
  ----
  8938
    16   Multiply by 16 again,
 -----     because 16 dr. = 1 oz.
 53640
 8938
------
143020
```

THE SAME, REVERSED.

```
16)143020
  16)8938 — 12 dr. rem
     28)558 — 10 oz. "
        4)19 — 26 lb. "
           4 — 3 qr. "
```

Ans. 4 cwt. 3 qr. 26 lb. 10 oz. 12 dr.

2. Reduce 7 cwt. 3 qr. 19 lb. to ounces. *Ans.* 14192 ounces.

3. Reduce 14192 ounces to pounds, quarters, &c. *Ans.* 7 cwt. 3 qr. 19 lb.

4. Reduce 29548 ounces to pounds, quarters, &c. *Ans.* 16 cwt. 1 qr. 26 lb. 12 oz.

5. Reduce 480 drams to pounds, &c. *Ans.* 1 lb. 14 oz.

6. Reduce 12 tons to ounces. *Ans.* 430080 oz.
7. Reduce 430080 ounces to tons. *Ans.* 12 tons.

APPLICATION OF TABLE IV. — APOTHECARIES' WEIGHT.

Ex. 1. Reduce 8 ℔. 8 ℥. 5 ʒ. 2 ℈. 12 gr. to grains. *Ans.* 50272.
2. Reduce 27 ℔. 9 ℥. 6 ʒ. 2 ℈. to scruples. *Ans.* 8012 scruples.
3. Reduce 477816 grains to pounds, &c. *Ans.* 82 ℔. 11 ℥. 3 ʒ. 1 ℈. 16 gr.
4. Reduce 6 ℔. 7 ℥. 6 ʒ. 1 ℈. 12 gr. to grains. *Ans.* 38312 grains.
5. Reduce 6348 scruples to pounds. *Ans.* 22 ℔. 4 ʒ.
6. Reduce 480 drams to pounds. *Ans.* 5 ℔.
7. Reduce 1 ℔. 1 ℥. 1 ʒ. 1 ℈. 1 gr. to grains. *Ans.* 6321 grains.

APPLICATION OF TABLE V. — CLOTH MEASURE.

Ex. 1. Reduce 30 yd. 3 qr. 3 na. to nails. *Ans.* 495 na.
2. Reduce 450 e. E. 3 qr. 2 na. to nails. *Ans.* 9014 nails.
3. Reduce 678 ells Flemish to nails. *Ans.* 8136 nails.
4. Reduce 8136 nails to ells Flemish. *Ans.* 678 ells.
5. Reduce 9014 nails to ells English. *Ans.* 450 e. E. 3 qr. 2 na.
6. Reduce 12 ells French, 5 qr. 3 na. to nails. *Ans.* 311 nails.
7. Reduce 622 nails to ells French. *Ans.* 25 ells, 5 qr. 2 na.

APPLICATION OF TABLE VI. — WINE MEASURE.

Ex. 1. Reduce 3 hhd. 42 gal. 2 qt. 1 pt. to pints. *Ans.* 1853 pints.
2. Reduce 6 pipes, 1 hogshead, and 1 gill, to gills. *Ans.* 26209 gills.
3. Reduce 30000 gills to pipes. *Ans.* 7 pipes, 55 gal. 2 qt.
4. Reduce 20 tuns to gills. *Ans.* 161280 gills.
5. Reduce 5 hhd. 56 gal. 2 qt. to pints. *Ans.* 2972 pints.
6. Reduce 16 barrels 21 gal. to quarts. *Ans.* 2100.

APPLICATION OF TABLE VII. — ALE OR BEER MEASURE.

1. Reduce 4 hhd. 45 gal. 3 qt. to pints. *Ans.* 2094.
2. Reduce 47 barrels of beer to pints. *Ans.* 13536 pints.
3. Reduce 13672 pt. to barrels, &c. *Ans.* 47 bar. 17 gal.
4. Reduce 451 bar. 7 gal. to quarts. *Ans.* 64972 quarts.
5. Reduce 21 hhd. to quarts. *Ans.* 4536 quarts.
6. Reduce 6562 pints to hogsheads. *Ans.* 15 hogsheads, 10 gal. 1 qt.

APPLICATION OF TABLE VIII. — DRY MEASURE.

Ex. 1. Reduce 6 ch. 9 bu. 3 pk. to gills. *Ans.* 57792 gills.
2. Reduce 87762 gills to chaldrons.
Ans. 9 ch. 18 bu. 3 pk. 2 qt. 2 gi.
3. In 136 bushels, how many pecks, quarts, and pints?
Ans. 544 pk. 4352 qt. 8704 pt.
4. Reduce 10640 pints to bushels. *Ans.* 166 bu. 1 pk.
5. Reduce 3 pecks to gills. *Ans.* 192 gills.
6. Reduce 720 quarts to bushels. *Ans.* 22 bu. 2 pk.

APPLICATION OF TABLE IX. — LONG MEASURE.

1. Reduce 8 leagues, 2 miles, 6 furlongs, 16 rods, 3 yards, 2 feet, 9 inches, and 2 barley-corns, to barley-corns.
Ans. 5094569 b. c.
2. How many barley-corns will reach round the earth, it being 360 degrees? *Ans.* 4755801600.
3. Reduce 48765000 barley-corns to miles, &c.
Ans. 256 m. 4 fur. 15 rods, 15 ft. 10 inches.
4. Reduce 26431 rods to miles, &c.
Ans. 82 m. 4 fur. 31 rods.
5. Reduce 1710720 inches to miles. *Ans.* 27 miles.
6. Reduce 7 fur. 36 rods, and 9 ft. to inches. *Ans.* 62676.

APPLICATION OF TABLE X. — LAND OR SQUARE MEASURE.

1. Reduce 500 acres to square rods. *Ans.* 80000 rods.
2. Reduce 32000 rods or poles to acres. *Ans.* 200 acres.
3. Reduce 3 square miles to square rods. *Ans.* 307200.
4. Reduce 458000 square rods to square miles, &c.
Ans. 4 sq. m. 302 acres, 2 roods.
5. Reduce 6272640 square inches to acres. *Ans.* 1 acre.

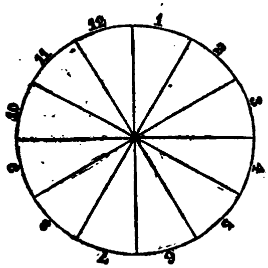
NOTE. — For figures to illustrate square and solid measure, the scholar is referred to Square and Cube Root.

APPLICATION OF TABLE XI. — SOLID MEASURE.

1. In a pile of wood containing 2 cords and 64 feet, how many solid inches? *Ans.* 552960.
2. Reduce 884736 solid inches of wood to cords.
Ans. 4 cords.
3. Reduce 6117120 cubic inches to tons of round timber.
Ans. 88 tons, 20 ft.
4. Reduce 72 tons of hewn timber to cubic inches.
Ans. 6220800.

APPLICATION OF TABLE XII. — CIRCULAR MOTION.

1. Reduce 6 signs, 21 degrees, 40 minutes, to seconds.
Ans. 726000 seconds.
2. Reduce 432000 seconds to signs. *Ans.* 4 signs.
3. Reduce 1 circle, 6 signs, 25 degrees, to minutes.
Ans. 33900 m.
4. Reduce 45200 minutes to circles, &c.
Ans. 2 cir. 1 S. 3°. 20′.



It is desirable that the scholar should obtain correct views of this measure. The adjoining figure will illustrate its application. The circle is regarded as an integral object. Its first division is into 12 signs. These are represented by the figures 1, 2, 3, 4, &c. Each sign is divided into 30 equal parts: these constitute degrees; and, as the circle contains 12 signs, it is evident the whole circle must contain 360 degrees. These again are divided into minutes, and the minutes into seconds. The degrees, minutes, and seconds, are not represented in the figure. This measure is obviously applied to bodies moving in a circle; such as all wheels in machinery, the revolutions of the heavenly bodies, &c.

APPLICATION OF TABLE XIII. — TIME.

1. Reduce 360 years, 300 days, 20 hours, 50 minutes, and 37 seconds, to seconds. *Ans.* 11386731037.

In the above sum, 365 days and 6 hours are allowed to the year.

2. Reduce 662709600 seconds to years, &c., allowing the year to be as above. *Ans.* 21 years.
3. Reduce 49 weeks to seconds. *Ans.* 29635200 sec.
4. Reduce 59270400 seconds to years, &c.
Ans. 1 year, 46 weeks.
5. Suppose my age to be 21 years; how many seconds have I lived? *Ans.* 662709600.
6. Reduce 1325419200 seconds to hours.
Ans. 368172 hours.

The following particulars require to be introduced here: — The year, as given above, contains 365 days and 6 hours. In four years, these six hours amount to 24 hours, or one day. Hence, every fourth year has 366 days. This is called Bissextile or Leap year. As it requires four years to gain this one day, it is obvious that the Leap year may be found by dividing the given year by 4. If it be Leap year, it will divide without remainder. If, in dividing, there be 1 remaining, the year under consideration is the first after Leap year; if there be 2 remaining, it is the second; and, if 3 remain, it is the third after Leap year. Thus, 1824 is Leap year; for $1824 \div 4 = 456$, and *nothing remains*. The same operation shows 1826 to

be the second after Leap year. In the table, the year is divided into 13 months. These are lunar months. It is much more usually divided into 12 calendar months, containing each the following number of days, viz., April, June, September, and November, 30; January, March, May, July, August, October, December, 31; and February, 28. To this last month (February) the additional day of the Leap year is added; so that, every fourth year, this month has 29 days.

PROMISCUOUS EXAMPLES.

1. In 64126 gills, how many bushels? *Ans.* 250 bu. 1 pk. 7 qt. 1 pt. 2 gi.
2. In 26709912 barley-corns, how many leagues? *Ans.* 46 l. 2 m. 4 fur. 6 rods, 1 yd.
3. In 161280 gills, how many tuns of wine? *Ans.* 20.
4. In 10 cords of wood, how many solid inches? *Ans.* 2211840.
5. In 20 hhd. of sugar, each 12 cwt., how many pounds? *Ans.* 26880.
6. How many gills in 250 bu. 1 pk. 7 qt. 1 pt. 2 gills? *Ans.* 64126.
7. How many pence are there in 16 bags, containing each 24 guineas, 16 shillings, and 8 pence, the guinea being 28 s.? *Ans.* 132224.
8. In 15840 yards, how many leagues? *Ans.* 3.
9. In 1876742 solid inches, how many cords, &c.? *Ans.* 8 cords, 62 feet, 134 in.

§ **43.** Thus far, examples have been avoided which, in their solution, require both multiplication and division. They will here be introduced. Let us take the following: — How many ells French in 15 pieces of cloth, containing each 20 yards? Now, it is evident that yards cannot be changed to ells French, at a single step. When the question is, How many times one quantity is contained in another, a simple operation of division is often all that is required to obtain the answer. But that will not suffice here, because the two quantities are not in the same denomination. The first step, then, is, to bring the two given quantities to the same kind. The scholar will therefore turn to Table V., Cloth Measure. He will there find that the ell French and the yard may both be brought into quarters; the former, by being multiplied by 6, and the latter, by 4. Therefore,

```
         1 5 pieces.
         2 0 = yd. in a piece.
       -----
       3 0 0 = yd. in 15 pieces.
           4 = qr. in a yard.
     -------
 6 ) 1 2 0 0 = qr. in 300 yards, or 15 pieces.
     -------
       2 0 0 = ells French, the quantity required.
```

There are evidently as many ells French as there are 6's in 1200 qr., as 6 qr. make one ell French.

The scholar will perceive that the given quantity must first be brought

into a denomination from which it may be changed to the required denomination.

The above work may be abbreviated by applying to the solution the principle explained in the rule for canceling. The numbers 15, 20, and 4, are multiplied together, and produce the dividend 1200. This is then divided by 6, and the result is the answer. The scholar may therefore turn back to the rule for canceling, and he will see that the following statement is in accordance with it, viz., $\frac{15.\ 20.\ 4}{6}$. This statement, by sec. 5th of the same rule, may be reduced to $\frac{5.\ 20.\ 4}{2}$; and again, by sec. 4th, to $\frac{5.\ 20.\ 2}{0}$; and by 6th and 7th sec. to 200, *Ans.*, as before.

We will now give the solution at a single statement, thus:

$$\frac{\overset{5}{\cancel{15}}.\ 20.\ \overset{2}{\cancel{4}}}{\underset{\cancel{2}}{\cancel{6}}}$$, and $5 \times 20 \times 2 = 200$ ells French, the *Ans.*

2. How many barrels, each holding 2 bushels and 3 pecks, are required, to contain 880 bushels of corn?

The 2 bushels and 3 pecks equal 11 pecks. The question, then, is, How many times are 11 pecks contained in 880 bushels? By the preceding solution, it will be seen that the bushels, as they are to be divided by 11 pecks, must also be brought into pecks. Therefore,

$$\begin{array}{rrl} & 880 & \\ & 4 & = \text{pk. in one bush.} \\ \hline 1 \text{ bbl.} = 11 \text{ pk.; therefore, } 11) & 3520 & = \text{pk. in 880 bush.} \\ \hline & 320 & = \text{No. bbl. required.} \end{array}$$

The same, canceled. The scholar will read over the explanation of the preceding sum, if he does not yet understand the work.

Statement: $\frac{880.\ 4}{11}$. Canceled: (see rule for canceling, sec. 4:) $\frac{\overset{80}{\cancel{880}}.\ 4}{\cancel{11}}$, and $80 \times 4 = 320$ barrels, the same as before.

3. In 33 guineas, at 28 shillings each, how many pistoles, each 22 shillings?

The simple question is, How many pistoles are there in 33 guineas? The guineas cannot be divided by the pistoles, for they are of different value. They must be brought upon some common ground, and the nearest is that of shillings; therefore,

$$\begin{array}{rrl} & 33 & \\ & 28 & \\ \hline & 264 & \\ & 66 & \\ \hline 22\text{s.} = 1 \text{ pistole; therefore, } 22) & 924 & = \text{shillings in 33 guineas.} \\ \hline & 42 & = \text{number of pistoles in the guineas.} \end{array}$$

The same, canceled: $\frac{33.28}{22}$. (See rule for canceling, sec. 4th and 5th.) Performed: $\frac{\overset{3}{\cancel{33}}.\overset{14}{\cancel{28}}}{\underset{2}{\cancel{22}}}$, and $14 \times 3 = 42$, the number of pistoles, as before.

4. Purchased 24 hogsheads of wine, at 1s. 8 d. per quart, and paid for the same with cloth, for which I was allowed 3 s. 4 d. per yard. How much cloth was required?

The price of the quart being given, it is obvious that the 24 hogsheads must first be brought to quarts. This is done by multiplying the 24 by 63 and by 4. Each quart is worth 1 s. 8 d. = 20 d. Therefore, multiplying the quarts by 20 d. gives the cost of the wine in pence. Again, each yard of cloth is worth 3 s. 4 d. = 40 d. If, then, the pence the wine cost be divided by the price of one yard of cloth, the quotient obtained must be the number of yards required; therefore, statement for canceling: $\frac{24.63.4.20}{40}$. Let this statement be compared with the above analysis of the sum, and with the rule for stating sums for canceling.

Statement repeated, and canceled: $\frac{24.63.\overset{2}{\cancel{4}}.\cancel{20}}{\underset{2}{\cancel{40}}}$; therefore, $24 \times 63 \times 2 =$ the number of yards required, viz., 3024 yd.

We will now give the usual solution of this sum, and observe the increased facility of canceling.

$$\begin{array}{r l}
24 & \\
63 & \\
\hline
72 & \\
144 & \\
\hline
1512 & = \text{the gallons in 24 hhd.} \\
4 & \\
\hline
6048 & = \text{qt. in the same.} \\
20 & \\
\hline
4,0)\,1209\,6,0 & = \text{cost of the same in pence.} \\
\hline
3024 & = \text{yd. of cloth required, the same as before.}
\end{array}$$

40 d. = price of 1 yd. of cloth; therefore, I divide by 40.

The scholar will readily perceive the object to be attained in sums of this character, viz., to exchange dissimilar quantities, or quantities of different denominations, but of equal value.

§ 44. **Rule for the common Mode of Operation.** — *Reduce the quantity to be exchanged, to the denomination in*

which the price, or the equivalent of exchange, of the other kind, is given; then divide by this price, or equivalent of exchange, and perform such operations of reduction as the nature of the case may require.)

§ **45. Rule** FOR CANCELING.—*Consider what is the quantity to be exchanged, and place it over a horizontal line towards the left. Then, on the right of this, also above the line, place such numbers as are required to reduce this quantity to the denomination in which the price, or equivalent of exchange, of the other kind, is given. Write also under the line those numbers which are necessary to reduce this price, or equivalent of exchange, to the required denomination. Proceed to cancel, multiply, and divide, as directed in the rule for canceling, and the number obtained will be the one sought.*

NOTE 1.—In stating for canceling, care should be taken to introduce into the statement every number required for the complete solution of the same, including all operations of reduction, &c., since each number introduced increases the opportunity for canceling, and thus abbreviates the operation.

NOTE 2.—If any one term consists of more than one denomination, it should be reduced to the lowest given denomination before stating.

5. How many times will a wheel, 18 feet in circumference, turn round in traveling 84 miles?

The thing to be done is, to change 84 miles into revolutions of the wheel. To do this, 84 miles must be reduced to feet, because the distance required for one revolution is given in feet, viz., 18. Therefore,

$84 \times 8 \times 40 \times 16\frac{1}{2} = 443520$, the feet in 84 miles; then, $443520 \div 18 = 24640$, revolutions required.

The preceding solution is by the first rule. We will now solve the sum by canceling. Statement: $\dfrac{84.\ 8.\ 40.\ 16\frac{1}{2}}{18}$.

Observe that the numbers above the line are those multiplied together for a dividend in the preceding operation, and that the one below the line was the divisor.

To avoid the fraction in the statement, $16\frac{1}{2}$ may be written $\frac{33}{2}$; for, since 1 unit $=$ 2 halves or $\frac{2}{2}$, 16 units $=$ 32 halves or $\frac{32}{2}$, and $16\frac{1}{2} = 33$ halves, or $\frac{33}{2}$. The preceding statement may, therefore, be written, $\dfrac{84.\ 8.\ 40.\ 33}{18.\ 2}$. Canceled: $\dfrac{\overset{14.}{\cancel{84}}.\ \overset{4.}{\cancel{8}}.\ 40.\ \overset{11}{\cancel{33}}}{\underset{\cancel{3}}{\cancel{18}}.\ \cancel{2}}$.

$$40 \times 11 \times 4 \times 14 = 24640, \textit{ Ans.}$$

The scholar will observe, that the 4th and 5th sections of the rule for canceling have been applied in the preceding solution.

6. In 30 purses, containing 20 guineas each, how many pounds? *Ans.* 840.

Statement for canceling: $\frac{30.\ 20.\ 28}{20}$.

The 28 above the line expresses the shillings in a guinea, and the 20 below it, the shillings in a pound. The scholar may perform the solution.

7. How many pounds, in money, will 9 tuns of wine cost, at 3 s. 4 d. per gallon? 3 s. 4 d. = 40 d.

Statement for canceling: $\frac{9.\ 2.\ 2.\ 63.\ 40}{12.\ \ 20}$. *Ans.* £378.

For the terms in this statement, the scholar is referred to Tables I. and VI. of the Compound Numbers.

8. How many times will a wheel 12 feet 6 inches in circumference, revolve in traveling 124 miles? *Ans.* $52377\frac{3}{5}$.

Statement: 12 ft. 6 in. = 150 in. $\frac{124.\ 8.\ 40.\ 16\frac{1}{2}.\ 12}{150}$. (See Table IX., Compound Numbers.)

The last three sums have been stated by the rule for canceling only, because that is regarded as *superior* to the common mode of solution. The scholar will feel at liberty to adopt either mode of operation. The following sums are not stated, that the scholar may exercise his own judgment.

9. How long will it take to count 6000000, at the rate of 75 per minute? *Ans.* $55\frac{5}{9}$ days.

10. In 107520 pounds of sugar, how many hogsheads, each containing 6 cwt.? *Ans.* 160.

11. If one quart of melasses cost 10 pence, how much will 12 hogsheads cost? *Ans.* 126 £.

12. How many dollars, each 8 s., will it cost to ride 45 leagues, at 6 pence a mile? *Ans.* \$8.437 +.

13. How much will 540 yards of cloth cost, at 3 s. 4 d. per yard, in dollars, at 6 shillings each? *Ans.* \$300.

14. How many dozen of gallon, quart, and pint bottles, of each an equal number, may be filled from a cistern holding 144 gallons? *Ans.* $8\frac{8}{11}$ dozen.

15. How many casks, each containing 1 bushel, 1 peck, are required to hold 145 bushels? *Ans.* 116.

16. I have five hogsheads of wine, 63 gallons each, which I wish to put into gallon, quart, and pint bottles, of each an equal number. How many will be required?

Ans. 229; and 1 pint of wine will be left.

17. In 16 cwt. 3 qr. 20 lb., how many parcels, each containing 36 lb.? *Ans.* $52\frac{2}{3}$.

18. In 56 ells Flemish, how many yards? *Ans.* 42.

19. In 144 yards, how many ells French? *Ans.* 96.

20. In 472 parcels of sugar, each 72 pounds, how many cwt.? *Ans.* 303 cwt. 1 qr. 20 lb.

21. If 15 casks of flour contain 4000 lb., how many cwt. are there in each? *Ans.* 2 cwt. 1 qr. 14$\frac{2}{3}$ lb.

22. In 8 lb. of drugs, how many parcels, each 12 drams? *Ans.* 64.

23. In 80 parcels, each 15 drams, how many pounds? *Ans.* 12$\frac{1}{2}$.

24. How many revolutions will a wheel 18 feet 4 inches in circumference, make in traveling 300 miles? *Ans.* 86400.

25. How many cups, each weighing 22 oz., may be made of 25 lb. 6 oz. of silver? *Ans.* 13 cups, and 20 oz. silver remain.

26. How much would 1008 nails of cloth cost, at 10 pence per yard, in dollars, at 6 shillings each? *Ans.* $8.75.

27. In 4 bales of cloth, each 15 pieces, and each piece 16 ells English, how many ells French? *Ans.* 800.

28. In 6 bales, each 12 pieces, and each piece 18 yards, how many ells Flemish? *Ans.* 1728.

29. In 4 ingots of silver, each weighing 2 lb. 6 oz. 11 pwt., how many grains? *Ans.* 58656.

30. How many hours, minutes, and seconds, in one year? *Ans.* 8766 h., 525960 min., and 31557600 sec.

31. In 1597 quarts, how many bushels, &c.? *Ans.* 49 bu., 3 pk., and 5 qt.

QUESTIONS.—What are compound numbers? How do they increase? What are included under this head? Let the fourteen tables of Compound Numbers be made familiar, before the scholar proceeds with Reduction.

What is Reduction? How are high denominations brought into low denominations? And how are low denominations brought into high? What is the rule when high denominations are brought into low? And what is it, when low denominations are brought into high? What should the scholar notice before commencing to reduce any quantity? In Circular Measure, how is the circle regarded? What are the divisions of the circle? To what is this measure applied? How many days and hours does the year contain? To what do the six hours amount in four years? How many days does every fourth year contain? What is this fourth year called? How may it be found? In dividing the given year by 4, what does the figure that remains (if any) show? What is the more usual division of the year? How many days are contained in each of the twelve months? When quantities are to be exchanged, what is the rule for the common mode of operation? What is the rule for canceling? What is Note 1st? What is Note 2d?

COMPOUND RULES.

§ 46. The scholar will recollect, that, in simple numbers, the denominations increase in value in the constant ratio of 10.

The peculiarity of the numbers in the preceding rule, and in the four next following, is, that they have no uniform ratio of increase, common to all denominations; but each denomina-

tion has its own peculiar ratio. These ratios are all represented in the tables of Compound Numbers.

In simple numbers, 10 units make 1 ten, 10 tens make 1 hundred, 10 hundred make 1 thousand, &c. In operations with these numbers, we therefore carry for 10.

In the table of English Money, 4 farthings make 1 penny, 12 pence 1 shilling, and 20 shillings 1 pound. For the same reason, therefore, that we carry for 10 in simple numbers, we carry for 4, 12, and 20, in operations with pounds, shillings, pence, and farthings; that is, from farthings to pence, we carry for 4, because 4 farthings make 1 penny; from pence to shillings, for 12, for a similar reason; and from shillings to pounds, for 20. The same general principle and reasoning may be applied to the other compound tables.

There is one peculiarity noticeable in *writing* compound numbers. In simple numbers, we always know that any figure sustains a *tenfold* relation to the figures next it; that is, the one on the left of it is of 10 times more value, and the one on the right, of 10 times less value, than they would be in *its place*. Hence, all that is here necessary is, that the figures preserve their proper order. In *compound numbers*, each denomination is known *only* by its own appropriate mark. There is, therefore, an obvious necessity for each denomination to be separately written.

COMPOUND ADDITION.

§ **47.** Compound Addition is an operation by which several numbers of different denominations, as pounds, shillings, pence, &c., are united together. The rule to be observed in writing down these numbers is, to place those of the same name under each other.

Let it be required to add together 3 £. 15 s. 9 d. 3 qr.; 5 £. 6 s. 8 d. 2 qr.; 8 £. 13 s. 11 d. 3 qr.; and 10 £. 12 s. 8 d. 2 qr. The following is a convenient mode of writing them:

1.

£.	*s.*	*d.*	*qr.*
3	15	9	3
5	6	8	2
8	13	11	3
10	12	8	2
28	9	2	2

The amount of the right-hand column is 10 farthings = 2 d. and 2 qr. Like simple numbers, the 2 qr. are set down, and the 2 d. added to the column of pence, the amount of which = 38 d. = 3 s. and 2 d. Setting down the 2 d. and carrying the 3 s. to the column of shillings, we make this column = 49 s. = 2 £. and 9 s. Lastly, setting down and carrying as before, we find the amount of the column of pounds to be 28, which we write at the foot of the column. We therefore find the amount of the four given numbers to be 28 £. 9 s. 2 d. 2 qr.

From the preceding example, the scholar will see the appropriateness of the following rule: —

§ 48. **Rule.** — *Write the numbers so that each denomination shall occupy a separate column. Then, commencing with the lowest denomination, add each column by itself.*

Notice, at the addition of each column, to how many of the denomination next above, the amount obtained is equal, and how many remain. Write down those that remain, and carry the other number to the next column. Proceed thus through all the denominations.

NOTE. — The whole amount of the left-hand column must be written down, if it be in the highest denomination. If it be not in the highest denomination, it should be reduced as far as practicable.

2.

£.	s.	d.	qr.
27	15	6	3
13	7	8	1
24	16	9	0
3	4	11	2
69	4	11	2

The column of farthings amounts to 6 qr. = 1 d. and 2 qr. The column of pence is 35 d. = 2 s. 11 d. The column of shillings is 44 s. = 2 £. and 4 s.; and the column of pounds = 69£. Carrying and setting down, agreeably to rule, we obtain the annexed amount.

3.

£.	s.	d.	qr.
0	15	7	0
0	14	6	3
0	8	3	1
0	18	11	2
2	17	4	2

The farthings are 6 = 1 d. 2 qr. The pence are 28 = 2 s. 4 d. The shillings are 57 = 2 £. 17 s., which is written agreeably to the above note

4.

£.	s.	d.	qr.
8	12	9	2
31	6	11	0
42	18	3	1
2	3	8	3
85	1	8	2

5.

£.	s.	d.	qr.
57	11	11	1
27	13	2	3
48	9	6	1
73	10	9	2

6.

£.	s.	d.
67	18	10
150	19	6
175	16	8
37	14	7

7. Add 1 £. 1 s. 1 d.; 10 £. 10 s. 10 d.; 100 £. 0 s. 7 d.; 73 £. 4 s. 9 d.; 43 £. 8 s. 11 d. *Ans.* 228 £. 6 s. 2 d.

8. Add 54 £. 7 s. 9 d.; 19 s. 11 d.; 144 £. 3 s. 10 d.; 132 £. 18 s.; 43 £. 6 s. 8 d. *Ans.* 375 £. 16 s. 2 d.

9. Add 444 £. 4 s. 11 d. 3 qr.; 26 £. 16 s. 4 d. 1 qr.; 372 £. 10 d. 2 qr.; 1780 £. 14 s. 6 d. 2 qr.; 200 £. 10 s. 10 d. 3 qr. *Ans.* 2824 £. 7 s. 7 d. 3 qr.

10. Add 76 £. 19 s. 1 d.; 86 £. 11 s. 7 d.; 43 £. 4 s. 8 d.; 750 £. 18 s. 6 d. *Ans.* 957 £. 13 s. 10 d.

11. Add 56 £. 18 s. 8 d.; 73 £. 11 s. 11 d.; 22 £. 12 s. 2 d.; 77 £. 17 s. 7 d.; 88 £. 18 s. 8 d. *Ans.* 319 £. 19 s.

12. Add 875 £. 16 s. 10 d. 3 qr.; 783 £. 19 s. 7 d. 2 qr.; 59 £. 17 s. 7 d. 3 qr.; 85 £. 13 s. 11 d. 1 qr.; 387 £. 14 s. 9 d. 3 qr. *Ans.* 2193 £. 2 s. 11 d.

13. Bought a horse, for 26 £. 12 s.; a yoke of oxen, for 31 £. 17 s. 8 d.; a cow, for 7 £. 16 s. 9 d.; and paid 15 s. 8 d. for a bridle. How much did they all cost me? *Ans.* 67 £. 2 s. 1 d.

14. Bought cloth, for 29 £. 6 s. 10 d.; ribbon, for 3 s. 4 d. 3 qr.; a pair of boots, for 1 £. 6 s. 3 d.; and paid 2 s. 8 d. 2 qr. for mending a pair of shoes. What was my bill for the whole? *Ans.* 30 £. 19 s. 2 d. 1 qr.

15. Bought at one time, goods to the amount of 175 £. 16 s. 11 d.; at another, to the amount of 35 £. 19 s. 8 d.; paid for carting, 7 £. 8 s. 9 d. 3 qr.; and for insurance, 3 £. 9 s. 7 d. What was the amount of my bills? *Ans.* 222 £. 14 s. 11 d. 3 qr.

16. Sold at one time, goods to the amount of 35 £. 11 s. 6 d. 3 qr.; at another, to the amount of 56 £. 19 s. 7 d. 1 qr.; at a third time, to the amount of 75 £. 1 s. 3 d.; and at a fourth, to the amount of 63 £. 13 s. 4 d. 2 qr. What was the whole amount of my sales? *Ans.* 231 £. 5 s. 9 d. 2 qr.

17. Bought a quantity of corn, for 113 £. 11 s. 11 d.; of rye, for 32 £. 19 s. 3 d.; of wheat, for 136 £. 16 s. 8 d.; and of oats, for 22 £. 14 s. 9 d. What was the whole amount? *Ans.* 306 £. 2 s. 7 d.

18. A man sold his farm for 856 £.; his sheep, for 67 £. 17 s.; his swine, for 19 £. 19 s. 11 d.; and his grain, for 36 £. 13 s. 2 d. How much money did he receive? *Ans.* 980 £. 10 s. 1 d.

TROY WEIGHT.

1.

lb.	*oz.*	*pwt.*	*gr.*
15	6	13	14
11	3	11	19
17	4	3	21
42	11	17	15
87	2	6	21

The column of grains amounts to 69 gr. = 2 pwt. 21 gr. The pwt. amount to 46 = 2 oz. and 6 pwt. The ounces amount to 26 = 2 lb. and 2 oz. The pounds amount to 87; the whole of which is to be written down.

The scholar will observe, that, as we have left the table of English Money, we have no longer to carry by 20, 12, and 4. Our carrying numbers now are 12, 20, and 24.

2.

lb.	oz.	pwt.	gr.
10	9	11	16
17	9	6	8
28	11	16	21
36	7	17	22
94	2	12	19

3.

lb.	oz.	pwt.	gr.
29	7	13	19
17	6	11	11
31	6	16	23
71	1	18	7
149	11	00	12

4. Add 1 lb. 8 oz. 18 pwt. 12 gr.; 9 lb. 6 oz. 19 pwt. 9 gr.; 11 lb. 5 oz. 3 pwt. 21 gr.; 22 lb. 6 oz. 10 pwt. 9 gr.; 15 lb. 5 oz. 12 pwt. 21 gr. *Ans.* 60 lb. 9 oz. 5 pwt.

5. Add 9 oz. 11 pwt. 16 gr.; 12 lb. 7 oz. 16 pwt. 11 gr.; 3 lb.; 71 lb. 16 pwt. 9 gr.; 16 lb. 9 oz. 17 pwt. 23 gr. *Ans.* 104 lb. 4 oz. 2 pwt. 11 gr.

6. Add 76 lb. 11 oz. 21 gr.; 3 lb. 3 oz. 3 pwt. 3 gr.; 11 lb. 7 oz. 19 pwt.; 13 lb. 9 oz. 11 pwt. 19 gr.; 14 lb. 11 oz. 17 pwt. 12 gr. *Ans.* 120 lb. 7 oz. 12 pwt. 7 gr.

7. Purchased, at one time, 4 lb. 3 oz. 16 pwt. 15 gr. of silver, and at another, 7 lb. 8 oz. 18 pwt. 23 gr.; besides a quantity of jewelry, weighing 5 lb. 11 oz. and 13 pwt. What was the whole weight? *Ans.* 18 lb. 8 pwt. 14 gr.

8. Add 3 lb. 9 oz. 13 pwt. 19 gr.; 2 lb. 10 oz. 9 pwt. 17 gr.; 6 lb. 11 oz. 18 pwt. 22 gr.; 9 oz. 11 pwt. 12 gr.; and 8 lb. 3 oz. 6 pwt. 20 gr. *Ans.* 22 lb. 9 oz. 18 gr.

9. Again, add 6 lb. 2 oz. 16 pwt. 14 gr.; 3 lb. 8 pwt. 2 gr.; 12 lb. 4 oz. 15 pwt. 22 gr.; 8 oz. 16 gr.; 5 lb. 13 gr.; and 23 gr. *Ans.* 27 lb. 4 oz. 2 pwt. 18 gr.

AVOIRDUPOIS WEIGHT.

1.

T.	cwt.	qr.	lb.	oz.
7	16	3	20	13
4	12	1	25	11
3	9	2	16	9
12	18	0	14	12
28	17	0	21	13

The amount of the ounces is 45 = 2 lb. 13 oz. The pounds amount to 77 = 2 qr. 21 lb. The qr. are 8 = 2 cwt. The cwt. are 57 = 2 tons, 17 cwt.; and the tons are 28.

2. Add 3 T. 19 cwt. 16 lb. 15 oz.; 9 T. 3 qr. 24 lb. 12 oz.; 1 T. 18 cwt. 1 qr. 26 lb. 14 oz.; 14 T. 5 cwt. 2 qr. 12 lb. 9 oz. *Ans.* 29 T. 4 cwt. 25 lb. 2 oz.

3. Add 16 cwt. 3 qr. 24 lb. 15 oz. 12 dr.; 32 cwt. 18 lb. 9 oz. 14 dr.; 7 cwt. 2 qr. 21 lb 7 oz. 9 dr.; 12 cwt. 3 qr. 11 lb. 11 oz. 12 dr. *Ans.* 69 cwt. 2 qr. 20 lb. 12 oz. 15 pwt.

4. Add 6 cwt. 2 qr. 27 lb. 12 oz. 9 dr.; 12 cwt. 3 qr. 16 lb. 9 oz. 12 dr.; 14 cwt. 1 qr. 24 lb. 14 oz. 6 dr.; 17 cwt. 3 qr. 8 lb. 15 oz. 8 dr. *Ans.* 51 cwt. 3 qr. 22 lb. 4 oz. 3 pwt.

5. Add 17 cwt. 9 lb. 4 oz. 8 dr.; 13 cwt. 3 qr. 27 lb. 15 oz. 12 dr.; 18 cwt. 2 qr. 17 lb. 13 oz. 8 dr.; 29 cwt. 1 qr. 23 lb. 12 oz. 13 dr. *Ans.* 79 cwt. 22 lb. 14 oz. 9 dr.

6. Purchased, at one time, 16 tons, 3 qr. 21 lb. of hay; at another, 9 tons, 16 cwt. 2 qr. 17 lb.; and at another, 27 tons, 13 cwt. 1 qr. 17 lb. How much did I purchase? *Ans.* 53 tons, 10 cwt. 3 qr. 27 lb.

7. Bought 36 cwt. 3 qr. 24 lb. of wool; but finding the demand large, I made three successive purchases, at each of which I bought 45 cwt. 2 qr. 16 lb. What was the amount of my purchases? *Ans.* 173 cwt. 3 qr. 16 lb.

APOTHECARIES' WEIGHT.

1.

℔.	℥.	ʒ.	℈.
2	8	3	1
5	6	2	2
6	4	7	2
8	6	5	2
23	2	3	1

2. Add 6 ℥. 2 ʒ. 2 ℈. 12 gr.; 9 ℥. 6 ʒ. 1 ℈. 15 gr.; 3 ℥. 4 ʒ. 9 gr.; 2 ℥. 7 ʒ. 2 ℈. 13 gr. *Ans.* 1 ℔. 10 ℥. 5 ʒ. 1 ℈. 9 gr.

3. Add 8 ℔. 9 ℥. 4 ʒ. 1 ℈. 14 gr.; 14 ℔. 6 ℥. 7 ʒ. 12 gr.; 1 ℔. 9 ℥. 5 ʒ. 1 ℈. 6 gr.; 8 ℔. 6 ℥. 7 ʒ. 2 ℈. 5 gr. *Ans.* 33 ℔. 9 ℥. 2 ℈. 17 gr.

4. A physician purchased the following quantities of medicine, at three different times, viz., 1 pound, 4 ounces, 5 drams; 3 pounds, 11 ounces, 6 drams, 2 scruples, 15 grains; and 7 drams, 1 scruple, and 12 grains. What was their whole weight? *Ans.* 5 ℔. 5 ℥. 3 ʒ. 1 ℈. 7 gr.

CLOTH MEASURE.

1. Add 7 yd. 3 qr. 2 na.; 9 yd. 2 qr. 3 na.; 6 yd. 1 qr.; and 8 yd. 3 qr. 3 na. *Ans.* 32 yd. 3 qr.

2. Add 1 yd. 1 qr. 1 na.; 6 yd. 3 qr. 2 na.; 12 yd. 2 qr. 3 na.; 15 yd. 1 qr. 2 na. *Ans.* 36 yd. 1 qr.

3. Add 16 e. E. 4 qr. 3 na.; 12 e. E. 3 qr. 3 na.; 18 e. E. 2 qr. 2 na.; 20 e. E. 3 qr. 1 na. *Ans.* 68 e. E. 4 qr. 1 na.

4. Add 21 e. F. 3 qr.; 13 e. F. 5 qr. 2 na.; 16 e. F. 4 qr. 3 na.; 19 e. F. 2 qr. 1 na. *Ans.* 71 e. F. 3 qr. 2 na.

5. Bought, at one purchase, 32 yd. 3 qr.; at another, 16 yd. 2 qr. 2 na.; and 24 yd. 1 qr. at another. How many yards did I purchase? *Ans.* 73 yd. 2 qr. 2 na.

6. Bought of one man, 12 ells English, 4 qr. 3 na.; and of each of two others, 27 ells English, 3 qr. and 3 na. How many ells did I purchase? *Ans.* 68 e. E. 2 qr. 1 na.

7. Received from France 12 ells, 4 qr. of broadcloth; 17

ells, 5 qr. 3 na. of cassimere; and 19 ells, 2 qr. 3 na. of silks. How many ells were there in the three articles purchased?

Ans. 50 ells, 0 qr. 2 na.

DRY MEASURE

1. Add 3 bu. 2 pk. 5 qt. 1 pt.; 7 bu. 3 pk. 7 qt. 2 pt.; 11 bu. 2 pk. 4 qt. 1 pt.; 6 bu. 1 pk. 3 qt. *Ans.* 29 bu. 2 pk. 5 qt.

2. Add 8 bu. 3 pk. 7 qt.; 9 bu. 2 pk. 4 qt. 1 pt.; 6 bu. 3 pk. 6 qt. 1 pt.; 4 bu. 2 pk. 2 qt. *Ans.* 30 bu. 4 qt.

3. Add 15 bu. 1 pk. 6 qt. 1 pt.; 127 bu. 3 pk. 7 qt. 1 pt.; 12 bu. 2 pk. 5 qt.; 16 bu. 2 pk. 7 qt. 1 pt.

Ans. 172 bu. 3 pk. 2 qt. 1 pt.

WINE MEASURE.

1. Add 4 T. 1 P. 1 hhd. 42 gal. 3 qt.; 6 T. 1 hhd. 24 gal. 2 qt.; 8 T. 1 P. 18 gal. 3 qt.; 12 T. 1 hhd. 23 gal.

Ans. 32 T. 46 gal.

2. Add 15 hhd. 27 gal. 3 qt. 1 pt. 2 gi.; 20 hhd. 13 gal. 2 qt. 3 gi.; 12 hhd. 16 gal. 1 qt. 1 pt. 1 gi.; 132 hhd. 54 gal. 3 qt. 3 gi. *Ans.* 180 hhd. 49 gal. 3 qt. 1 gi.

3. Add 140 hhd. 46 gal. 2 qt. 1 pt. 1 gi.; 127 hhd. 15 gal. 3 qt. 3 gi.; 263 hhd. 29 gal. 1 qt. 1 pt. 2 gi.; 42 hhd. 27 gal. 3 qt. 3 gi *Ans.* 573 hhd. 56 gal. 3 qt. 1 gi.

LONG MEASURE.

1. Add 12 L. 2 m. 5 fur. 36 rd.; 9 L. 1 m. 7 fur. 24 rd.; 15 L. 6 fur. 17 rd.; 30 L. 2 m. 4 fur. 26 rd. *Ans.* 68 L. 2 m. 23 rd.

2. Add 4 m. 4 fur. 23 rd. 5 yd. 2 ft. 9 in. 2 b. c.; 9 m. 3 fur. 30 rd. 6 yd. 1 ft. 10 in. 1 b. c.; 6 m. 2 fur. 36 rd. 4 yd. 2 ft. 8 in. 2 b. c.; 5 m. 2 fur. 27 rd. 4 yd. 2 ft. 8 in. 2 b. c.

Ans. 25 m. 6 fur. 1 ft. 1 in. 1 b. c.

3. Add 9 m. 6 fur. 12 rd. 3 yd.; 4 m. 7 fur. 26 rd. 2 yd.; 12 m. 4 fur. 32 rd. 5 yd.; 7 m. 1 fur. 38 rd. 4 yd.

Ans. 34 m. 4 fur. 30 rd. 3 yd.

4. Add 18 L. 2 m. 5 fur. 18 rd. 3 yd. 2 ft. 7 in. 1 b. c.; 21 L. 4 fur. 30 rd. 4 yd. 1 ft. 9 in. 2 b. c.; 32 L. 1 m. 7 fur. 31 rd. 5 yd. 2 ft. 4 in. 1 b. c.; 76 L. 2 m. 3 fur. 39 rd. 4 yd. 2 ft. 10 in. 2 b. c. *Ans.* 149 L. 1 m. 6 fur. 1 rd. 2 yd. 2 ft. 2 in.

LAND OR SQUARE MEASURE.

1. Add 7 A. 2 roods, 36 rd. 24 yd. 6 ft. 72 in.; 8 A. 3 roods, 23 rd. 20 yd. 4 ft. 91 in.; 5 A. 1 rood, 15 rd. 17 yd. 8 ft. 108 in.;

12 A. 3 roods, 12 rd. 13 yd. 6 ft. 22 in.; 15 A. 17 rd. 9 yd. 7 ft. 136 in. *Ans.* 49 A. 3 roods, 25 rd. 25½ yd. 6 ft. 141 in.

2. Add 9 A. 3 roods, 21 rd. 6 yd.; 12 A. 2 roods, 37 rd. 11 yd.; 1 A. 1 rood, 39 rd. 12 yd.; 15 A. 3 roods, 12 rd. 16 yd.; 8 A. 1 rood, 9 rd. 12 yd.

Ans. 48 A. 39 rd. 26 yd. 6 ft. 108 in.

3. Add 46 A. 29 rd. 11 yd. 7 ft.; 27 A. 3 roods, 26 rd. 6 yd. 4 ft.; 18 A. 2 roods, 32 rd. 6 yd. 4 ft.; 25 A. 3 roods, 30 rd. 7 yd. 5 ft.; 15 A. 17 rd. 9 yd. 7 ft.

Ans. 133 A. 3 roods, 15 rd. 11 yd. 6 ft. 108 in.

4. Add 34 A. 2 roods, 33 rds. 7 yd. 6 ft.; 44 A. 30 rd. 7 yd. 5 ft.; 15 A. 3 roods, 29 rd. 10 yd. 5 ft.; 33 A. 3 roods, 36 rd. 8 yd. 7 ft.; 44 A. 3 roods, 37 rd. 8 yd. 7 ft.

SOLID MEASURE.

1. Add 99 ft. 420 in.; 78 ft. 864 in.; 320 ft. 740 in.; 950 ft. 222 in.; 48 ft. 12 in. *Ans.* 11 C. 88 ft. 530 in.

2. Add 11 C. 72 ft. 726 in.; 12 C. 16 ft. 317 in. 113 C. 17 ft. 36 in.; 4 C. 117 ft. 1372 in.; 116 C. 8 ft. 96 in.

Ans. 257 C. 103 ft. 819 in.

3. Add 3 C. 99 ft. 777 in.; 66 C. 77 ft. 333 in.; 122 C. 116 ft. 1240 in.; 372 C. 108 ft. 1617 in.; 12 C. 96 ft. 456 in.

Ans. 578 C. 114 ft. 967 in.

TIME.

1. Add 2 yr. 9 mo. 27 d. 13 h. 22 m. 56 sec.; 8 yr. 11 mo. 3 d. 21 h. 43 m. 21 sec.; 1 yr. 11 mo. 18 d. 23 h. 69 m. 52 sec.; 2 yr. 8 mo. 26 d. 17 h. 36 m. 8 sec.

Ans. 16 yr. 5 mo. 17 d. 4 h. 52 m. 17 sec.

2. Add 15 d. 21 h. 43 m. 51 sec.; 23 d. 17 h. 55 m. 56 sec.; 6 d. 19 h. 59 m. 49 sec.; 16 d. 15 h. 43 m. 36 sec.

Ans. 63 d. 3 h. 23 m. 12 sec.

3. Add 5 w. 6 d. 9 h. 7 m. 59 sec.; 6 w. 5 d. 21 h. 39 m. 27 sec.; 22 w. 4 d. 23 h. 36 m. 42 sec.; 11 w. 3 d. 16 h. 17 m. 18 sec. *Ans.* 46 w. 6 d. 22 h. 41 m. 26 sec.

4. Add 14 w. 3 d. 17 h. 8 m. 47 sec.; 15 w. 2 d. 20 h. 40 m. 43 sec.; 16 w. 1 d. 22 h. 45 m. 23 sec.; 17 w. 4 d. 23 h. 18 m. 22 sec. *Ans.* 63 w. 6 d. 11 h. 53 m. 15 sec.

CIRCULAR MOTION.

1. Add 6 S. 23°. 42′. 39″; 8 S. 26°. 54′. 36″; 3 S. 19°. 11′. 9″; 2 S. 21°. 13′. 23″. *Ans.* 22 S. 1°. 1′. 47″.

2. Add 4 S. 9°. 29′. 41″; 6 S. 7°. 43′. 2″; 11 S. 8°. 51′. 59″; 12 S. 13°. 27′. 17″. *Ans.* 34 S. 9°. 31′. 59″.

3. Add 3 S. 22°. 40′. 37″; 11 S. 29°. 59′. 57″; 4 S. 23°. 18′. 38″; 8 S. 6°. 13′. 15″. *Ans.* 28 S. 22°. 12′. 27″.

APPLICATION.

Ex. 1. What is the amount of 23 £. 11 d.; 13 £. 17 s. 3 qr.; 16 s. 8 d.; and 11 d. 3 qr.? *Ans.* 37 £. 15 s. 7 d. 2 qr.

2. Bought the following quantities of oil, viz., 12 gal. 3 qt.; 2 hhd. 42 gal. 2 qt. 1 pt.; and 13 hhd. 56 gal. What was the whole amount? *Ans.* 16 hhd. 48 gal. 1 qt. 1 pt.

3. Add together 250 £. 18 s. 9 d. 3 qr.; 16 £. 7 s. 2 qr.; 21 £. 19 s. 3 d.; 18 s. 6 d.; and 36 £.
Ans. 326 £. 3 s. 7 d. 1 qr.

4. What is the amount of 5 cwt. 3 qr. 27 lb.; 2 qr. 29 lb.; 12 cwt. 1 qr. 17 lb.; and 36 cwt. 16 lb.? *Ans.* 55 cwt. 1 qr. 5 lb.

5. Bought, at one time, 7 bu. 3 pk. of wheat; at another, 9 bu. 1 pk.; and had, previously, in each of two bins, 6 bu. and 3 pk. What was the whole amount? *Ans.* 30 bu. 2 pk.

6. Sold one cow, for 10 £. 15 s. 6 d.; another, for 6 £. 19 s. 11 d.; and a colt, for 12 £. 6 s. 4 d. How much did they all amount to? *Ans.* 30 £. 1 s. 9 d.

7. Bought four casks of wine, of which the first contained 42 gal. 2 qt. 1 pt.; the second, 65 gal. 1 pt.; the third, 50 gal. 3 qt.; and the fourth, 55 gal. 1 qt. 1 pt. How many gallons did I purchase? *Ans.* 213 gal. 3 qt. 1 pt.

8. Purchased three pieces of land. The first contained 17 acres, 1 rood, and 35 rods; the second, 36 acres, 2 roods, 21 rods; and the third, 46 acres and 37 rods. How much land did I purchase? *Ans.* 100 acres, 1 rood, 13 rods.

QUESTIONS.—In what ratio do simple numbers increase? What is the peculiarity of Compound Numbers? Where are the ratios of increase and decrease of compound numbers given? Why do you carry for 10 in simple numbers? Why do you carry for 4, 12, and 20, in the table of English Money? What peculiarity noticeable in *writing* compound numbers? What only is necessary in writing simple numbers? How are compound numbers known? How must each denomination, therefore, be written? What is Compound Addition? How are numbers to be written? What is the rule? What is the note following the rule?

COMPOUND SUBTRACTION.

§ **49.** The scholar has now become acquainted with Compound Addition; and he was previously acquainted with the Simple rules. He needs, therefore, to be informed only, that

Compound Subtraction sustains the same relation to Compound Addition, that Simple Subtraction does to Simple Addition. It is the subtracting of numbers of different denominations.

In this rule, instead of constantly borrowing 10, when the lower figure is the larger, he must borrow as many units as are required of the denomination he is subtracting, to make a unit of the next higher denomination; that is, when it becomes necessary to borrow a number in subtracting *farthings*, 4 is the number always required; in subtracting *pence*, 12 is the number; and in *shillings*, 20; and in like manner in other denominations.

§ **50. Rule.**—*Place the less quantity under the greater, so that each denomination shall stand under one of its own name or kind. Begin at the right, and proceed in all respects as in Simple Subtraction, except in borrowing when the lower figure is the larger; in doing which, instead of* 10, *(the number borrowed in Simple Subtraction,) borrow as many units as make one of the next higher denomination. Whenever a number is borrowed,* 1 *is to be carried to the next lower figure.*

		£.	s.	d.	qr.
Ex. 1.	From	16	18	8	1
	Take	8	16	11	3
	Ans.	8	1	8	2

First, I cannot take 3 qr. from 1 qr. I therefore add 4 qr. to the upper figure, and take the 3 from the amount, 5, and obtain a remainder of 2. I carry 1 to the next lower figure, viz., 11, which makes it 12, and proceed to take it from the figure above, but find it impracticable. I therefore add 12 to the upper figure, 8, making it 20; and from this amount, subtract 12, and obtain the 8 in the answer. Again, I carry 1 to the next number, 16, which makes it 17, and take this from the number above, and obtain a remainder of 1. I here borrowed nothing, and have nothing to carry; therefore, 8 from 16 leaves 8.

2.				3.				4.			
£.	s.	d.	qr.	£.	s.	d.	qr.	£.	s.	d.	qr.
35	11	9	3	46	13	7	1	74	0	9	2
17	9	11	2	21	17	9	3	72	19	11	3

5. From 99 £. 16 s. 8 d. 3 qr., take 77 £. 17 s. 7 d. 1 qr.
Ans. 21 £. 19 s. 1 d. 2 qr.

6. From 33 £. 12 s. 3 d. 1 qr., take 13 £. 8 s. 9 d. 3 qr.
Ans. 20 £. 3 s. 5 d. 2 qr.

7. From 94 £. 11 s. 8 d., take 72 £. 9 s. 11 d.
Ans. 22 £. 1 s. 9 d.

8. A certain man owed 75 £. 13 s. 9 d., and paid of this sum 39 £. 19 s. 11 d. How much remained due?
Ans. 35 £. 13 s. 10 d.

9. Received of three individuals the following sums of money, viz., of A, 16 £. 12 s. 8 d. 3 qr.; of B, 21 £. 17 s. 9 d.; and of C, 46 £. 19 s. I afterwards paid D 58 £. 13 s. 9 d. 2 qr. How much had I left? *Ans.* 26 £. 15 s. 8 d. 1 qr.

10. The following sums are due to A, viz., 136 £. 15 s. 11 d.; 450 £. 8 s. 6 d.; 356 £. 17 s. 10 d. 2 qr.; and 12 £. 9 s. 4 d. He is indebted to B, 67 £. 14 s. 9 d. 2 qr.; to C, 24 £. 11 s. 3 d.; and to D, 571 £. 11 s. 11 d. How much is due to him, more than he owes? *Ans.* 292 £. 13 s. 8 d.

TROY WEIGHT.

1. From 14 lb. 9 oz. 19 pwt. 16 gr., take 10 lb. 11 oz. 16 pwt. 23 gr. *Ans.* 3 lb. 10 oz. 2 pwt. 17 gr.

2. From 46 lb. 11 oz. 13 pwt., take 13 lb. 9 oz. 17 pwt. *Ans.* 33 lb. 1 oz. 16 pwt.

3. From 9 lb. 11 oz. 11 pwt. 21 gr., take 4 lb. 3 oz. 19 pwt. 23 gr. *Ans.* 5 lb. 7 oz. 11 pwt. 22 gr.

4. From 36 lb. 7 oz. 14 pwt. 17 gr., take 17 lb. 9 oz. 17 pwt. 22 gr. *Ans.* 18 lb. 9 oz. 16 pwt. 19 gr.

5. From 8 lb. 9 oz. 16 pwt. 11 gr., take 2 lb. 11 oz. 19 pwt. 23 gr. *Ans.* 5 lb. 9 oz. 16 pwt. 12 gr.

6. From 11 oz. 9 pwt. 18 gr., take 10 oz. 16 pwt. 23 gr. *Ans.* 12 pwt. 19 gr.

AVOIRDUPOIS WEIGHT.

1. From 6 T. 13 cwt. 11 lb. 12 oz. 13 dr., take 4 T. 17 cwt. 3 qr. 5 lb. 13 oz. 14 dr. *Ans.* 1 T. 15 cwt. 1 qr. 5 lb. 14 oz. 15 dr.

2. From 12 cwt. 3 qr. 19 lb. 13 oz. 14 dr., take 9 cwt. 2 qr. 21 lb. 11 oz. 6 dr. *Ans.* 3 cwt. 26 lb. 2 oz. 8 dr.

3. From 31 cwt. 1 qr. 23 lb. 14 oz. 15 dr., take 26 cwt. 3 qr. 25 lb. 15 oz. 8 dr. *Ans.* 4 cwt. 1 qr. 25 lb. 15 oz. 7 dr.

4. From 9 T. 17 cwt. 3 qr. 20 lb. 15 oz. 8 dr., take 2 T. 15 cwt. 2 qr. 26 lb. 12 oz. 15 dr. *Ans.* 7 T. 2 cwt. 22 lb. 2 oz. 9 dr.

5. Having in my possession 45 cwt. 3 qr. 17 lb. of cheese, I sold 32 cwt. 27 lb. How much remained? *Ans.* 13 cwt. 2 qr. 18 lb.

APOTHECARIES' WEIGHT.

1. From 5 ℔. 9 ℥. 5 ʒ. 2 ℈. 16 gr., take 3 ℔. 11 ℥. 6 ʒ. 2 ℈. 15 gr. *Ans.* 1 ℔. 9 ℥. 7 ʒ. 1 gr.

2. From 12 ℔. 6 ℥. 5 ʒ. 2 ℈. 15 gr., take 8 ℔. 9 ℥. 4 ʒ. 1 ℈. 17 gr. *Ans.* 3 ℔. 9 ℥. 1 ʒ. 18 gr.

3. From 31 ℔. 5 ℥. 1 ʒ. 1 ℈. 12 gr., take 20 ℔. 10 ℥. 5 ʒ. 2 ℈. 15 gr. *Ans.* 10 ℔. 6 ℥. 3 ʒ. 1 ℈. 17 gr.

CLOTH MEASURE.

1. From 9 yd. 3 qr. 3 na., take 4 yd. 2 qr. 3 na. *Ans.* 5 yd. 1 qr.
2. From 10 e. F. 5 qr. 1 na., take 6 e. F. 3 qr. 3 na. *Ans.* 4 e. F. 1 qr. 2 na.
3. From 21 e. E. 1 qr. 2 na., take 16 e. E. 4 qr. 3 na. *Ans.* 4 e. E. 1 qr. 2 na.
4. From 16 e. F. 1 qr. 3 na., take 8 e. F. 2 qr. 2 na. *Ans.* 7 e. F. 5 qr. 1 na.

DRY MEASURE.

1. From 15 bu. 3 pk. 3 qt. 2 gi., take 12 bu. 2 pk. 5 qt. 1 pt. 3 gi. *Ans.* 3 bu. 5 qt. 3 gi.
2. From 26 bu. 5 qt. 1 gi., take 23 bu. 3 pk. 7 qt. 1 pt. 2 gi. *Ans.* 2 bu. 5 qt. 3 gi.
3. From 30 bu. 2 pk. 7 qt. 1 pt. 3 gi., take 16 bu. 3 pk. 5 qt. 1 pt. 3 gi. *Ans.* 13 bu. 3 pk. 2 qt.

WINE MEASURE.

1. From 3 T. 1 P. 1 hhd. 27 gal. 3 qt. 1 pt. 1 gi., take 2 T. 2 hhd. 47 gal. 1 qt. 1 pt. 3 gi. *Ans.* 1 T. 1 hhd. 43 gal. 1 qt. 1 pt. 2 gi.
2. From 27 hhd. 19 gal. 3 qt. 1 gi., take 16 hhd. 43 gal. 1 qt. 1 pt. 3 gi. *Ans.* 10 hhd. 39 gal. 1 qt. 2 gi.
3. From 137 hhd. 42 gal. 1 qt. 3 gi., take 128 hhd. 56 gal. 3 qt. 1 pt. 1 gi. *Ans.* 8 hhd. 48 gal. 1 qt. 1 pt. 2 gi.
4. From 175 hhd. 59 gal. 1 pt. 3 gi., take 21 hhd. 37 gal. 3 qt. 1 pt. 2 gi. *Ans.* 154 hhd. 21 gal. 1 qt. 1 gi.

LONG MEASURE.

1. From 15 m. 4 fur. 27 rd. 4 yd. 2 ft. 11 in. 1 b.c., take 12 m. 3 fur. 36 rd. 3 yd. 1 ft. 9 in. 2 b. c. *Ans.* 3 m. 31 rd. 1 yd. 1 ft. 1 in. 2 b. c.
2. From 32 m. 5 fur. 39 rd. 1 yd. 2 ft. 3 in., take 27 m. 2 fur. 4 rd. 3 yd. 1 ft. 11 in. *Ans.* 5 m. 3 fur. 34 rd. 3 yd. 1 ft. 10 in.
3. From 17 L. 2 m. 3 fur. 19 rd. 3 yd. 1 ft. 7 in., take 12 L. 1 m. 7 fur. 35 rd. 4 yd. 2 ft. 8 in. *Ans.* 5 L. 3 fur. 23 rd. 3½ yd. 1 ft. 11 in.
4. From 31 L. 3 fur. 15 rd. 4 yd. 2 ft. 7 in. 2 b. c., take 27 L. 2 m. 5 fur. 17 rd. 1 yd. 2 ft. 9 in. 2 b. c. *Ans.* 3 L. 5 fur. 38 rd. 2 yd. 2 ft. 10 in.

LAND OR SQUARE MEASURE.

1. From 9 A. 36 rd. 14 yd. 8 ft., take 4 A. 39 rd. 6 yd. 4 ft.
Ans. 4 A. 157 rd. 8 yd. 4 ft.

2. From 74 A. 3 R. 27 rd. 16 yd., take 64 A. 2 R. 31 rd. 12 yd. *Ans.* 10 A. 36 rd. 4 yd.

3. From 12 A. 1 R. 16 rd. 15 yd., take 9 A. 2 R. 17 rd. 16 yd. *Ans.* 2 A. 2 R. 38 rd. 29 yd. 2 ft. 36 in.

SOLID MEASURE.

1. From 21 C. 62 ft. 856 in., take 16 C. 115 ft. 972 in.
Ans. 4 C. 74 ft. 1612 in.

2. From 56 C. 110 ft. 1462 in., take 19 C. 36 ft. 472 in.
Ans. 37 C. 74 ft. 990 in.

3. From 8 C. 100 ft. 8 in., take 1 C. 101 ft. 1560 in.
Ans. 6 C. 126 ft. 176 in.

TIME.

1. From 16 yr. 8 mo. 3 w. 5 d. 13 h., take 7 yr. 9 mo. 2 w. 6 d. 21 h. *Ans.* 8 yr. 11 mo. 5 d. 16 h.

2. From 12 mo. 2 w. 1 d. 15 h. 21 m. 35 sec., take 9 mo. 3 w. 2 d. 16 h. 22 m. 36 sec. *Ans.* 2 mo. 2 w. 5 d. 22 h. 58 m. 59 sec.

3. From 19 yr. 152 d. 13 h. 42 m. 21 sec., take 16 yr. 256 d. 19 h. 36 m. 56 sec. *Ans.* 2 yr. 260 d. 18 h. 5 m. 25 sec.

4. From 45 yr. 67 d. 17 h. 50 m. 15 sec., take 36 yr. 36 d. 22 h. 46 m. 45 sec. *Ans.* 9 yr. 30 d. 19 h. 3 m. 30 sec.

CIRCULAR MOTION.

1. From 8 S. 18°. 42′. 36″., take 6 S. 26°. 11′. 52″.
Ans. 1 S. 22°. 30′. 44″.

2. From 9 S. 27°. 36′. 51″., take 1 S. 29°. 42′. 52″.
Ans. 7 S. 27°. 53′. 59″.

3. From 8 S. 19°. 38′. 46″., take 6 S. 21°. 42′. 50″.
Ans. 1 S. 27°. 55′. 56″.

4. From 11 S. 21°. 49′. 59″., take 6 S. 27°. 13′. 21″.
Ans. 4 S. 24°. 36′. 38″.

PROMISCUOUS EXAMPLES.

Ex. 1. I have in my possession 46 £. 19 s. 11 d. How much shall I have left, after paying a debt of 27 £. 13 s. 9 d.?
Ans. 19 £. 6 s. 2 d.

2. Received 156 £. 3 s. 8 d., after which I paid out 137 £ 15 s. 10 d. How much remained in my possession?
Ans. 18 £. 7 s. 10 d.

3. Lent a friend 16 £. 17 s. 6 d. On the following day, he paid me 5 £. 13 s. 11 d.; one week after, he made another payment of 7 £. 5 s. 10 d. How much then remained due?

Ans. 3 £. 17 s. 9 d.

4. Bought a wagon, for 9 £. 11 s., and sold the same for 13 £. 5 s. How much did I gain? *Ans.* 3 £. 14 s.

5. Bought a horse, for 56 £. 15 s., and exchanged the same for a colt, and received 46 £. 11 s. in money. How much did the colt cost me? *Ans.* 10 £. 4 s.

6. A man, having 15 tons, 13 cwt. 3 qr. of hay, sold 5 tons, 10 cwt., and gave 3 cwt. 3 qr. to a friend. How much had he left? *Ans.* 10 tons.

7. Bought 7 cwt. 3 qr. 16 lb. of rice, at one purchase, and 9 cwt. 1 qr. 27 lb. at another; of this, 2 cwt. 16 lb. was stolen, and of the remainder, I sold 11 cwt. 2 qr. 21 lb. How much had I left? *Ans.* 3 cwt. 2 qr. 6 lb.

8. Three men bought a piece of land, for 450 £. 16 s. 10 d., of which two of them paid, each, 69 £. 17 s. 6 d. What did the third man pay? *Ans.* 311 £. 1 s. 10 d.

9. I owned a tract of land, containing 356 acres, 3 roods, and 30 rods; from this I sold to A 127 acres, 2 roods, and to B 27 acres, 1 rood, and 36 rods. How much remained?

Ans. 201 acres, 3 roods, 34 rods.

10. A father, 46 years, 9 months, and 27 days old, has two sons; the elder of whom is 19 years, 3 months, and 13 days old; and the younger, 7 years, 10 months, and 21 days. How much does the father's age exceed the sum of his sons'?

Ans. 19 y. 7 m. 23 d.

11. Bought a quantity of cotton, which, at the price agreed upon, came to 20 £. 4 s. In pay for this I gave a quantity of rice, worth 15 £. 18 s., and the balance in cash. How much money did I pay? *Ans.* 4 £. 6 s.

12. A merchant bought a piece of cloth, containing 40 yards, from which he sold 36 yd. 1 qr. 2 na. How much had he left?

Ans. 3 yd. 2 qr. 2 na.

13. Sold from a pile of wood, containing 40 cords, 64 feet, 39 cords, 32 feet. How much remained? *Ans.* 1 cord, 32 ft.

14. Bought 560 acres of land, for 940 £. From this I sold to A 120 acres, 2 roods, and 16 rods, for 300 £.; and to B 150 acres, 1 rood, and 24 rods, for 297 £. 10 s. and 6 d. How much land remains in my possession, and how much has it cost me?

Ans. 289 acres; cost, 342 £. 9 s. 6 d.

QUESTIONS.—What is Compound Subtraction? Instead of 10, how many are to be borrowed here? What is the number borrowed in subtracting farthings? Why? What in subtracting pence? Shillings? And why? What is the rule?

COMPOUND MULTIPLICATION.

§ 51. The peculiarity of Compound Numbers having been fully explained, and multiplication of Simple Numbers being also understood, no other definition of this rule is needed than is conveyed by the *name*.

A simple inquiry presents itself, viz., Are both the given numbers compound? To answer this, the scholar needs only to be informed, that, in multiplication, the *multiplier* is always a simple number, showing how many times the multiplicand is to be repeated. The multiplicand, therefore, *only*, is compound. The product, as it is formed by repeating the multiplicand, must necessarily be of the same denomination with it.

Take the following illustration:—Multiply 8 s. 9 d. 2 qr. by 4.

PERFORMED.

	8 s.	9 d.	2 qr.
			4
1 £.	15 s.	2 d.	0 qr.

In this example, we say, four times 2 farthings are 8 farthings, and 8 qr. = 2 d. and 0 qr. remain. We therefore write down a cipher, and carry 2. Again, four times 9 d. are 36 d., and 2 d. to carry make 38 d. = 3 s. and 2 d. The 2 d. is written down, and the 3 s. carried. Lastly, four times 8 s. are 32 s., and 3 s. to carry make 35 s. = 1 £. 15 s., which is written down, as seen in the answer. If, now, in the above example, 8 s. 9 d. 2 qr. had been given as the price of one yard of cloth, and the scholar had been required to find the price of 4 yards, the operation would have been the same. The number of yards only decides how many *times* the price of one yard is to be repeated.

CASE I.

§ 52. When the Multiplier or Simple Number is not greater than 12.

Rule.—*Multiply the compound number by the simple one, commencing with the lowest denomination, and carrying from one denomination to another, as in the preceding Compound Rules.*

Ex. 1. Multiply 9 £. 16 s. 8 d. 2 qr. by 9.

PERFORMED.

9 £.	16 s.	8 d.	2 qr.	
			9	
88 £.	10 s.	4 d.	2 qr.	*Ans.*

Explanation.—9 times 2 qr. = 18 qr. = 4 d. 2 qr.; 9 times 8 d. = 72 d., and 4 d. from the farthings added, make 76 d. = 6 s. 4 d. Again, 9 times 16 s. = 144 s., and the 6 s. obtained from the pence, make 150 s. = 7 £. 10 s.; and, lastly, 9 times 9 £. = 81 £., and 81 £. + 7 £. = 88. The product, therefore, is as given above, viz., 88 £. 10 s. 4 d. 2 qr.

2. Multiply 16 £. 11 s. 9 d. 3 qr. by 3.

PERFORMED.

16 £.	11 s.	9 d.	3 qr.	
			3	
49 £.	15 s.	5 d.	1 qr.	*Ans.*

3. Multiply 1 £. 11 s. 6 d. 2 qr. by 5. *Ans.* 7 £. 17 s. 8 d. 2 qr.
4. Multiply 11 s. 9 d. by 3. *Ans.* 1 £. 15 s. 3 d.
5. Multiply 15 £. 10 s. 8 d. by 2. *Ans.* 31 £. 1 s. 4 d.
6. Multiply 5 s. 6 d. by 9. *Ans.* 2 £. 9 s. 6 d.
7. What cost 4 gallons of wine, at 8 s. 7 d. per gallon? *Ans.* 1 £. 14 s. 4 d.
8. What cost 5 cwt. of raisins, at 1 £. 7 s. 9 d. 2 qr. per cwt.? *Ans.* 6 £. 18 s. 11 d. 2 qr.
9. What cost 8 yards of broadcloth, at 1 £. 2 s. 3 d. per yard? *Ans.* 8 £. 18 s.
10. What cost 11 tons of hay, at 2 £. 1 s. 10 d. per ton? *Ans.* 23 £. 0 s. 2 d.
11. What cost 12 bushels of wheat, at 9 s. 10 d. per bushel? *Ans.* 5 £. 18 s.

CASE II.

§ 53. When the Multiplier, or Simple Number, is a Composite Number greater than 12.

Rule.—*Separate the simple number or multiplier into its component parts, and multiply first by one of these parts, and the product of this multiplication by the others in succession. The last product will be the answer required.*

Note.—It will generally be found more expeditious to divide the multiplier into two parts only. Should it, however, be large, as 125 or 1728, it may be divided into more than two parts, viz., 125 into three 5's, and 1728 into three 12's.

Ex. 1. Multiply 6 s. 10 d. by 28. 28 = 4 × 7. Therefore,

	6 s.	10 d.	
		7	
2 £.	7 s.	10 d.	= product of 7.
		4	
9 £.	11 s.	4 d.	= prod. of 4 times 7 = 28.

2. What cost 27 yards of cloth, at 7 s. 6 d. per yard? 27 = 9 × 3. Therefore,

	7 s.	6 d.	
		9	
3 £.	7 s.	6 d.	= price of 9 yards.
		3	
10 £.	2 s.	6 d.	= price of 3 times 9 yd. = 27 yd

3. What cost 32 yards, at 9 s. 9 d. per yard? *Ans.* 15 £. 12 s.

4. What cost 20 yards, at 3 s. 6 d. per yard? *Ans.* 3 £. 10 s.

5. What cost 36 gallons, at 5 s. 8 d. per gallon? *Ans.* 10 £. 4 s.

6. What cost 63 yards, at 7 s. 6 d. 2 qr. per yard? *Ans.* 23 £. 15 s. 1 d. 2 qr.

7. What cost 72 yards, at 3 s. 11 d. per yard? *Ans.* 14 £. 2 s

8. What cost 144 yards, at 1 £. 4 s. 2 d. per yard? *Ans.* 174 £.

9. Sold 21 men, each, 3 qr. 16 lb. of sugar. How many cwt. did I sell? *Ans.* 18 cwt. 3 qr.

10. Suppose 27 young lads to have lived each 6 years, 9 months, 8 days, and 11 hours; how many days, &c., have they all lived, allowing 30 days to a month? *Ans.* 182 yr. 10 mo. 18 d. 9 h.

CASE III.

§ 54. When the Multiplier is more than 12, and is not a Composite Number.

Rule. — *Take any two or more numbers, whose product will come as near as possible to the given number or multiplier, without exceeding it, and, having multiplied the given price of one yard, pound, &c., by them, retain their product. Then multiply the same given price by the number wanting to make up the entire multiplier, and add the product to the preceding product. Their sum will be the product required.*

Note. — If preferred, the given quantity, or multiplier, may be multiplied by each denomination of the compound quantity separately, and the several products, reduced to the highest denomination, may be added together.

Ex. 1. What will 51 yards of cloth cost, at 3 s. 6 d. per yard?

The two numbers whose product comes nearest to 51, are 7 and 7, and their product is 49. Consequently, if we multiply 3 s. 6 d by 7, and their product by 7 again, we shall obtain the cost of 49 yards. There will then be the cost of two yards wanting. This will be obtained by multiplying 3 s. 6 d., the price of 1 yard, by 2. The operation is thus performed: —

	3 s.	6 d.
		7
1 £.	4 s.	6 d. = the price of 7 yards.
		7
8 £.	11 s.	6 d. = price of 49 yards.
	7 s.	0 d. = price of 2 yards.
8 £.	18 s.	6 d. = price of 51 yards.

Or, the sum may be solved by the note, thus: 51 yards, at 6 d. per yard, = 306 d. = 1 £. 5 s. 6 d.; and 51 yards, at 3 s. per yard, = 153 s. = 7 £. 13 s.; and 1 £. 5 s. 6 d. added to 7 £. 13 s., as before, gives 8 £. 18 s. 6 d.

2. What cost 47 yards of cloth, at 17 s. 9 d. per yard?

Ans. 41 £. 14 s. 3 d.

Rule 2d. — *Observe how many units, tens, and hundreds, compose the given quantity;* then,

1st. *Multiply the given price by the units, and place the product by itself.*

2d. *Multiply the given price by* 10, *and the product of this by the number of tens in the given quantity, and write the product under the product of units.*

3d. *Multiply the product of* 10 *already obtained by* 10, (by which we obtain the price of 100,) *and this product by the number of hundreds, and place the result under the product of units and tens.*

4th. *Add the three products together; their sum will be the required answer.*

3. What is the value of 327 lb. of butter, at 1 s. 3 d. per lb.? The quantity given is 7 units, 2 tens, and 3 hundreds.

Therefore,

1 s. 3 d.	
7	
8 s. 9 d. = price of 7 lb.	

	1 s.	3 d.
		10
	12 s.	6 d.
		2
1 £.	5 s.	0 d. = price of 20 lb.

	12 s.	6 d. = price of 10 lb.
		10
6 £.	5 s.	0 d. = price of 100 lb.
		3
18 £.	15 s.	0 d. = price of 300 lb.

	£.	s.	d.	
Then,				
		8	9	= prod. of 7 units.
	1	5	0	= prod. of 2 tens.
	18	15	0	= prod. of 3 hundreds.
	20 £.	8 s.	9 d.	= prod. of 327.

4. What would 463 yards of cloth cost, at 3 s. 4 d. per yard? *Ans.* 77 £. 3 s. 4 d.
5. What would 222 cwt. of sugar cost, at 18 s. 8 d. per cwt.? *Ans.* 207 £. 4 s.
6. What cost 23 gallons of melasses, at 3 s. 6 d. per gallon? *Ans.* 4 £. 6 d.
7. What cost 94 yards of cloth, at 1 £. 9 s. 4 d. per yard? *Ans.* 137 £. 17 s. 4 d.
8. What cost 59 yards of baize, at 3 s. 4 d. per yard? *Ans.* 9 £. 16 s. 8 d.
9. What cost 29 cwt. of sugar, at 17 s. 8 d. per cwt.? *Ans.* 25 £. 12 s. 4 d.
10. What cost 78 yards of cloth, at 9 s. 3 d. per yard? *Ans.* 36 £. 1 s. 6 d.
11. What cost 65 cwt. of sugar, at 19 s. 3 d. per cwt.? *Ans.* 62 £. 11 s. 3 d.

12. Seventeen men brought each a load of hay to market, weighing 17 cwt. 3 qr. and 21 lb., and received each for his load, 5 £. 8 s. 3 d. What quantity of hay did they all bring? and how much money did they all receive? *Ans.* They brought 15 T. 4 cwt. 3 qr. 21 lb., and received 92 £. 3 d.

EXAMPLES OF WEIGHTS AND MEASURES.

1. What is the weight of 5 hogsheads of sugar, each weighing 7 cwt. 3 qr. 16 lb.? *Ans.* 39 cwt. 1 qr. 24 lb.
2. What is the weight of 9 chests of tea, each weighing 3 cwt. 2 qr. 9 lb.? *Ans.* 32 cwt. 25 lb.
3. In 8 piles of wood, each containing 4 cords and 56 feet, how many cords and feet? *Ans.* 35 cords, 64 feet.
4. Multiply 15 yards, 3 qr. and 2 nails, by 9. *Ans.* 142 yd. 3 qr. 2 nails.
5. Multiply 20 years, 5 months, 3 weeks, and 6 days, by 14. *Ans.* 286 yr. 11 mo. 2 w.
6. In 10 fields, containing each 12 acres, 2 roods, and 16 rods, how many acres? *Ans.* 126.
7. In 7 casks, containing each 42 gallons, 3 quarts, and 1 pint, how many gallons, &c.? *Ans.* 300 gal. 1 pint.

QUESTIONS. — What is always the nature of the multiplier? Which is the compound number, the multiplier or multiplicand? What is the nature of the product? In case the quantity by which you multiply is yards, what does the

number of yards decide? What is Case I.? What is the rule? What is Case II.? What is the rule? What is the note under Case II.? What is Case III.? What is the rule? What note follows the rule? What is Rule 2d, Case III.? No direct definition has been given of Compound Multiplication: will the scholar give one?

COMPOUND DIVISION.

§ **55.** This is the last of the Compound rules, and is the reverse of the preceding. In this rule a compound number is given as a dividend, and a simple number as a divisor; and by the operation, the dividend is resolved into as many equal parts as there are units in the divisor. The quotient is always one of these equal parts, and is therefore a compound number, each figure of which is of the same denomination as the figure or figures in the dividend from which it was obtained.

CASE I.

§ **56.** When the Divisor or Simple Number is 12, or less than 12.

Rule.—*Divide the highest denomination first. If, after dividing this, there be a remainder, reduce it to the next lower denomination, adding the figures of the dividend in that denomination to it, and divide again. Proceed in the same manner through all denominations; the number obtained will be the one required.*

Ex. 1. Divide 17 £. 11 s. 5 d. by 8.

PERFORMED.

8) 17 £. 11 s. 5 d.
———————————
2 £. 3 s. $11\frac{1}{8}$ d.

17 £. ÷ 8 = 2, and 1 remains. What remains of any number or quantity, must obviously be of the same kind as the quantity itself. Therefore, the 1 is one pound, = 20 s.; and 20 s. + 11 s. = 31 s., and 31 ÷ 8 = 3 s., and 7 s. remain. Again, 7 s. = 84 d., and 84 d. + 5 d. = 89 d.; and 89 ÷ 8 = 11, and 1 remainder, = $11\frac{1}{8}$ d. Therefore the answer is as given above, viz., 2 £. 3 s. $11\frac{1}{8}$ d.

2. Divide 25 £. 18 s. 9 d. by 6.

PERFORMED.

6) 25 £. 18 s. 9 d.
———————————
4 £. 6 s. 5 d. 2 qr.

3. Divide 13 £. 19 s. 4 d. by 4. *Ans.* 3 £. 9 s. 10 d.

4. Divide 140 £. 12 s. 9 d. by 12.
Ans. 11 £. 14 s. 4 d. 3 qr.

5. Divide 73 £. 16 s. 11 d. 3 qr. by 9.
Ans. 8 £. 4 s. 1 d. 1$\frac{2}{9}$ qr.

6. Divide 12 cwt. 3 qr. 12 lb. by 10. *Ans.* 1 cwt. 1 qr. 4 lb.

7. Eleven men own equal shares of 36 hhd. 42 gal. and 2 qt. of wine. What is each man's share?
Ans. 3 hhd. 21 gal. 1$\frac{5}{11}$ gills.

8. Seven men bought 16 hhd. 24 gal. 3 qt. of wine, for which they paid 45 £. 18 s. 6 d.; each man paying the same money, and, consequently, entitled to an equal share of wine. What was each man's share? and how much money did he pay? *Ans.* His share was 2 hhd. 21 gal. 2$\frac{1}{7}$ qt., and he paid 6 £. 11 s. 2 d. 2$\frac{2}{7}$ qr.

CASE II.

§ 57. When the Divisor is a Composite Number greater than 12.

Rule. — *Resolve the divisor into its component parts, and divide the compound number by each of these parts in succession. The quotient arising from the first division, will form a dividend for the second; and so on.*

Ex. 1. Divide 26 £. 16 s. 8 d. by 21. The factors of 21 are 7 and 3, because $7 \times 3 = 21$. Therefore,

```
7 ) 26 £.   16 s.   8 d.
    -------------------
3 )  3 £.   16 s.   8 d.
    -------------------
     1 £.    5 s.   6⅔ d. = the quotient of 26 £. 16 s. 8 d.
                            ÷ 21, and is the Ans.
```

2. Divide 47 £. 15 s. 8 d. by 24. *Ans.* 1 £. 19 s. 9 d. 3$\frac{1}{3}$ qr.
3. Divide 85 £. 11 s. 11 d. by 16. *Ans.* 5 £. 6 s. 11 d. 3$\frac{3}{4}$ qr.
4. Divide 128 £. 9 s. by 42. *Ans.* 3 £. 1 s. 2 d.
5. Divide 15 £. 18 s. 9 d. by 72. *Ans.* 4 s. 5$\frac{1}{8}$ d.
6. Divide 5 £. 10 s. 3 d. by 81. *Ans.* 1 s. 4 d. 1$\frac{1}{3}$ qr.
7. Divide 7 £. 19 s. 9 d. by 96. *Ans.* 1 s. 7 d. 3$\frac{7}{8}$ qr.
8. Divide 27 £. 16 s. by 32. *Ans.* 17 s. 4 d. 2 qr.

CASE III.

§ 58. When the Divisor is large, and not a Composite Number.

Rule. — *Divide the whole compound quantity by the whole divisor; reducing the remainders, after the division of each denomination, to the next lower denomination, as directed in Case I.*

Ex. 1. Divide 8 £. 18 s. 6 d. by 51.

PERFORMED.

```
51 ) 8 £. 18 s. 6 d. ( 0 £. 3 s. 6 d.
     20
    ---
    178 = the shillings in 8 £. 18 s.
    153
    ---
     25 = shillings remaining.
     12
    ---
    306 = pence in 25 s. 6 d.
    306
    ---
    000
```

The pounds are first reduced to shillings, and the given shillings are added. The 178 shillings are thus produced. This, divided by 51, gives 3 as a quotient figure, and 25 as a remainder. After a second reduction, 306 pence are obtained, which contains the divisor six times. Thus the answer obtained is 3 s. 6 d.

2. Divide 41 £. 14 s. 3 d. by 47. *Ans.* 17 s. 9 d.
3. Divide 4 £. 1 s. 5 d. 2 qr. by 23. *Ans.* 3 s. 6 d. 2 qr.
4. Divide 137 £. 17 s. 4 d. by 94. *Ans.* 1 £. 9 s. 4 d.
5. Divide 36 £. 1 s. 6 d. by 78. *Ans.* 9 s. 3 d.
6. Divide 10 £. 5 s. 8 d. by 59. *Ans.* 3 s. 5 d. 3 qr.+
7. Divide 25 £. 12 s. 4 d. by 29. *Ans.* 17 s. 8 d.
8. Divide 61 £. 12 s. by 65. *Ans.* 18 s. 11 d. 1 qr. +

EXAMPLES IN WEIGHTS AND MEASURES.

1. Divide 5 hhd. 42 gal. 3 qt. equally among 4 men.
Ans. 1 hhd. 26 gal. 1 qt. 1 pt. 2 gi.
2. Divide 14 cwt. 1 qr. 12 lb. by 5.
Ans. 2 cwt. 3 qr. 13 lb. 9 oz. 9$\frac{3}{5}$ dr.
3. Divide 27 yd. 1 qr. 2 na. by 7. *Ans.* 3 yd. 3 qr. 2$\frac{4}{7}$ na.
4. Divide 156 bu. 3 pk. 6 qt. by 18. *Ans.* 8 bu. 2 pk. 7 qt.
5. Divide 9 hhd. 28 gal. 2 qt. by 12. *Ans.* 49 gal. 2 qt. 1 pt.
6. Divide 16 cwt. 3 qr. 18 lb. by 32. *Ans.* 2 qr. 3 lb. 3 oz.
7. If 27 loads of hay weigh 30 tons, 8 cwt. 2 qr. 23 lb., what is the weight of one load? *Ans.* 1 ton, 2 cwt. 2 qr. 5 lb.
8. A man traveled 17 leagues, 1 mile, 4 furlongs, and 21 poles, in 21 hours. At what rate did he travel per hour?
Ans. 2 m. 4 fur. and 1 pole.
9. Nine men own 56 lb. 6 oz. and 17 pwt. of silver. What will each man receive if the whole quantity be equally divided among them? *Ans.* 6 lb. 3 oz. 8 pwt. 13$\frac{1}{3}$ gr.
10. Bought 15 loads of hay, the whole weight of which was 12 tons, 15 cwt. 3 qr. 16 lb. Supposing them all to have been equal, what was the weight of each? *Ans.* 17 cwt. 6$\frac{2}{3}$ lb.
11. If a man's income be 86 £. 18 s. 10 d. per year, what is it per calendar month? *Ans.* 7 £. 4 s. 10$\frac{5}{6}$ d.
12. If I pay 15 £. 3 s. 8 d. for 56 pairs of gloves, what is one pair worth? *Ans.* 5 s. 5 d. $\frac{4}{7}$ qr.

13. If a hogshead of wine cost 33£. 12 s., what is the price of a gallon? *Ans.* 10 s. 8 d.

14. If 42 yards of cloth cost 21£. 18 s. 8 d., what was the cost per yard? *Ans.* 10 s. 5 d. $1\frac{1}{3}$ qr.

15. If 16 men cut 53 cords 69 feet of wood in 2 days, what did each man cut per day? *Ans.* 1 cord, $86\frac{5}{32}$ feet.

APPLICATION OF THE FOUR PRECEDING RULES.

1. A silversmith sold to his customer 3 dozen silver spoons, each weighing 3 oz. 3 pwt. 16 gr.; $1\frac{1}{2}$ dozen tea-spoons, each weighing 14 pwt. 20 gr.; 3 silver cups, each weighing 20 oz. 18 pwt. In return, he received old silver to the amount of 8 lb. 11 oz. 19 pwt. For how much ought he to receive pay? *Ans.* 6 lb. 10 oz. 14 pwt.

2. Bought the following articles at the prices mentioned, viz.:

	£.	s.	d.
4 cwt. of sugar, at 2£. 4 s. 8 d. per cwt., . .			
3 hhd. of melasses, at 2 s. 4 d. per gallon, .			
5 lb. of green tea, at 7 s. 6 d. per pound, . .			
12 lb. of raisins, at 2 s. per pound, ...			
42 yd. of cotton cloth, at 1 s. 6 d. per yard, .			
27 lb. of ham, at 1 s. 3 d. per pound, ...			
What was the amount of my bill? *Ans.*	38£.	17 s.	11 d.

3. Bought of James Rankin,	£.	s.	d.
27 yards of broadcloth, at 1£. 9 s. per yard,			
42 yards of Irish linen, at 4 s. 6 d. per yard,			
36 hats, valued each at 18 s., ...			
30 pairs of shoes, at 7 s. 8 d. per pair, ...			
What was the amount of my bill? *Ans.*	92£.	10 s.	0 d.

4. A, owning 100 acres of land, divided it into 8 equal parts, and sold each part for $22.50 per acre. How many acres were there in each part? and what was the value of the same? *Ans.* 12 A. 2 R.; value, $281.25.

5. Out of a pipe of wine, a merchant sold 36 gallons, 3 quarts, and 1 pint at each of three different times. He then filled 15 bottles, holding 1 pint and 2 gills each. How much remained? *Ans.* 12 gal. 2 qt. and 2 gills.

6. Bought 144 pairs of shoes, for 96£. What was the price of one pair? *Ans.* 13 s. 4 d.

7. A person, dying, left real estate to the amount of 2356£. 19 s. 9 d., and personal property to the amount of 3184£. 12 s. 8 d. In his will, he directed that his wife should receive one

sixth of the whole, and that the remainder should be equally divided among his four daughters. What was the share of each?

Ans. The widow, 923 £. 12 s. $\frac{4}{8}$ d.; and the daughters, each, 1154 £. 10 s. $1\frac{1}{24}$ d.

QUESTIONS.—How does Compound Division compare with Compound Multiplication? What is given as a dividend? What as a divisor? Into what is the dividend resolved by the operation? What is always one of these equal parts? Is the quotient a simple or compound number? How may you know the denomination of each figure in the quotient? What is Case I.? What is the rule? What is Case II.? What is the rule? What is Case III.? What rule?

VULGAR FRACTIONS.

§ **59.** 1. When a unit, or single object, is divided into a number of equal parts, each of these parts is a fraction.

If it be divided into two equal parts, each part is called a *half*, and is thus written: $\frac{1}{2}$.

If it be divided into three equal parts, each is called a *third*, and is thus written: $\frac{1}{3}$.

If the whole be separated into six equal parts, each part is called a *sixth*, and if into eight equal parts, an *eighth*, of the whole, and is thus written: $\frac{1}{6}$, $\frac{1}{8}$.

When more parts than one are to be expressed, the figure above the line designates their number, thus, $\frac{5}{6}$; by which expression we are to understand that the unit is divided into six equal parts, and that five of these parts are included in the fraction.

The fraction, therefore, is used to express *parts of units*, and is represented by two numbers, one standing below, and the other above, a short horizontal line. The number below the line is called the *denominator*, and shows the number of equal parts into which the unit is divided. The number above the line is called the *numerator*, and shows how many of these equal parts are included in the fraction, or make up its value. Thus, of the fraction $\frac{5}{9}$, the lower number shows a unit to be divided into nine equal parts; and the upper number, that five of these parts are included in the fraction.

These two numbers, when spoken of collectively, are called the *terms* of the fraction.

§ **60.** 2. Fractions are divided into six kinds, viz., Proper, Improper, Simple, Compound, Mixed, and Complex.

A Proper Fraction is one whose numerator is less than its denominator: as, $\frac{3}{8}$.

An Improper Fraction is one whose numerator equals or exceeds its denominator; as, $\frac{7}{5}$.

A Simple Fraction consists of one expression, and is either proper or improper: as, $\frac{4}{5}$, or $\frac{6}{5}$.

A Compound Fraction is the fraction of a fraction; as, $\frac{1}{3}$ of $\frac{2}{5}$ of $\frac{4}{5}$. It may consist of any number of simple fractions.

A Mixed Number consists of a whole number and fraction written together; as, $6\frac{2}{3}$, $25\frac{1}{7}$, &c.

A Complex Fraction is one that has a fraction in its numerator or denominator, or both; as, $\frac{2\frac{1}{4}}{3}$, $\frac{4}{7\frac{1}{4}}$, $\frac{9\frac{2}{3}}{8\frac{1}{4}}$, &c.

§ 61. 3. The denominator shows the number of equal parts into which the unit is divided; and the numerator how many of these parts are expressed in the fraction. Consequently, the greater the numerator, the denominator being given, the greater the value of the fraction; and the less the numerator, the less the value of the fraction. If the denominator be 8, and the numerator 1, the value expressed is $\frac{1}{8}$, or one eighth part of a unit; if the numerator be 2, the value expressed is $\frac{2}{8}$, or two eighth parts of a unit; if it be 4, the value is $\frac{4}{8}$, or four eighth parts of a unit; and if it be 6, the value is $\frac{6}{8}$, or six eighth parts of a unit.

The value of a fraction is, therefore, the quotient arising from dividing the numerator by the denominator, and always increases in the same ratio as the numerator, so long as the denominator remains unaltered.

We may therefore express any value, not only less than a unit, but equal to, and even greater than a unit, by a fraction. Thus, if we take 9 as the denominator of a fraction, and any number less than 9 as a numerator of the same, the value expressed is always less than a unit; as, $\frac{4}{9}$; or, if 9 be taken as the numerator, we obtain the fraction $\frac{9}{9}$, which, as the unit was divided into 9 parts only, is obviously equal to 1. Again, we may suppose more than a single unit of the same kind to be divided in the same manner, and their parts united in one fraction, and thus obtain fractions of any value more than a unit. If two units be thus divided into seven equal parts, and three parts of the one be united to all the parts of the other, the fraction would be $\frac{10}{7}$; or, if all the parts of each be united, it would be $\frac{14}{7}$, which is equal to 2; or, if three units were thus divided, all their parts would produce the fraction $\frac{21}{7} = 3$.

The only consideration which limits the value of a fraction, is the number of equal parts united in the same expression.

From the preceding, it is obvious that the *value* of a fraction is increased in the same ratio as the numerator; hence,

§ **62.** 4. *A fraction is multiplied by a whole number, by multiplying the numerator only.*

In accordance with the above principle, the scholar may multiply the following examples: —

1. Multiply $\frac{1}{2}$ by 3. *Ans.* $\frac{3}{2}$.	7. Multiply $\frac{7}{13}$ by 7.
2. Multiply $\frac{2}{3}$ by 5. *Ans.* $\frac{10}{3}$.	8. Multiply $\frac{5}{6}$ by 12.
3. Multiply $\frac{3}{4}$ by 3. *Ans.* $\frac{9}{4}$.	9. Multiply $\frac{11}{12}$ by 12.
4. Multiply $\frac{1}{6}$ by 9. *Ans.* $\frac{9}{6}$.	10. Multiply $\frac{7}{8}$ by 8.
5. Multiply $\frac{8}{12}$ by 6. *Ans.* $\frac{48}{12}$.	11. Multiply $\frac{1}{3}$ by 3. *Ans.* $\frac{3}{3}=1$.
6. Multiply $\frac{11}{12}$ by 9. *Ans.* $\frac{99}{12}$.	12. Multiply $\frac{1}{7}$ by 7. *Ans.* $\frac{7}{7}=1$.

From the last two examples, it is obvious that a fraction is multiplied by a number equal to its own denominator, by rejecting that denominator, and retaining only the numerator.

It should always be an object with the scholar, to preserve the terms of a fraction *as small as is possible, and express the true value.* This was not regarded in the above examples. A little experience will show, that to increase or diminish the value of a fraction, it is only necessary to make the numerator larger or smaller, *compared with the denominator.* Suppose it be required to multiply the fraction $\frac{1}{6}$ by 2. By the above rule, the product would be $\frac{2}{6}$, which is equal in value to $\frac{1}{3}$, and this is at once obtained by dividing the denominator by 2, instead of multiplying the numerator as above, thus: $\frac{1}{6 \div 2 = 3}$; therefore,

§ **63.** *A fraction is multiplied by a whole number, by dividing the denominator by that number.*

The following examples will illustrate this principle: —

1. Multiply $\frac{3}{4}$ by 2. *Ans.* $\frac{3}{2}$.	6. Multiply $\frac{2}{21}$ by 7. *Ans.* $\frac{2}{3}$.
2. Multiply $\frac{2}{6}$ by 3. *Ans.* $\frac{2}{2}$, or 1.	7. Multiply $\frac{5}{32}$ by 4.
3. Multiply $\frac{5}{8}$ by 4. *Ans.* $\frac{5}{2}$.	8. Multiply $\frac{12}{27}$ by 9.
4. Multiply $\frac{9}{10}$ by 5. *Ans.* $\frac{9}{2}$.	9. Multiply $\frac{1}{20}$ by 5.
5. Multiply $\frac{3}{16}$ by 8. *Ans.* $\frac{3}{2}$.	10. Multiply $\frac{9}{128}$ by 8.

§ **64.** *The value of a fraction may, therefore, be multiplied by a whole number, either by multiplying the numerator, or dividing the denominator by that number.*

Note. — **The denominator should always be divided, whenever it can be done without a *remainder*.**

§ **65.** 5. *A fraction is divided by a whole number, by dividing the numerator by that number.*

This needs no explanation. If we divide a number by 2, we take a half, and if by 3, a third, of that number; that is, the divisor always shows what part of the dividend is taken; therefore, $\frac{12}{13} \div 2 = \frac{6}{13}$, and $\frac{12}{13} \div 3 = \frac{4}{13}$.

The following examples will illustrate the operation of the above principle:—

1. Divide $\frac{3}{6}$ by 3.	*Ans.* $\frac{1}{6}$.	5. Divide $\frac{16}{17}$ by 4.
2. Divide $\frac{4}{7}$ by 2.	*Ans.* $\frac{2}{7}$.	6. Divide $\frac{20}{23}$ by 5.
3. Divide $\frac{8}{15}$ by 8.	*Ans.* $\frac{1}{15}$.	7. Divide $\frac{3}{4}$ by 7.
4. Divide $\frac{12}{13}$ by 6.	*Ans.* $\frac{2}{13}$.	

In this last example, the scholar will find a difficulty. He cannot divide the numerator in any way, except to place it over the 7, in the form of a fraction, as will hereafter be explained; and this would make one fraction the numerator of another fraction. When, therefore, the divisor will not divide the numerator without a remainder, a more convenient mode of operating is desirable. It will be remembered that division is the reverse of multiplication; and since we can multiply fractions by dividing the denominator, we will try the effect of dividing fractions by multiplying the denominator. Let it be required to divide $\frac{3}{12}$ by 3. By dividing as above, we obtain $\frac{1}{12}$ as the quotient, viz., $\frac{3}{12} \div 3 = \frac{1}{12}$. By the mode we propose to try, we obtain $\frac{3}{36}$. It therefore remains to show that $\frac{1}{12} = \frac{3}{36}$. If any object be first divided into 12 equal parts, and then each of these 12 parts be divided into 3 equal parts, it is plain that the whole would be divided into 36 equal parts, and that each twelfth part would make 3 thirty-sixth parts; therefore, $\frac{1}{12} = \frac{3}{36}$; hence,

§ **66.** *A fraction is divided by a whole number, by multiplying its denominator by that number.*

This principle may be applied to the following sums:—

1. Divide $\frac{3}{5}$ by 6.	*Ans.* $\frac{3}{30}$.	5. Divide $\frac{9}{10}$ by 8.
2. Divide $\frac{1}{8}$ by 2.	*Ans.* $\frac{1}{16}$.	6. Divide $\frac{11}{21}$ by 7.
3. Divide $\frac{7}{8}$ by 3.	*Ans.* $\frac{7}{24}$.	7. Divide $\frac{13}{11}$ by 11.
4. Divide $\frac{1}{12}$ by 5.	*Ans.* $\frac{1}{60}$.	8. Divide $\frac{15}{16}$ by 6.

By uniting the two preceding principles, we have the following more comprehensive principle, viz.—

§ **67.** *The value of a fraction is divided by a whole number, by dividing the numerator, or by multiplying the denominator by that number.*

NOTE. — The numerator should always be divided, when it can be done *without a remainder.* In all other cases, the denominator should be multiplied.

From the preceding remarks and illustrations, we learn *that whatever operation is performed on the numerator of a fraction, the* SAME OPERATION IS PERFORMED *on the* VALUE *of the fraction; but that the effect produced on the* VALUE *of any fraction is the* REVERSE *of the* OPERATION PERFORMED ON ITS DENOMINATOR.

§ **68.** 6. *A fraction is multiplied by a fraction, by multiplying the numerators together for a new numerator, and the denominators for a new denominator.*

For example: Let it be required to multiply $\frac{1}{3}$ by $\frac{3}{4}$. Agreeably to the principles already explained, if I multiply the denominator of the fraction $\frac{1}{3}$ by 4, the other denominator, I shall obtain $\frac{1}{4}$ of that quantity, viz., $\frac{1}{12}$; and if I multiply this quantity, viz., $\frac{1}{12}$, by 3, the other numerator, I shall make this value three times as large, that is, it will become $\frac{3}{12}$; therefore, $\frac{3}{12}$ is $\frac{3}{4}$ of $\frac{1}{3}$, or $\frac{1}{3} \times \frac{3}{4} = \frac{3}{12}$.

In accordance with the above, the scholar may multiply the following fractions: —

1. Multiply $\frac{1}{3}$ by $\frac{5}{6}$. *Ans.* $\frac{5}{18}$.
2. Multiply $\frac{3}{4}$ by $\frac{6}{7}$. *Ans.* $\frac{18}{28}$.
3. Multiply $\frac{2}{5}$ by $\frac{7}{9}$. *Ans.* $\frac{14}{45}$.
4. Multiply $\frac{9}{11}$ by $\frac{3}{4}$. *Ans.* $\frac{27}{44}$.
5. Multiply $\frac{8}{9}$ by $\frac{1}{12}$. *Ans.* $\frac{8}{108}$.
6. Multiply $\frac{1}{13}$ by $\frac{1}{3}$. *Ans.* $\frac{1}{39}$.
7. Multiply $\frac{11}{13}$ by $\frac{6}{7}$.
8. Multiply $\frac{8}{11}$ by $\frac{2}{3}$.
9. Multiply $\frac{15}{16}$ by $\frac{1}{10}$.
10. Multiply $\frac{3}{7}$ by $\frac{9}{10}$.

§ **69.** 7. *A fraction is divided by another fraction, by inverting the divisor, and multiplying them together, as before.*

A unit is contained in the fraction $\frac{3}{4}$, three fourths of once; $\frac{1}{2}$ is consequently contained in the same fraction twice as often, viz., $\frac{6}{4}$ of a time; and $\frac{1}{3}$, three times as often, viz., $\frac{9}{4}$ of a time; which fractions are obviously obtained by multiplying $\frac{3}{4}$ by $\frac{1}{2}$ and $\frac{1}{3}$ inverted. Again, suppose it be required to find how many times $\frac{3}{4}$ is contained in $\frac{7}{8}$. As before, a unit, or 1, is contained in $\frac{7}{8}$ seven eighths of a time; $\frac{1}{4}$ would be contained in it four times as often, viz., $\frac{28}{8}$ of a time, and $\frac{3}{4}$ would be contained in the same only one third as often as $\frac{1}{4}$, viz., $\frac{28}{24}$ of a time, $= 1\frac{4}{24}$, or $1\frac{1}{6}$. This result is obtained by inverting the divisor $\frac{3}{4}$, and multiplying it into the dividend, $\frac{7}{8}$; thus, $\frac{7}{8} \times \frac{4}{3} = \frac{28}{24}$.

The following examples may now be performed: —

1. Divide $\frac{1}{2}$ by $\frac{2}{3}$. *Ans.* $\frac{3}{4}$.
2. Divide $\frac{5}{6}$ by $\frac{7}{8}$. *Ans.* $\frac{40}{42}$.
3. Divide $\frac{4}{5}$ by $\frac{9}{10}$. *Ans.* $\frac{40}{45}$.
4. Divide $\frac{3}{4}$ by $\frac{11}{12}$. *Ans.* $\frac{36}{44}$.

5. Divide $\frac{1}{12}$ by $\frac{5}{8}$. *Ans.* $\frac{8}{60}$.
6. Divide $\frac{1}{40}$ by $\frac{1}{20}$. *Ans.* $\frac{20}{40}$.
7. Divide $\frac{3}{5}$ by $\frac{4}{7}$.
8. Divide $\frac{6}{7}$ by $\frac{9}{11}$.
9. Divide $\frac{6}{7}$ by $\frac{1}{2}$.
10. Divide $\frac{12}{13}$ by $\frac{1}{6}$.

§ **70.** 8. *If the numerator and denominator of any fraction be both multiplied or both divided by the same number, the value of the fraction will not be altered.*

Of this principle no explanation is necessary. The value of the fraction being the quotient arising from dividing the numerator by the denominator, it is obvious that, if both the terms be doubled, or repeated any number of times, the value of the quotient will not be affected.

REDUCTION OF FRACTIONS.

CASE I.

§ 71. TO REDUCE FRACTIONS TO THEIR LOWEST TERMS; OR, TO FIND THE LOWEST TERMS BY WHICH THE VALUE OF A GIVEN FRACTION CAN BE EXPRESSED.

Rule.—*Divide both numerator and denominator by any number that will divide them both* WITHOUT REMAINDER; *then divide the quotients obtained, in the same manner, and so continue to do till there is no number greater than* 1 *that will divide them. The last quotient will be the numerator and denominator required.*

Ex. 1. Reduce $\frac{48}{64}$ to its lowest terms. Operation: $\frac{48}{64} \div 8 = \frac{6}{8}$; and $\frac{6}{8} \div 2 = \frac{3}{4}$, its lowest term.

2. Reduce $\frac{124}{196}$ to its lowest terms. *Ans.* $\frac{31}{49}$.
3. Reduce $\frac{1296}{5184}$ to its lowest terms. *Ans.* $\frac{1}{4}$.
4. Reduce $\frac{96}{108}$ to its lowest terms. *Ans.* $\frac{8}{9}$.
5. Reduce $\frac{49}{56}$ to its lowest terms. *Ans.* $\frac{7}{8}$.
6. Reduce $\frac{960}{1240}$ to its lowest terms. *Ans.* $\frac{24}{31}$.
7. Reduce $\frac{76}{80}$ to its lowest terms. *Ans.* $\frac{19}{20}$.
8. Reduce $\frac{7465}{67185}$ to its lowest terms. *Ans.* $\frac{1}{9}$.
9. Reduce $\frac{78943}{315772}$ to its lowest terms. *Ans.* $\frac{1}{4}$.
10. Reduce $\frac{6786}{67860}$ to its lowest terms. *Ans.* $\frac{1}{10}$.
11. Reduce $\frac{468}{1060}$ to its lowest terms. *Ans.* $\frac{117}{265}$.
12. Reduce $\frac{104}{56}$ to its lowest terms. *Ans.* $\frac{13}{7}$.
13. Reduce $\frac{65}{91}$ to its lowest terms. *Ans.* $\frac{5}{7}$.

CASE II.

§ 72. To reduce a Whole Number, or a Mixed Quantity, to an Improper Fraction.

Rule.—*If the given quantity be a whole number, multiply it by the proposed denominator; the product will be the numerator; but, if it be a mixed quantity, multiply the whole number by the denominator of the fraction, and to the product add the given numerator; then, under the number thus produced, write the denominator.*

Ex. 1. Reduce 21 to a fraction whose denominator is 9. Operation: $21 \times 9 = 189$, the numerator; the fraction, therefore, is $\frac{189}{9}$.

2. Reduce $8\frac{1}{3}$ to an improper fraction. Operation: $8 \times 3 = 24$, and $24 + 1 = 25$, the numerator; therefore, $\frac{25}{3}$ is the answer.

3. Reduce $16\frac{3}{4}$ to an improper fraction. *Ans.* $\frac{67}{4}$.
4. Reduce $17\frac{13}{14}$ to an improper fraction. *Ans.* $\frac{251}{14}$.
5. Reduce $47\frac{1}{16}$ to an improper fraction. *Ans.* $\frac{753}{16}$.
6. Reduce $135\frac{1}{6}$ to an improper fraction. *Ans.* $\frac{811}{6}$.
7. Reduce $1\frac{15}{22}$ to an improper fraction. *Ans.* $\frac{37}{22}$.
8. Reduce $1728\frac{21}{27}$ to an improper fraction. *Ans.* $\frac{46677}{27}$.
9. Reduce $9\frac{1}{2}$ to an improper fraction. *Ans.* $\frac{19}{2}$.
10. Reduce $12\frac{5}{6}$ to an improper fraction. *Ans.* $\frac{77}{6}$.
11. Reduce 8 to a fraction whose denominator shall be 9. *Ans.* $\frac{72}{9}$.
12. Reduce 16 to fraction whose denominator shall be 12. *Ans* $\frac{192}{12}$.

CASE III.

§ 73. To reduce an Improper Fraction to a Whole or Mixed Number.

Rule.—*Divide the numerator by the denominator; the quotient will be the whole number. If there be any remainder, place it over the denominator, at the right of the whole number.*

Note.—The true quotient includes both the whole number and fraction. In all cases of division, therefore, the remainder, if any, constitutes the numerator of a fraction of which the divisor is the denominator.

Ex. 1. Reduce $\frac{141}{17}$ to a mixed number.

OPERATION.

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17)141(8
   136
   ---
     5 rem.; therefore, 8 5/17 is the answer.
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2. Reduce $\frac{346}{23}$ to a mixed quantity. *Ans.* $15\frac{1}{23}$.
3. Reduce $\frac{79}{6}$ to a mixed quantity. *Ans.* $13\frac{1}{6}$.
4. Reduce $\frac{49}{7}$ to a mixed quantity or whole number. *Ans.* 7.
5. Reduce $\frac{456}{79}$ to a mixed quantity. *Ans.* $5\frac{61}{79}$.
6. Reduce $\frac{3564}{346}$ to a mixed number. *Ans.* $10\frac{104}{346}$.
7. Reduce $\frac{847}{15}$ to a mixed number. *Ans.* $56\frac{7}{15}$.
8. Reduce $\frac{1246}{22}$ to a mixed number. *Ans.* $56\frac{7}{11}$.
9. Reduce $\frac{108}{9}$ to a whole number. *Ans.* 12.
10. Reduce $\frac{73}{12}$ to a mixed number. *Ans.* $6\frac{1}{12}$.
11. Reduce $\frac{1728}{6}$ to a whole or mixed number. *Ans.* 288.
12. Reduce $\frac{56789}{21}$ to its proper number. *Ans.* $2704\frac{5}{21}$.

CASE IV.

§ 74. To reduce Compound Fractions to Simple ones.

Rule 1st.—*Multiply all the numerators together for a new numerator, and all the denominators for a new denominator, and reduce the new fraction to its lowest terms, by Case I.*

Ex. 1. Reduce $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{5}{6}$ to a simple fraction.

Performed: $2 \times 3 \times 5 = 30$, the new numerator; and $3 \times 4 \times 6 = 72$, the new denominator; therefore, $\frac{30}{72}$ is the fraction required, but susceptible of being expressed in lower terms; therefore, $\frac{30}{72} \div 6 = \frac{5}{12}$, *Ans.*

Compound fractions may be reduced to simple ones, however, much more expeditiously, by canceling. The labor of reducing to lower terms is thereby avoided.

Rule 2d.—*Draw a horizontal line, and place all the numerators above the line, and all the denominators below it. Cancel the numbers as far as practicable, as taught in the Rule for Canceling; then make the product of the numbers remaining above the line the new numerator, and the product of those remaining below, the new denominator.*

Note 1.—If there be nothing remaining above the line, after canceling, 1 will always be the numerator of the new fraction. The same is true of the denominators.

Ex. 2. Reduce $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{4}{5}$ to a simple fraction.

Statement: $\dfrac{1.\ 3.\ 4}{2.\ 4.\ 5}$. Canceled: $\dfrac{1.\ 3.\ \cancel{4}}{2.\ \cancel{4}.\ 5}$. *Ans.* $\frac{3}{10}$.

Example 1st stated and solved by canceling:

$\dfrac{2.\ 3.\ 5}{3.\ 4.\ 6}$. Canceled: $\dfrac{\cancel{2}.\ \cancel{3}.\ 5}{\cancel{3}.\ \underset{2}{\cancel{4}}.\ 6}$. *Ans.* $\frac{5}{12}$.

3. Reduce $\frac{6}{7}$ of $\frac{1}{2}$ of $\frac{14}{24}$ to a simple fraction.

Statement: $\dfrac{6.\ 1.\ 14}{7.\ 2.\ 24}$. Canceled: $\dfrac{6.\ 1.\ \cancel{14}}{\cancel{7}.\ \cancel{2}.\ \underset{4}{\cancel{24}}}$. *Ans.* $\frac{1}{4}$.

NOTE 2.—Whenever the product of any two numbers on one side of the line will cancel any number on the opposite side, they may be so canceled; as, in the last example, 7 and 2 below the line cancel 14 above it.

4. Reduce $\frac{5}{9}$ of $\frac{12}{13}$ of $\frac{4}{5}$ of $\frac{7}{8}$ to a simple fraction.

Statement: $\dfrac{5.\ 12.\ 4.\ 7}{9.\ 13.\ 5.\ 8}$. Canceled: $\dfrac{\cancel{5}.\ \overset{\cancel{6}}{\cancel{12}}.\ \overset{2}{\cancel{4}}.\ 7}{\underset{3}{\cancel{9}}.\ 13.\ \cancel{5}.\ \underset{\cancel{2}}{\cancel{8}}}$,

and $7 \times 2 = 14$, numerator; and $13 \times 3 = 39$, denominator; therefore the new fraction is $\frac{14}{39}$.

5. Reduce $\frac{6}{7}$ of $\frac{13}{8}$ of $\frac{1}{12}$ of $\frac{8}{15}$ of $\frac{7}{13}$ to a simple fraction. *Ans.* $\frac{1}{30}$.

6. Reduce $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$ of $\frac{7}{8}$ of $\frac{8}{9}$ of $\frac{9}{10}$ to a simple fraction. *Ans.* $\frac{1}{10}$.

7. Reduce $\frac{16}{18}$ of $\frac{5}{7}$ of $\frac{9}{10}$ of $\frac{8}{5}$ of $\frac{6}{9}$ of $\frac{4}{5}$ to a simple fraction. *Ans.* $\frac{256}{525}$.

NOTE 3.—If any term of a compound fraction be a mixed number, it must be reduced to an improper fraction before stating.

8. Reduce $\frac{1}{3}$ of $\frac{6}{7}$ of $4\frac{2}{3}$ of $\frac{4}{5}$ of $\frac{3}{4}$ to a simple fraction. $4\frac{2}{3} = \frac{14}{3}$; therefore, statement: $\dfrac{1.\ 6.\ 14.\ 4.\ 3}{3.\ 7.\ \ 3.\ 5.\ 4}$; which, canceled, will give the *Ans.* $\frac{4}{5}$.

9. Reduce $\frac{2}{3}$ of $7\frac{1}{5}$ of $\frac{2}{4}$ of $2\frac{4}{7}$ to a simple fraction. *Ans.* $\frac{216}{35} = 6\frac{6}{35}$.

10. Reduce $\frac{1}{3}$ of $\frac{3}{6}$ of $\frac{2}{7}$ of $\frac{8}{3}$ of $\frac{6}{7}$ to a simple fraction. *Ans.* $\frac{16}{147}$.

11. Reduce $\frac{9}{10}$ of $\frac{6}{7}$ of $\frac{14}{15}$ to a simple fraction. *Ans.* $\frac{18}{25}$.

12. Reduce $\frac{9}{10}$ of $\frac{8}{6}$ of $\frac{7}{5}$ of $\frac{1}{12}$ to a simple fraction. *Ans.* $\frac{7}{50}$.

13. Reduce $\frac{11}{12}$ of $\frac{16}{17}$ of $\frac{51}{64}$ to a simple fraction. *Ans.* $\frac{11}{16}$.

14. Reduce $\frac{8}{9}$ of $\frac{4}{5}$ of $\frac{9}{10}$ of $\frac{1}{8}$ to a simple fraction. *Ans.* $\frac{2}{25}$.

15. Reduce $\frac{3}{2}$ of $8\frac{1}{2}$ of $5\frac{3}{6}$ to a simple fraction. *Ans.* $\frac{561}{8}$, or $70\frac{1}{8}$.

CASE V.

§ 75. TO CHANGE FRACTIONS FROM ONE DENOMINATION TO ANOTHER, WITHOUT ALTERING THE VALUE.

First. To reduce Fractions of low Denominations to those of higher Value.

Rule.—*Divide the fraction, or, what is the same thing, multiply the denominator by such numbers as are required to*

reduce the given quantity from the GIVEN *to the* REQUIRED DENOMINATION.)

Ex. 1. Reduce $\frac{5}{6}$ of a penny to the fraction of a pound.

The numbers required to reduce pence to pounds, are 12 and 20; therefore, $\frac{5}{6}$ of a penny is to be divided by these numbers; and since this can be effected, in the present case, only by multiplying the denominator, the operation will be, $\frac{5}{6 \times 12 \times 20} = \frac{5}{1440}$; and this, by Case I., is reduced to $\frac{1}{288}$, which is the fraction required. Hence, $\frac{5}{6}$ of a penny equals $\frac{1}{288}$ of a pound.

The canceling principle may, however, be successfully applied in the solution of sums of this character.

Rule FOR CANCELING.—*Place the numerator of the given fraction above a horizontal line, and its denominator below it; then place also* BELOW *the line, such numbers as are necessary to reduce the denomination given to that required. Cancel, &c., as before.*

We will solve the above example by this rule also.

$$\text{Statement: } \frac{5}{6.\ 12.\ 20}.$$

The scholar should compare the statement with the rule, to see that he understands its application. The above statement canceled: $\frac{\cancel{5}}{6.\ 12.\ \underset{4}{\cancel{20}}}$; and $6 \times 12 \times 4 = 288$, the denominator, as before, and nothing remains as a numerator; therefore, as before, $\frac{1}{288}$ of a pound is the answer. (See Note 1, Case IV.)

2. Reduce $\frac{3}{4}$ of a farthing to the fraction of a shilling. By the common rule, $\frac{3}{4 \times 4 \times 12 = 192}$, which, by Case I., equals $\frac{1}{64}$, *Ans.* By the rule for canceling, $\frac{3}{4.\ 4.\ 12}$. The same, canceled, $\frac{\cancel{3}}{4.\ 4.\ \underset{4}{\cancel{12}}} = \frac{1}{64}$, *Ans.*

3. Reduce $\frac{4}{5}$ of a penny to the fraction of a pound. Statement: $\frac{4}{5.\ 12.\ 20}$. Canceled: $\frac{\cancel{4}}{5.\ \underset{3}{\cancel{12}}.\ 20}$; and $5 \times 3 \times 20 = 300$; therefore, *Ans.* $\frac{1}{300}$.

4. Reduce $\frac{6}{7}$ of a gallon to the fraction of a hogshead. *Ans.* $\frac{2}{147}$.

Statement: $\frac{6}{7.\ 63}$. The 63 below the line reduces the gallons to hogsheads.

5. Reduce $\frac{3}{8}$ of an ounce Troy to the fraction of a pound. *Ans.* $\frac{1}{32}$.

6. Reduce $\frac{16}{17}$ of a minute to the fraction of a day. *Ans.* $\frac{1}{1530}$.

7. Reduce $\frac{8}{11}$ of a pound avoirdupois to the fraction of a cwt. *Ans.* $\frac{1}{154}$.

8. Reduce $\frac{3}{8}$ of a nail to the fraction of an ell English. *Ans.* $\frac{3}{160}$.

9. Reduce $\frac{5}{12}$ of a penny to the fraction of a pound. *Ans.* $\frac{1}{576}$.

10. Reduce $\frac{12}{13}$ of an hour to the fraction of a year. *Ans.* $\frac{1}{9490}$.

§ **76.** *Secondly.* To reduce Fractions of high Denominations to equivalent Fractions of lower Denominations.

Rule.—*Multiply the numerator by such numbers as are required to reduce the given quantity from the given to the required denomination, and then, by Case I., reduce the result to its lowest terms.*

Ex. 1. Reduce $\frac{1}{96}$ of a shilling to the fraction of a farthing.

To reduce shillings to farthings, we multiply by 12 and 4; therefore, $\frac{1}{96} \times 12 \times 4 = \frac{48}{96}$; and, by Case I., $\frac{48}{96} = \frac{1}{2}$, *Ans.*

Rule for Canceling.—*Place the numerator of the given fraction above a horizontal line, and the denominator below, as before; then place above the line such numbers as are necessary to reduce the denomination given to that required. Cancel, &c., as before.*

The above sum solved by this rule. Statement: $\frac{1.\ 12.\ 4}{96}$. The same canceled: $\frac{1.\ \cancel{12}.\ \cancel{4}}{\cancel{96}}$ with 8 and 2 beneath; therefore, $\frac{1}{2}$ of a farthing is the answer.

The scholar will carefully observe the difference between the statement here, and the one given for reducing low denominations to high.

2. Reduce $\frac{1}{360}$ of a pound to the fraction of a penny. *Ans.* $\frac{2}{3}$.

Statement: $\frac{1.\ 20.\ 12}{360}$.

3. Reduce $\frac{1}{108}$ of a pound Troy to the fraction of a pwt. *Ans.* $\frac{20}{9}$, or $2\frac{2}{9}$ pwt. Statement: $\frac{1.\ 12.\ 20}{108}$.

4. Reduce $\frac{1}{15}$ of a pound Troy to the fraction of an ounce. *Ans.* $\frac{4}{5}$.

5. Reduce $\frac{1}{12}$ of a penny to the fraction of a farthing. *Ans.* $\frac{1}{3}$.

6. Reduce $\frac{1}{880000}$ of a mile to the fraction of a barley-corn. *Ans.* $\frac{9}{500}$.

7. Reduce $\frac{1}{154}$ of a cwt. to the fraction of a pound avoirdupois. *Ans.* $\frac{8}{11}$.

8. Reduce $\frac{3}{160}$ of an ell English to the fraction of a nail. *Ans.* $\frac{3}{8}$.

9. Reduce $\frac{1}{5475}$ of a year to the fraction of an hour. *Ans.* $1\frac{3}{5}$.

CASE VI.

§ 77. To reduce Fractions of a higher Denomination to their Value in Whole Numbers of a lower Denomination.

Rule. — *Reduce the fraction to its next lower denomination, by multiplying the numerator by the requisite number, and divide the product by the denominator; the quotient thus obtained will be a whole number of the lower denomination, and the remainder, if any, may be reduced and divided as before. This process may be continued till nothing remains, or till the fraction is reduced to the lowest denomination.*

Ex. 1. Reduce $\frac{2}{3}$ of a pound sterling to its value in shillings and pence.

OPERATION.

$$\begin{array}{rl} & 2 = \text{numerator.} \\ & \underline{20} \\ \text{Div. by denom. } 3\,) & \underline{40} = \text{shillings.} \\ & 13 \text{ s., and 1 s. remains, which equals 12d.,} \end{array}$$

and 12 d. $\div$ 3 = 4d. Therefore, 13 s. 4 d. is the required number.

It is evident that $\frac{2}{3}$ of a pound sterling is 20 times as many thirds of a shilling, viz., $\frac{40}{3} = 13\frac{1}{3}$ shillings; and $\frac{1}{3}$ of a shilling is 12 thirds of a penny, that is, $\frac{12}{3} = 4$ d. Hence, $\frac{2}{3}$ of a pound is 13 s. 4 d.

2. Reduce $\frac{1}{60}$ of a pound sterling to its value in lower denominations.

Solution : $\frac{1}{60}$ of a pound $= \frac{20}{60}$ of a shilling, and $\frac{20}{60}$ of a shilling $= \frac{240}{60}$ of a penny $= 4$ d., *Ans.*

3. Reduce $\frac{3}{4}$ of a pound Troy to its integral value. *Ans.* 9 oz.

4. Reduce $\frac{1}{18}$ of a day to its integral value. *Ans.* 1 h. 20 m.

5. Reduce $\frac{9}{10}$ of an hour to its value in whole numbers. *Ans.* 54 m.

6. Reduce $\frac{1}{96}$ of a hogshead to its value in whole numbers. *Ans.* 2 qt. 1 pt. 1 gi.
7. Reduce $\frac{5}{8}$ of a quart to its integral value. *Ans.* 1 pt. 1 gi.
8. Reduce $\frac{3}{5}$ of an ell English to a whole number. *Ans.* 3 qr.
9. Reduce $\frac{4}{5}$ of a yard to a whole number. *Ans.* $3\frac{1}{5}$ qr.
10. Reduce $\frac{2}{3}$ of an ell French to a whole number. *Ans.* 4 qr.

CASE VII.

§ 78. TO REDUCE THE LOWER DENOMINATIONS OF A COMPOUND NUMBER TO FRACTIONS OF A HIGHER DENOMINATION.

Rule. — *Reduce the given quantity to the lowest denomination in that quantity, for a numerator of the fraction; and then reduce a unit of the higher denomination to the same denomination with the numerator, for the denominator of the fraction. This fraction reduced to its lowest terms by Case I. will be the one required.*

Ex. 1. Reduce 2 qr. 2 na. to the fraction of a yard.

Operation: 2 qr. 2 na. = 10 nails, the numerator; and 1 yd. = 4 qr., and 4 qr. = 16 nails, the denominator. Therefore, $\frac{10}{16}$ is the fraction, which equals $\frac{5}{8}$, *Ans.*

The operation may be much abbreviated by canceling; for which the following will be found a convenient rule: —

Rule FOR CANCELING. — *Reduce the given quantity to the lowest denomination mentioned, (if it consist of different denominations,) and place it above a horizontal line; and beneath the same line place the numbers required to reduce this denomination to the required denomination. Cancel, multiply, &c., and the terms of the required fraction will be obtained.*

Ex. 2. Reduce 3 s. 4 d. to the fraction of a pound.

Operation: 3 s. 4 d. = 40 d.; and pence are reduced to pounds by dividing by 12 and 20. Therefore,

$$\text{Statement: } \frac{40}{12.\ 20}. \quad \text{Canceled: } \frac{\overset{2}{\cancel{40}}}{\underset{6}{\cancel{12}}.\ \cancel{20}}.$$

Nothing remains above the line; the numerator of the required fraction, therefore, is 1; and 6 remains below the line, and consequently is the denominator. Therefore, $\frac{1}{6}$ is the fraction required; and 3s. 4d. is $\frac{1}{6}$ of a pound.

3. Reduce 2 roods and 30 rods to the fraction of an acre.

2 roods and 30 rods $=$ 110 rods; and rods are reduced to acres, by dividing by 40 and by 4. The statement, therefore, is, $\frac{110}{40.\ 4}$; and the same, canceled, is, $\frac{11\cancel{0}}{4\cancel{0}.\ 4} = \frac{11}{16}$, *Ans.*

4. Reduce 12 ounces to the fraction of a pound avoirdupois. *Ans.* $\frac{3}{4}$.

5. Reduce 26 gal. 2 qt. to the fraction of a hogshead. *Ans.* $\frac{53}{126}$.

6. Reduce 3 fur. 20 rods to the fraction of a mile. *Ans.* $\frac{7}{16}$.

7. Reduce 3 qr. 2 na. to the fraction of an ell English. *Ans.* $\frac{7}{10}$.

8. Reduce 2 dr. 2 sc. to the fraction of an ounce. *Ans.* $\frac{1}{3}$.

9. Reduce 2 qr. 24 lb. to the fraction of a cwt. *Ans.* $\frac{5}{7}$.

10. Reduce 6 oz. 10 pwt. to the fraction of a pound Troy. *Ans.* $\frac{13}{24}$.

11. Reduce 6 qt. to the fraction of a bushel. *Ans.* $\frac{3}{16}$.

12. Reduce 12 h. 30 m. to the fraction of a day. *Ans.* $\frac{25}{48}$.

13. Reduce 4 d. 12 h. to the fraction of a week. *Ans.* $\frac{9}{14}$.

14. Reduce 15 deg. 30 m. to the fraction of a sign. *Ans.* $\frac{31}{60}$

CASE VIII.

§ 79. TO REDUCE FRACTIONS HAVING DIFFERENT DENOMINATORS, TO EQUIVALENT FRACTIONS HAVING A COMMON DENOMINATOR.

Rule.—*Multiply all the denominators together, for a new denominator, and each numerator into all the denominators except its own, for a new numerator to each fraction. The several numerators, placed over the common denominator, will give the required fractions.*

NOTE.—The fractions should be reduced to their lowest terms before multiplying.

If the scholar looks carefully into the nature of this rule, he will see that the operation consists simply in multiplying the numerators and denominators by the same numbers; and he has already learned that this does not affect the value of the fraction.

Ex. 1. Reduce $\frac{6}{7}$, $\frac{4}{5}$, and $\frac{8}{11}$, to a common denominator.

PERFORMED.

$7 \times 5 \times 11 = 385$, the common denominator.

$6 \times 5 \times 11 = 330$, the num. for $\frac{6}{7}$, which therefore $= \frac{330}{385}$.

$4 \times 7 \times 11 = 308$, the numerator for $\frac{4}{5}$; therefore, $\frac{4}{5} = \frac{308}{385}$.
$8 \times 5 \times 7 = 280$, the numerator for $\frac{8}{11}$; therefore, $\frac{8}{11} = \frac{280}{385}$.
The required fractions, therefore, are, $\frac{330}{385}$, $\frac{308}{385}$, $\frac{280}{385}$.

2. Reduce $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, to a common denominator.
Ans. $\frac{60}{120}$, $\frac{40}{120}$, $\frac{30}{120}$, and $\frac{24}{120}$.
3. Reduce $\frac{2}{3}$, $\frac{2}{7}$, $\frac{1}{3}$, and $\frac{2}{8}$, to a common denominator.
Ans. $\frac{168}{252}$, $\frac{72}{252}$, $\frac{84}{252}$, $\frac{63}{252}$.
4. Reduce $\frac{4}{5}$, $\frac{11}{3}$, $\frac{1}{12}$, and $\frac{9}{16}$, to a common denominator.
Ans. $\frac{2304}{2880}$, $\frac{10560}{2880}$, $\frac{240}{2880}$, $\frac{1620}{2880}$.
5. Reduce $\frac{1}{2}$, $\frac{2}{3}$, $\frac{8}{9}$, and $\frac{3}{10}$, to a common denominator.
Ans. $\frac{810}{1620}$, $\frac{1080}{1620}$, $\frac{1440}{1620}$, $\frac{486}{1620}$.
6. Reduce $\frac{3}{5}$, $\frac{1}{3}$, $\frac{7}{9}$, and $\frac{8}{11}$, to a common denominator.
Ans. $\frac{891}{1485}$, $\frac{495}{1485}$, $\frac{1155}{1485}$, $\frac{1080}{1485}$.

At the commencement of this rule, the scholar was instructed relative to the peculiar form and nature of fractions, and made acquainted with certain principles of universal application. In the course of the preceding eight cases, he has been taught the various changes of which fractions are susceptible, while their *value* remains unaffected. His attention will now be directed to those operations by which their *value is affected.*

ADDITION OF FRACTIONS.

§ 80. If the scholar will turn back to Simple Addition, he will there find it stated, that numbers or quantities of the same kind only, can be reduced to a single number or quantity by adding. The same is true of fractions. It is obvious that $\frac{1}{3}$ of a shilling and $\frac{1}{3}$ of a penny make neither $\frac{2}{3}$ of a shilling nor $\frac{2}{3}$ of a penny. But $\frac{1}{3}$ of a shilling makes $\frac{12}{3}$ of a penny; and to this we can add $\frac{1}{3}$ of a penny, and the amount will be $\frac{13}{3}$ of a penny.

Therefore, before we can add fractions, they must be reduced to the same denomination. (See Case V.)

It is equally impossible to add fractions whose denominators are unlike. $\frac{1}{2}$ of a shilling added to $\frac{1}{4}$ of a shilling makes neither $\frac{2}{6}$ of a shilling nor $\frac{2}{4}$ of a shilling. But $\frac{1}{2}$ of a shilling $= \frac{2}{4}$ of a shilling; and $\frac{2}{4} + \frac{1}{4} = \frac{3}{4}$ of a shilling.

Fractions must therefore be reduced to a common denominator, before they can be united. (See Case VIII.) Hence we have the following rule: —

Rule.—*(Reduce all the fractions to the same denomination, and also to a common denominator; then add their numerators, and place their sum over the common denominator. If the fractions produced be improper, reduce them to a whole number or mixed quantity.)*

NOTE.—If any of the fractions are compound, they must be reduced to simple ones, before they can be reduced to a common denominator. (See Case IV.)

Ex. 1. What is the sum of $\frac{1}{3}$ and $\frac{6}{7}$?—These fractions, reduced to a common denominator by Case VIII., become $\frac{7}{21}$ and $\frac{18}{21}$, and $\frac{7}{21}+\frac{18}{21}=\frac{25}{21}$, or $1\frac{4}{21}$, *Ans.*

2. What is the sum of $\frac{3}{4}$ of $\frac{7}{9}$, and $\frac{4}{5}$ of $\frac{5}{9}$?—By Case IV. $\frac{3}{4}$ of $\frac{7}{9}=\frac{7}{12}$, and $\frac{4}{5}$ of $\frac{5}{9}=\frac{4}{9}$, and $\frac{7}{12}+\frac{4}{9}=\frac{63}{108}+\frac{48}{108}$, (see Case VIII.) and $\frac{63}{108}+\frac{48}{108}=\frac{111}{108}$, or $1\frac{3}{108}$.

3. What is the sum of $\frac{1}{2}$ and $\frac{7}{8}$? *Ans.* $\frac{11}{8}$, or $1\frac{3}{8}$.

4. What is the amount of $\frac{3}{4}$ of $\frac{1}{9}$, and $\frac{3}{7}$ of $\frac{14}{15}$? *Ans.* $\frac{29}{60}$.

5. What is the amount of 18 and 16, and $\frac{1}{2}$ of $\frac{3}{4}$? *Ans.* $34\frac{3}{8}$.

NOTE 2.—When whole numbers are combined in the same operation with fractions, add each separately, and unite them, as in the above sum.

6. What is the amount of 21, 7, $\frac{7}{9}$, and $\frac{4}{5}$ of $\frac{1}{3}$? *Ans.* $29\frac{2}{45}$.

7. What is the amount of $\frac{3}{4}$ of a penny added to $\frac{1}{9}$ of a £.?

EXPLANATION.—$\frac{1}{9}$ of a £.$=\frac{80}{3}$ of a penny; and $\frac{3}{4}$ of a penny $+\frac{80}{3}$ of a penny $=\frac{329}{12}$ of a penny, and this equals 2 s. 3 d. $1\frac{2}{3}$ qr. *Ans.*

8. What is the amount of $\frac{3}{4}$ of a yard and $\frac{1}{4}$ of a nail?

Ans. 3 qr. $0\frac{1}{4}$ na.

9. What is the sum of $\frac{5}{6}$ of a pound added to $\frac{1}{2}$ of a shilling?

Ans. $\frac{103}{6}$ s. $=$ 17 s. 2 d.

10. What is the sum of $\frac{1}{2}$ of $\frac{4}{5}$, and $\frac{6}{7}$ of $\frac{11}{12}$? *Ans.* $\frac{83}{70}$, or $1\frac{13}{70}$.

11. What is the sum of $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$ of $\frac{4}{5}$ of $\frac{5}{6}$ of $\frac{6}{7}$, and $\frac{1}{3}$?

Ans. $\frac{10}{21}$.

12. What is the sum of $\frac{5}{9}$ of a ton added to $\frac{3}{7}$ of a cwt.?

Ans. $11\frac{34}{63}$ cwt.

13. What is the sum of $\frac{4}{5}$ of a day added to $\frac{1}{2}$ of an hour?

Ans. $19\frac{7}{10}$ hours.

14. What is the sum of $\frac{1}{8}$ of a pound, $\frac{2}{3}$ of a shilling, and $\frac{1}{2}$ of a penny? *Ans.* 3 s. 2 d. 2 qr.

15. What is the sum of $\frac{1}{6}$ of a week, $\frac{3}{4}$ of a day, and $\frac{1}{4}$ of an hour? *Ans.* 1 day, 22 hours, 15 m.

SUBTRACTION OF FRACTIONS.

§ **81.** As we can subtract a quantity from another of the same kind only, it is obvious that the same preparations are necessary to perform operations in this rule as in the preceding; therefore,

Rule. — *Prepare the fraction as in addition, then substract the less numerator from the greater, and place the remainder over the common denominator.*

It will be obvious that the difference of the numerators is the difference sought.

Ex. 1. From $\frac{7}{8}$ take $\frac{1}{3}$. Operation: $\frac{7}{8} - \frac{1}{3} = \frac{21}{24} - \frac{8}{24} = \frac{13}{24}$, *Ans.*

2. From $\frac{8}{9}$ take $\frac{8}{21}$. $\frac{8}{9} - \frac{8}{21} = \frac{168}{189} - \frac{72}{189} = \frac{96}{189}$, *Ans.*

3. From $\frac{6}{7}$ take $\frac{2}{7}$. *Ans.* $\frac{4}{7}$.

In this last example, it is evident that, as the denominators are the same, the operation consists in subtracting the numerators only. The same is true of all similar examples, provided only that the fractions are of the same denomination.

4. From $\frac{11}{12}$ take $\frac{5}{12}$. *Ans.* $\frac{6}{12}$, or $\frac{1}{2}$.
5. From $\frac{15}{16}$ take $\frac{12}{16}$. *Ans.* $\frac{3}{16}$.
6. From $\frac{19}{20}$ take $\frac{7}{20}$. *Ans.* $\frac{12}{20}$, or $\frac{3}{5}$.
7. From $\frac{27}{28}$ take $\frac{19}{28}$. *Ans.* $\frac{8}{28}$, or $\frac{2}{7}$.
8. From $\frac{1}{2}$ of a pound take $\frac{5}{6}$ of a shilling. $\frac{1}{2}$ of a pound $= \frac{20}{2}$ of a shilling, and $\frac{20}{2} = \frac{60}{6}$ of a shilling. Therefore, $\frac{60}{6} - \frac{5}{6} = \frac{55}{6}$ of a shilling, and $\frac{55}{6}$ of a shilling $= 9$ s. 2 d.
9. From $\frac{3}{4}$ of a league take $\frac{1}{2}$ of a mile. 1 league $= 3$ miles; therefore, $\frac{3}{4}$ of a league $= \frac{9}{4}$ of a mile; and $\frac{1}{2}$ of a mile $= \frac{2}{4}$ of the same; hence, $\frac{9}{4} - \frac{2}{4} = \frac{7}{4}$ or $1\frac{3}{4}$ of a mile, which is the distance required.
10. From $\frac{1}{2}$ of a shilling take $\frac{3}{4}$ of a penny. *Ans.* $\frac{21}{4}$, or $5\frac{1}{4}$ pence.
11. From $\frac{7}{8}$ of a day take $\frac{2}{7}$ of an hour. *Ans.* $\frac{144}{7}$, or $20\frac{4}{7}$ hours.
12. From $\frac{9}{10}$ of a pound avoirdupois take $\frac{3}{4}$ of an ounce. *Ans.* $\frac{273}{20}$, or $13\frac{13}{20}$ ounces.
13. From $\frac{3}{4}$ take $\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4}$. *Ans.* $\frac{1}{2}$.
$\frac{1}{2}$ of $\frac{2}{3}$ of $\frac{3}{4} = \frac{1}{4}$, and $\frac{3}{4} - \frac{1}{4} = \frac{2}{4}$ or $\frac{1}{2}$.
14. From $\frac{11}{12}$ of an ell English take $\frac{1}{2}$ of a yard. *Ans.* $\frac{31}{12}$, or $2\frac{7}{12}$ qr.
15. From $9\frac{3}{4}$ take $6\frac{1}{2}$. *Ans.* $3\frac{1}{4}$.

16. From 19 yards take $5\frac{7}{8}$ yards. *Ans.* $13\frac{1}{8}$.

17. From 7 ells English take $4\frac{1}{2}$ yards.
Ans. $\frac{17}{4} = 4\frac{1}{4}$ yards, or $3\frac{2}{5}$ ells English.

18. From $\frac{1}{8}$ of a pound sterling take $\frac{3}{4}$ of a penny.
Ans. $\frac{311}{12}$ of a penny, or 2s. $1\frac{11}{12}$ d.

19. From $\frac{6}{9}$ of a rod take $\frac{2}{3}$ of a foot. *Ans.* $\frac{31}{3}$ or $10\frac{1}{3}$ feet.

20. From $\frac{6}{7}$ of an ounce take $\frac{3}{4}$ of a pwt.
Ans. $\frac{459}{28}$ pwt., or $16\frac{11}{28}$ pwt.

MULTIPLICATION OF FRACTIONS.

§ 82. A fraction may be multiplied by a whole number, either by multiplying the numerator or dividing the denominator by that number. This has been fully illustrated in section 4th of Fractions.

Ex. 1. Multiply $\frac{1}{3}$ by 16. *Ans.* $\frac{16}{3}$, or $5\frac{1}{3}$.
2. Multiply $\frac{5}{6}$ by 8. *Ans.* $\frac{40}{6}$, or $6\frac{4}{6}$.
3. Multiply $\frac{6}{7}$ by 9. *Ans.* $\frac{54}{7}$, or $7\frac{5}{7}$.
4. Multiply $\frac{8}{9}$ by 3. *Ans.* $\frac{24}{9}$, or $2\frac{6}{9}$.
5. Multiply $\frac{20}{21}$ by 7. *Ans.* $\frac{20}{3}$, or $6\frac{2}{3}$.
6. Multiply $\frac{21}{24}$ by 8. *Ans.* $\frac{21}{3}$, or 7.
7. Multiply $\frac{33}{36}$ by 12. *Ans.* $\frac{33}{3}$, or 11.
8. Multiply $\frac{29}{27}$ by 3. *Ans.* $\frac{29}{9}$, or $3\frac{2}{9}$.

A fraction is multiplied into a whole number equal to its denominator, by *rejecting that denominator*.

9. Multiply $\frac{15}{21}$ by 21. By dividing $\frac{15}{21}$ by 21, I take $\frac{1}{21}$ part of 15; if, then, this $\frac{1}{21}$ part be repeated 21 times, it is evident that the value of all the parts will equal 15.

10. Multiply $\frac{72}{82}$ by 82. *Ans.* 72.
11. Multiply $\frac{7}{9}$ by 9. *Ans.* 7.
12. Multiply $\frac{41}{73}$ by 73. *Ans.* 41.
13. Multiply $\frac{21}{43}$ by 43. *Ans.* 21.

§ 83. To multiply Fractions by Fractions.

Rule.—*Multiply the numerators of the given fractions together, for the required numerator, and the denominators, for the required denominator; then, by Case I., reduce the terms as far as practicable.*

NOTE. — Mixed numbers are to be reduced to improper fractions before multiplying; or we may first multiply the integers, and then the fractions, and add their products.

Ex. 1. Multiply $\frac{3}{5}$ by $\frac{7}{9}$. $\frac{3}{5} \times \frac{7}{9} = \frac{21}{45}$, and $\frac{21}{45} \div 3 = \frac{7}{15}$, *Ans.*

2. Multiply $\frac{3}{8}$ by $\frac{7}{9}$. $\frac{3}{8} \times \frac{7}{9} = \frac{21}{72} = \frac{7}{24}$, *Ans.*

3. Multiply $7\frac{1}{2}$ by $3\frac{1}{3}$. $7\frac{1}{2} = \frac{15}{2}$, and $3\frac{1}{3} = \frac{10}{3}$. and $\frac{15}{2} \times \frac{10}{3} = \frac{150}{6} = \frac{25}{1}$, or 25, *Ans.*

Rule FOR CANCELING. — *Place the numerators of the given fractions above a horizontal line, and their denominators below the same. Cancel, multiply, &c., as before.*

One important advantage of the above rule will be found in the fact, that it always gives the answer in its lowest possible terms.

4. Multiply $\frac{1}{6}$ by $\frac{3}{5}$.

Statement: $\frac{1.\ 3}{6.\ 5}$. Canceled: $\frac{1.\ \cancel{3}}{\underset{2}{\cancel{6}}.\ 5} = \frac{1}{10}$, *Ans.*

5. Multiply $\frac{3}{4}$ of $\frac{1}{6}$ of $\frac{2}{3}$ by $\frac{1}{2}$ of $\frac{2}{9}$ of $\frac{11}{12}$.

Statement: $\frac{3.\ 1.\ 2.\ 1.\ 2.\ 11}{4.\ 6.\ 3.\ 2.\ 9.\ 12}$. Canceled: $\frac{\cancel{3}.\ 1.\ \cancel{2}.\ 1.\ \cancel{2}.\ 11}{\underset{2}{\cancel{4}}.\ 6.\ \cancel{3}.\ \cancel{2}.\ 9.\ 12}$;

therefore, $11 =$ numerator, and $2 \times 6 \times 9 \times 12 = 1296$, denominator, and $\frac{11}{1296}$, *Ans.*

6. Multiply $\frac{9}{15}$ by $\frac{3}{18}$.

Statement: $\frac{9.\ 3}{15.\ 18}$. *Ans.* $\frac{1}{10}$.

7. Multiply $\frac{7}{8}$ by $2\frac{1}{2}$. *Ans.* $\frac{35}{16}$, or $2\frac{3}{16}$.

8. Multiply $16\frac{1}{2}$ by $12\frac{1}{3}$. *Ans.* $203\frac{1}{2}$.

9. Multiply $13\frac{1}{4}$ by $9\frac{3}{8}$. *Ans.* $124\frac{7}{32}$.

10. Multiply $\frac{1}{2}$ of $\frac{5}{6}$ of $\frac{3}{8}$ of $\frac{3}{4}$ of $\frac{7}{9}$ of $\frac{2}{7}$ of $\frac{3}{4}$ of $\frac{2}{5}$, by $\frac{1}{2}$ of $\frac{9}{10}$ of $\frac{1}{3}$ of $\frac{1}{6}$. *Ans.* $\frac{1}{384}$.

In solving sums by canceling, like the above, the necessity of reducing compound fractions to simple ones is avoided.

11. Multiply $\frac{1}{2}$ of 19 by $\frac{3}{6}$.

Statement: $\frac{1.\ 19.\ 3}{2.\quad 6}$. *Ans.* $\frac{19}{4}$, or $4\frac{3}{4}$.

If any whole numbers are given as parts of the dividend, it is only necessary to write them above the line, as in the last example; or, if they are given as parts of the divisor, they only require to be placed below the line; the operation then proceeds as before.

12. Multiply $\frac{3}{4}$ of 10 by $\frac{2}{5}$. *Ans.* 3.

13. Multiply 144 by $\frac{1}{12}$. *Ans.* 12.

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14. Multiply 395 by $\frac{1}{3}$ of $\frac{2}{5}$. *Ans.* $\frac{158}{3}$, or $52\frac{2}{3}$.

15. Multiply $\frac{6}{8}$ of 3 times $\frac{9}{12}$ of 5 times $\frac{1}{2}$ of $\frac{1}{3}$ of $\frac{1}{4}$, by $\frac{6}{7}$ of $\frac{4}{5}$ of $\frac{5}{8}$ of $\frac{7}{12}$. *Ans.* $\frac{1}{8}$.

16. What will $2\frac{1}{2}$ tons of hay cost, at $16\frac{3}{4}$ dollars per ton? *Ans.* $41\frac{7}{8}$ dollars.

17. What will $2\frac{1}{3}$ barrels of sugar cost, at $18\frac{1}{4}$ dollars per barrel? *Ans.* $42\frac{7}{12}$ dollars.

18. What will $8\frac{3}{4}$ pounds of tea cost, at $1\frac{1}{4}$ dollars per pound? *Ans.* $10\frac{15}{16}$ dollars.

19. What will $4\frac{3}{4}$ cords of wood cost, at $3\frac{3}{4}$ dollars per cord? *Ans.* $17\frac{13}{16}$ dollars.

20. What will 9 yards of cloth cost, at $\frac{7}{8}$ dollar per yard? *Ans.* $7\frac{7}{8}$ dollars.

21. What will 34 gallons of wine cost, at $1\frac{3}{8}$ dollars per gallon? *Ans.* $46\frac{3}{4}$ dollars.

22. What will $12\frac{1}{2}$ barrels of sugar cost, at $15\frac{1}{4}$ dollars per barrel? *Ans.* $190\frac{5}{8}$ dollars.

23. What will $22\frac{1}{2}$ pounds of lard cost, at $\frac{1}{8}$ dollar per pound? *Ans.* $\$2\frac{13}{16}$.

DIVISION OF FRACTIONS.

§ 84. Division of Fractions is naturally divided into the three following kinds, viz., the division of a fraction by a whole number; the division of a whole number by a fraction; and the division of a fraction by a fraction.

Division of fractions by a whole number was fully illustrated in Sec. 5th of the remarks introductory to this rule. It is, therefore, necessary here merely to repeat, that a *fraction is divided by a whole number, either by dividing its numerator, or multiplying its denominator by that number.*

Ex. 1. Divide $\frac{18}{21}$ by 9. *Ans.* $\frac{2}{21}$.

2. Divide $\frac{3}{5}$ by 7. *Ans.* $\frac{3}{35}$.

3. Divide $\frac{13}{9}$ by 11. *Ans.* $\frac{13}{99}$.

4. Divide $\frac{21}{12}$ by 3. *Ans.* $\frac{7}{12}$.

5. Divide $\frac{81}{90}$ by 9. *Ans.* $\frac{9}{90}$, or $\frac{1}{10}$.

6. Divide $\frac{1}{53}$ by 8. *Ans.* $\frac{1}{424}$.

7. Divide $\frac{31}{12}$ by 6. *Ans.* $\frac{31}{72}$.

8. Divide $\frac{11}{7}$ by 5. *Ans.* $\frac{11}{35}$.

9. Divide $\frac{17}{40}$ by 12. *Ans.* $\frac{17}{240}$.

10. Divide $\frac{42}{49}$ by 7. *Ans.* $\frac{6}{49}$.
11. Divide $\frac{19}{8}$ by 40. *Ans.* $\frac{19}{320}$.
12. Divide $\frac{1}{6}$ by 72. *Ans.* $\frac{1}{432}$.

It is obvious that the quotient arising from dividing a whole number by a fraction, must be as much larger than the number itself, as a unit or 1 is greater than the fraction; or, in other words, the given dividend must bear the same ratio to the required quotient, as the numerator of the fraction bears to its denominator. A unit, or 1, is contained in 6, six times; $\frac{1}{2}$ is contained in the same number, twelve times, and $\frac{1}{3}$, eighteen times; and $\frac{2}{3}$, half as many times as $\frac{1}{3}$, viz., nine times. The operation to obtain this last quotient is as follows: $6 \times 3 = 18$, the number of thirds in 6; and $18 \div 2 = 9$.

For dividing a whole number by a fraction, we have, then, the following rule: —

§ **85.** *Multiply the whole number by the denominator of the fraction, and divide the product by the numerator.*

Ex. 1. Divide 9 by $\frac{3}{4}$. Operation: $9 \times 4 = 36$, and $36 \div 3 = 12$, the quotient.

2. Divide 15 by $\frac{5}{6}$. Operation: $15 \times 6 = 90$, and $90 \div 5 = 18$, quotient.

Operations of a similar character may be performed by canceling.

Rule for Canceling. — *Place the whole number above a horizontal line, and invert the fractional divisor; that is, place the denominator above the line, and the numerator below. Cancel, &c.*

3. Divide 21 by $\frac{3}{7}$. Statement: $\frac{21 . 7}{3}$. Canceled: $\frac{\overset{7}{\cancel{21}} . 7}{\cancel{3}}$, and $7 \times 7 = 49$, the quotient required.

4. Divide 42 by $\frac{6}{7}$. Statement: $\frac{42 . 7}{6}$. *Ans.* 49.

The following sums may be divided by either of the above rules: —

5. Divide 18 by $\frac{3}{7}$. *Ans.* 42.
6. Divide 63 by $\frac{9}{13}$. *Ans.* 91.
7. Divide 63 by $\frac{9}{11}$. *Ans.* 77.
8. Divide 42 by $\frac{8}{9}$. *Ans.* $47\frac{1}{4}$.
9. Divide 66 by $\frac{2}{3}$. *Ans.* 99.
10. Divide 121 by $\frac{7}{8}$. *Ans.* $138\frac{2}{7}$.
11. Divide 101 by $\frac{11}{12}$. *Ans.* $110\frac{2}{11}$.
12. Divide 32 by $\frac{4}{5}$. *Ans.* 40.

§ 86. That the scholar may be enabled to commence understandingly the division of fractions by fractions, he may turn back, and review the seventh section of the remarks introductory to Fractions. It is there said, that *a fraction is divided by another fraction by inverting the divisor, and then multiplying them together as in multiplication.*

1. Divide $\frac{7}{8}$ by $\frac{2}{3}$. *Ans.* $\frac{21}{16}$, or $1\frac{5}{16}$.
2. Divide $\frac{8}{9}$ by $\frac{1}{2}$. *Ans.* $\frac{16}{9}$, or $1\frac{7}{9}$.

Rule for Canceling.—*Proceed in all respects as in multiplication of fractions, in arranging the terms of the dividend; then invert the divisor; that is, place the numerators below, and the denominators above, the line. Proceed to cancel, &c.*

Ex. 3. Divide $\frac{15}{16}$ by $\frac{5}{12}$.

$$\text{Statement: } \frac{15.\ 12}{16.\ 5}. \quad \text{Canceled: } \frac{\overset{3}{\cancel{15}}.\ \overset{3}{\cancel{12}}}{\underset{4}{\cancel{16}}.\ \cancel{5}}.$$

$3 \times 3 = 9$, $\frac{9}{4}$ or $2\frac{1}{4}$, *Ans.*

Note.—When the divisor is a *compound fraction*, each fraction in the divisor must be inverted.

4. Divide $\frac{3}{4}$ of $\frac{4}{5}$ by $\frac{1}{4}$ of $\frac{3}{5}$. Statement: $\frac{3.\ 4.\ 4.\ 5}{4.\ 5.\ 1.\ 3}$. Canceled: $\frac{\cancel{3}.\ \cancel{4}.\ 4.\ \cancel{5}}{\cancel{4}.\ \cancel{5}.\ 1.\ \cancel{3}}$; therefore, $\frac{4}{1}$, or 4, is the answer required.

5. Divide $\frac{15}{17}$ of $\frac{11}{12}$ by $\frac{3}{5}$ of $\frac{6}{7}$. Statement: $\frac{15.\ 11.\ 5.\ 7}{17.\ 12.\ 3.\ 6}$

Ans. $\frac{1925}{1224}$, or $1\frac{701}{1224}$.

6. Divide $\frac{4}{5}$ of $\frac{6}{14}$ of $\frac{3}{4}$ by $\frac{1}{4}$ of $\frac{6}{7}$ of $\frac{9}{10}$. *Ans.* $\frac{4}{3}$, or $1\frac{1}{3}$.
7 Divide $\frac{1}{3}$ by $\frac{1}{6}$. *Ans.* 2.
8. Divide $\frac{9}{10}$ of $\frac{6}{7}$ of $\frac{14}{15}$ of $\frac{18}{19}$ of $\frac{28}{30}$ of $\frac{1}{2}$ by $\frac{2}{3}$ of $\frac{7}{8}$ of $\frac{9}{10}$ of $\frac{8}{14}$. *Ans.* $\frac{504}{475} = 1\frac{29}{475}$.
9. Divide $4\frac{3}{4}$ by $\frac{3}{4}$ of 6. *Ans.* $1\frac{1}{18}$.
10. Divide $6\frac{3}{4}$ by $\frac{3}{4}$ of 4. *Ans.* $2\frac{1}{4}$.
11. Divide $\frac{3}{4}$ of $\frac{2}{3}$ by $\frac{4}{5}$ of $6\frac{1}{3}$. *Ans.* $\frac{15}{152}$.
12. Divide 6 by $\frac{1}{3}$. *Ans.* 18.
13. Divide $3\frac{1}{3}$ by $2\frac{1}{2}$. *Ans.* $1\frac{1}{3}$
14. Divide $\frac{3}{4}$ by 12. *Ans.* $\frac{1}{16}$.
15. Divide $\frac{1}{3}$ of $\frac{5}{6}$ by $\frac{3}{5}$ of $\frac{1}{4}$. *Ans.* $1\frac{23}{27}$.
16. Divide $\frac{3}{4}$ of 72 by $\frac{1}{8}$ of 56. *Ans.* $7\frac{5}{7}$.
17. Divide $\frac{18}{19}$ by 6. *Ans.* $\frac{3}{19}$.
18. Divide $\frac{3}{5}$ by 7. *Ans.* $\frac{3}{35}$.
19. Divide $\frac{9}{10}$ by 3. *Ans.* $\frac{3}{10}$.
20. Divide $\frac{1}{2}$ by 8. *Ans.* $\frac{1}{16}$.
21. Divide 8 by $\frac{1}{2}$. *Ans.* 16.

22. Divide 12 by $\frac{2}{3}$. *Ans.* 18.
23. Divide 41 by $\frac{2}{5}$. *Ans.* $\frac{205}{2}$.
24. Divide 35 by $\frac{5}{7}$. *Ans.* 49.
25. Bought 8 lb. of coffee, for $\frac{24}{27}$ of a dollar. What was the cost of one pound? *Ans.* $\frac{3}{27}$, or $\frac{1}{9}$, of a dollar.
26. Bought 9 pounds of sugar, for $\frac{11}{12}$ of a dollar. What was the price of a pound? *Ans.* $\frac{11}{108}$ of a dollar.
27. In $8\frac{1}{3}$ weeks a family consumes $84\frac{1}{2}$ pounds of butter. How much is that per week? *Ans.* $10\frac{7}{50}$.

APPLICATION.

1. What are the smallest terms in which the fraction $\frac{525}{600}$ can be expressed? *Ans.* $\frac{7}{8}$.
2. Reduce $\frac{77}{88}$ to its lowest terms. *Ans.* $\frac{7}{8}$.
3. If \$42 be equally divided among 12 boys, what fraction of one dollar will each boy receive? *Ans.* $\frac{7}{2}$ of a dollar.
4. What fraction of one dollar is \$$3\frac{5}{8}$? *Ans.* $\frac{29}{8}$ of a dollar.
5. If a man purchase $\frac{462}{8}$ of a yard of cloth, how many yards does he purchase? *Ans.* $57\frac{3}{4}$ yd.
6. What is the difference between $\frac{375}{4}$ of a yard, and 93 whole yards? *Ans.* $\frac{3}{4}$ of a yd.
7. If a yard of cloth cost \$8.40, what will $\frac{3}{7}$ of $\frac{1}{2}$ of a yard cost? *Ans.* \$1.80.
8. Paid \$48 for a quantity of grain. How much did $\frac{3}{4}$ of $\frac{1}{3}$ of it cost? *Ans.* \$12.
9. Reduce $\frac{6}{7}$ of a penny to the fraction of a pound. *Ans.* $\frac{1}{280}$.
10. What fraction of a pound is 3 pwt. of gold? *Ans.* $\frac{1}{80}$.
11. Bought a quantity of oil, for $\frac{9}{11}$ of a pound sterling. How many shillings, pence, &c., did it cost? *Ans.* 16 s. 4 d. $1\frac{5}{11}$ qrs.
12. Reduce $\frac{4}{5}$, $\frac{5}{9}$, $\frac{8}{9}$, and $\frac{9}{10}$, to a common denominator. *Ans.* $\frac{72}{90}$, $\frac{50}{90}$, $\frac{80}{90}$, and $\frac{81}{90}$.
13. What is the amount of the fractions $\frac{4}{5}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$? *Ans.* $\frac{191}{60}$, or $3\frac{11}{60}$.
14. Bought a yard of cloth, for $\frac{6}{7}$ of a dollar. What would be the cost of 8 yards, at the same rate? *Ans.* \$$6\frac{6}{7}$.
15. If I pay $\frac{3}{4}$ of a dollar for a gallon of oil, what will $16\frac{1}{3}$ gallons cost? *Ans.* \$$12\frac{1}{4}$.
16. What would $\frac{8}{9}$ of an acre of land cost, if $\frac{1}{2}$ of an acre cost \$27? *Ans.* \$48.
17. Bought $\frac{3}{4}$ of a ten-acre lot, and sold $\frac{2}{5}$ of what I purchased. How much did I sell? *Ans.* 3 acres.
18. What would 395 lb. of butter cost, at $\frac{2}{5}$ of $\frac{1}{3}$ of a dollar per pound? *Ans.* \$$52\frac{2}{3}$.

19. Expended \$18 for cloth, worth $\frac{5}{6}$ of a dollar per yard. How many yards did I purchase? *Ans.* $21\frac{3}{5}$ yd.

20. Paid $\frac{18}{19}$ of a dollar for 6 yards of ribbon. What did I pay per yard? *Ans.* $\frac{3}{19}$ of 1 dollar.

21. If $\frac{3}{8}$ of an acre of land cost \$33, what will $\frac{1}{16}$ of an acre cost? *Ans.* \$$5\frac{1}{2}$.

22. If $\frac{5}{7}$ of a yard cost $\frac{11}{12}$ of a dollar, what will 2 yards cost? *Ans.* \$2.566 +.

23. If 5 loads of hay cost \$60, what will $\frac{8}{9}$ of a load cost? *Ans.* \$$10\frac{2}{3}$.

24. A man owning $\frac{4}{5}$ of a vessel, sold $\frac{1}{3}$ of his share for \$478.50. What would the whole vessel come to at the same rate? *Ans.* \$1794.375.

25. If 16 men do a piece of work in $56\frac{2}{3}$ days, in what time will 24 men do the same? *Ans.* $37\frac{7}{9}$ days.

26. Bought 20 yards of camlet, 5 qr. wide. How many yards will be required, to line the same, that is only 3 qr. wide? *Ans.* $33\frac{1}{3}$ yards.

27. If $\frac{4}{5}$ of a dollar will pay for $10\frac{1}{2}$ lb. of sugar, what is the price of 1 lb. *Ans.* $7\frac{13}{21}$ cents.

28. If \$$20\frac{1}{3}$ will buy $15\frac{1}{3}$ barrels of apples, what is the cost per barrel? *Ans.* \$$1\frac{15}{46}$.

29. Sold $\frac{3}{4}$ of my farm, for \$2700. What is the value of the whole at the same rate? *Ans.* \$3600.

30. If 6 lb. of tea cost \$$7\frac{4}{5}$, what is the value of 1 lb.? *Ans.* \$$1\frac{3}{10}$.

31. If $16\frac{6}{7}$ yards of cloth cost \$$33\frac{5}{7}$, what is the cost of 1 yd.? *Ans.* \$2.

32. If a piece of cloth, measuring $13\frac{5}{9}$ yards, cost \$$27\frac{1}{3}$, what is the value of 1 yd.? *Ans.* \$$2\frac{1}{61}$.

33. If 7 horses consume $2\frac{3}{4}$ tons of hay, how much does each consume? *Ans.* $\frac{11}{28}$ of a ton.

34. If 7 horses consume $2\frac{3}{4}$ tons of hay, how much will 9 horses consume in the same time? *Ans.* $3\frac{15}{28}$ tons.

35. Bought 6 lb. of hyson tea, and 10 lb. of black tea. The price per lb. of the latter was $\frac{3}{8}$ the price of the former, which was $\frac{5}{6}$ of a dollar. What was the price of the whole? *Ans.* \$$8\frac{1}{8}$.

36. Bought 8 yards of cloth of one kind, and 13 yards of another kind. For the former I paid \$$3\frac{3}{8}$ per yd., and for the latter, $\frac{7}{8}$ of that price. What did the whole cost? *Ans.* \$$65\frac{25}{64}$.

37. Bought a barrel of melasses, at $\frac{5}{12}$ of a dollar per gallon, for which I agreed to pay $\frac{3}{8}$ in cash, $\frac{1}{5}$ in eggs, at $\frac{1}{6}$ of a dollar per doz., and the remainder in butter, at $\frac{3}{16}$ of a dollar per lb. How much of each did I pay?

Ans. Cash, \$4.921$\frac{7}{8}$; 15$\frac{3}{4}$ doz. eggs; and 29$\frac{3}{4}$ lb. butter.

38. If $\frac{5}{8}$ of a yard cost $\frac{5}{7}$ £., what will $\frac{3}{5}$ of an ell English cost? *Ans.* 17 s. 1 d. 2$\frac{6}{7}$ qr.

39. Met a boy going to market with a basket containing 100 apples, of which he promised to give me one, provided I would tell him how to divide the remainder so as to comply with his father's directions, viz., that he should bring home the value of $\frac{1}{6}$ of the apples in pepper, $\frac{1}{4}$ in sugar, $\frac{1}{3}$ in cinnamon, and $\frac{1}{4}$ in starch. How did I divide them?

Ans. 16$\frac{1}{2}$ of them to be expended for pepper, 24$\frac{3}{4}$ for sugar, 33 for cinnamon, and 24$\frac{3}{4}$ for starch.

40. Took a journey of 872 miles; $\frac{11}{24}$ of which I traveled by steam-boat, $\frac{1}{4}$ by rail-road, $\frac{1}{6}$ by stage-coach, and walked the remainder. What distance did I travel in each mode?

Ans. By steam-boat, 399$\frac{2}{3}$ miles; by rail-road, 218 miles; by stage-coach, 145$\frac{1}{3}$; and on foot, 109 miles.

QUESTIONS.—What is a Fraction? If a unit be divided into two equal parts, what is each of these parts called? How is it written? If it be divided into three equal parts, what is each part called, and how is it written? Similar questions should be asked respecting other fractions. When more parts than one are to be expressed, how is it done? What do fractions, therefore, express? How are they represented? What is the number below the line called? What does the denominator show? What is the number above the line called? What does the numerator show? What are the two numbers called, when spoken of collectively? How many kinds of fractions are there? What are they? What is a proper fraction? Give an example. What is an improper fraction? Give an example. What is a simple fraction? Give an example. What is a compound fraction? Give an example. What is a mixed number? Give an example. What is a complex fraction? Give an example. What does the denominator show? If the denominator remain the same, how is the value of the fraction affected by increasing the numerator? How, by diminishing it? Give an illustration. What is, therefore, the value of a fraction? If the numerator of a fraction be less than the denominator, how is its value compared with a unit? If the numerator be equal to the denominator, how then does its value compare with a unit? And how, if the numerator be greater than the denominator? What is the only consideration which limits the value of a fraction? In what ratio is the value of a fraction increased? How, then, may fractions be multiplied? How is a fraction multiplied into a number equal to its denominator? In what form should the terms of the fraction always be preserved? What is necessary to increase or diminish the value of a fraction? How, then, may a fraction be multiplied by a whole number? In what two ways may fractions be multiplied by whole numbers? How may a fraction be divided by a whole number? How else may a fraction be divided by a whole number? In what two ways, then, may fractions be divided by whole numbers? In what case should the numerator always be divided? What operation is therefore performed on the value of a fraction, when the numerator is operated upon? And what, when the denominator is operated upon? How is a fraction multiplied by a fraction? How is a fraction divided by a fraction? If the numerator and denominator be both multiplied by the same number, how is the value of the fraction affected? What is the rule for reducing fractions to their lowest terms?

What is the rule for reducing a whole number or mixed quantity to an improper fraction? How is an improper fraction reduced to a whole or mixed number? In all cases of division, what disposition may be made of the remainder, when any occurs? How are compound fractions reduced to simple

ones? What is the rule for canceling? What is Note 1? What is Note 2? What is Note 3? What is Case V.? How are fractions of low denominations reduced to those of higher denominations? What is the rule for canceling?

How are fractions of high denominations reduced to those of a lower value? What is Case VI.? What is the rule for it? What is Case VII.? What is the rule for it? What is the rule for canceling? What is Case VIII.? What is the rule for it? What is the note? What must be done before fractions can be added? What else requires to be done? What is the rule for the addition of fractions? What note follows? What is Note 2? What preparations are necessary before fractions can be subtracted? What is the rule for subtracting fractions? How is a fraction multiplied by a whole number? How is a fraction multiplied into a quantity equal to its denominator. What is the rule for multiplying fractions by fractions? What is the rule for canceling? Into what three kinds is division of fractions naturally divided? How is a fraction divided by a whole number? What is the rule for dividing a whole number by a fraction? How are fractions divided by fractions? What is the rule for canceling? What note follows the rule?

DECIMAL FRACTIONS.

§ 87. In the preceding rule we have contemplated the unit as divided into *any number of equal parts*,

We are now to regard it as divided, first, into ten equal parts; then each of these into ten other equal parts, or the whole unit into one hundred equal parts; and these parts again, each into ten other parts, or the whole into a thousand equal parts, &c. The expressions obtained by these several divisions, therefore, *decrease in value in the constant ratio of ten, from the left to the right*, and are called DECIMALS. Whole numbers, as was shown in Numeration, *increase* in the same ratio from *the right to the left*, and both commence their enumeration with the unit figure. The connection between them is therefore so intimate as to render them susceptible of being written together and subjected to the same operations. The only important consideration in writing them, in addition to what has already been explained, is to *distinguish the one from the other*. This is effected by the period, called in decimals, *the point of separation*, which is always placed between them. In the expression 23.56, the 23 is the whole number, and the .56 the decimal.

It will be observed that decimals, although they express parts of units, do not, like vulgar fractions, require two terms to express them. The given decimal may, however, be regarded, as in truth it is, a numerator, with a denominator always understood. What this denominator is, shall be our next object to illustrate. The scholar may therefore, in the first place, carefully examine the following table of whole numbers and

decimals. The decimals, it will be observed, are read or numerated from the left to the right.

Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.		Tenths.	Hundredths.	Thousandths.	Tens of Thousandths.	Hundreds of Thousandths.	Millionths.
8	7	6	5	4	3	2	.	3	4	5	6	7	8

By an examination of the preceding table, the scholar will see that the 3 on the right of the separatrix is so many tenths. In the preceding rule, this would be thus expressed, $\frac{3}{10}$, which, by Section 8th of the introductory remarks of the same rule, equals $\frac{30}{100}$. He will also see, that the 4 on the same side of the separatrix, is so many hundredths, or $\frac{4}{100}$. Now, it is obvious that these two fractions, united, would make $\frac{34}{100}$. The same process of reasoning will show that the next figure, or 5, is so many thousandths; and, since $\frac{34}{100}=\frac{340}{1000}$, if the 5 be added, the amount will be $\frac{345}{1000}$. From the preceding, we therefore learn that, if the decimal consist of one figure only, it is so many tenths; if it consist of two figures, it is so many hundredths; and if of three, it is so many thousandths; and from this we derive the following conclusion, viz., *that the denominator of a decimal fraction always consists of a figure* 1, *with as many ciphers annexed to it, as there are figures in the given decimal.*

The scholar may write a denominator to each of the following decimals, viz., .6; .356; .26; .7426; .98654; .71639.

§ 88. From the preceding explanation of the nature of decimals, it is obvious that ciphers added to the right of a decimal do not affect its value, while those placed on the left diminish its value in a ten-fold ratio; for .5 is the same in value as $\frac{5}{10}$, or $\frac{1}{2}$. Now, if a cipher be added to the right of the .5, it becomes .50, which is of equal value with $\frac{50}{100}$, and this also equals $\frac{1}{2}$. (See Case I., Vulgar Fractions.) If, then, .5 and .50 are each equal to $\frac{1}{2}$, it is obvious that the value of a decimal is not affected by ciphers placed on the right of it; but, if they be placed on the *left, they diminish the value of the decimal in a ten-fold ratio.* The decimals .5, .05, and .005, will serve as an illustration. From the explanation given above, the denominators of these several decimals are 10, 100, and 1000; or the decimals may be thus written: $\frac{5}{10}$, $\frac{05}{100}$, $\frac{005}{1000}$. But five hundredths equal only

one tenth part of five tenths; and five thousandths, one tenth part of five hundredths. Hence, ciphers placed on the left of a decimal, diminish its value as above specified.

The following numbers may now be expressed by figures, and then read:

1. Seventy-six and six tenths. *Ans.* 76.6.
2. One and three hundredths. *Ans.* 1.03.
3. Eighty and fifty-eight thousandths. *Ans.* 80.058.
4. One hundred and fifty-six, and thirty-nine thousandths. *Ans.* 156.039.
5. One hundred and one and five thousandths. *Ans.* 101.005.

§ **89.** The scholar will observe, that, if there is but one decimal figure, and that tenths, it requires the point only to be placed at the left of it, to express its true value; if it be hundredths, it requires a cipher to be placed at the left of it; and if it be thousandths, it requires two ciphers to be thus placed, with the decimal point on the left of the ciphers; and so on, according to the denomination.

6. Write down nine, and three hundred thousandths. *Ans.* 9.00003.
7. Write down twelve, and one millionth. *Ans.* 12.000001.
8. Three hundred and seventy-five, and seven tens of thousandths. *Ans.* 375.0007.
9. Ninety-five hundredths.
10. Three hundred and sixteen thousandths.
11. Forty-five millionths.
12. Sixty-nine, and nine hundred and three thousandths.
13. Four hundred and fifty-six, and seventeen millionths.
14. Five hundred, and three tens of millionths.
15. One, and six hundred of millionths.
16. Eleven, and seven billionths.
17. Seven hundred and sixty-two billionths.
18. Four hundred and twenty-one, and nineteen thousandths.
19. Seven hundred and six, and one hundred and three millionths.
20. Twelve hundred and six trillionths.

The scholar is now requested to turn back to Federal Money, and compare the denominations there given with those here brought to view. He will there find the dollar given as the unit money; the dime as the tenth part of the dollar; the cent as the tenth part of the dime, or the hundredth part of the dollar; and the mill as the tenth part of the cent, the hundredth part of the dime, and the thousandth part of the dollar. It is therefore obvious, that Federal Money and decimals are operated upon by the same general principles.

1. Reduce \$21, 8 dimes, and 6 mills, to mills. *Ans.* 21806.
2. Reduce 21806 mills to dollars, cents, and mills. *Ans.* \$21.806.
3. Reduce \$12, 3 dimes, 4 cents, and 9 mills, to mills. *Ans.* 12349.
4. Reduce 12349 mills to dollars, cents, and mills. *Ans.* \$12.349.
5. Reduce \$25 to cents. *Ans.* 2500.
6. Reduce \$9 to mills. *Ans.* 9000.

To reduce dollars to cents, we therefore add two ciphers, and to reduce them to mills we add three.

7. Reduce 2567 cents to dollars. *Ans.* \$25.67, or \$25 and 67 cents.
8. Reduce 38679 mills to dollars, &c. *Ans.* \$38.679, or \$38, 67 cents, and 9 mills.

To reduce cents to dollars, we therefore cut off two figures, and to reduce mills to the same, we cut off three figures from the right of the given number.

9. Reduce \$2 to mills.
10. Reduce 99 cents to mills.
11. Reduce \$1.03 to mills.
12. Reduce 467 cents to dollars.
13. Reduce 12008 mills to dollars.
14. Reduce \$42 and 3 mills to mills.
15. Reduce 9000 mills to dollars.

ADDITION OF DECIMAL FRACTIONS.

§ **90.** The scholar must here exercise a good degree of caution in writing the numbers to be added. He will recollect that, in adding vulgar fractions, it was necessary to reduce them all to the same name or denominator, before the numerators could be added; and that, in Simple and Compound Addition, the same denominations only could be united. The same is true of Decimal Fractions. Hence, we have the following rule:—

Rule.—*Place the whole numbers as in Simple Addition, with units under units, and tens under tens, &c. Also place the decimals on the right of the whole numbers, with tenths under*

tenths, hundredths under hundredths, and thousandths under thousandths, &c.; then, beginning with the lowest denomination, add up and carry, as in Simple Addition. Lastly, from the amount point off as many decimals as are equal to the greatest number of decimals in any one of the given numbers.

Ex. 1. What is the amount of 3.56; 42.923; 125.6; 4.32; and 59.365?

ADDED.

```
  3.56
 42.923
125.6
  4.32
 59.365
-------
235.768
```

The greatest number of decimals in either of the numbers given, is three; therefore, three decimals are to be cut off from the sum. It will always be found correct to place the decimal point in the amount, directly below those of the given numbers.

2. Add the following numbers, 325.63; 275.215; 1.02; 17.653; 136.1. *Amount*, 755.618.

3. What is the amount of 72.5; 32.071; 2.1574; 371.4; 2.75? *Ans.* 480.8784.

4. What is the amount of 225.75; 25.50; 8.255; 27.225? *Ans.* 286.73.

5. What is the amount of 35.175; 75.15; 13.31; 25.755? *Ans.* 149.39.

6. What is the amount of 304.39; 291.09; 136.99; 12.10? *Ans.* 744.57.

7. What is the amount of 365.541; 487.06; 94.67; 472.5; 439.089? *Ans.* 1858.860.

8. What is the amount of 2.151; 375.422; .675; .4567? *Ans.* 378.7047.

9. Add together the following decimals, viz., sixteen hundredths; two hundred and thirty-five thousandths; six tenths; and one thousandth. *Ans* .996.

10. What is the sum of one tenth; two hundredths; three thousandths; four tens of thousandths; five hundreds of thousandths, and six millionths? *Ans.* .123456.

11. What is the amount of $72.375; $41.176; $1.47; $395.20; $56.65; $146.73; and $0.16? *Ans.* $713.761.

12. Bought a yoke of oxen, for $121.56; a horse, for $156.375; a cow, for $37.086; and a quantity of grain, for $95.739. What was the cost of the whole? *Ans.* $410.760.

SUBTRACTION OF DECIMALS.

§ **91.** The scholar will need no explanation of the nature of this rule. His knowledge of Subtraction, and of the peculiarity of decimals, will enable him at once to make a correct application of the following rule: —

Rule. — *Set down the less number under the greater, so that each figure in the lower number, or subtrahend, shall stand directly under one of its own name or denomination. Proceed to subtract as in simple numbers, and place the separatrix as in addition of decimals.*

Ex. 1. From 378.635 take 195.275.

OPERATION.

378.635
195.275
―――――
183.360, remainder.

2. From 462.3 take 218.15. *Rem.* 244.15.
3. From 16.705 take 7.6845. *Rem.* 9.0205.
4. From 132.4 take 36.36. *Rem.* 96.04.
5. From 127.05 take 66.006. *Rem.* 61.044.
6. From 100.001 take 77.77. *Rem.* 22.231.
7. From five hundred thirty-six, and fifteen hundredths, take two hundred thirty-six, and eighteen hundredths. *Rem.* 299.97.
8. From six dollars take fifty-five cents. *Rem.* $5.45.
9. From one dollar take one mill. *Rem.* .999 mills.
10. From 16 dollars take nineteen cents and one mill. *Rem.* $15.809.

MULTIPLICATION OF DECIMALS.

§ **92.** **Rule.** — *Multiply as in whole numbers, and point off as many decimals in the product as there are decimals in both factors. Whenever the decimals in the product are not as many as those of the factors, the deficiency must be supplied by placing ciphers on the left of them.*

Ex 1. Multiply 25.16 by 3.45.

```
  25.16
   3.45
  -----
  12580
 10064
 7548
 -------
 86.8020
```

Four figures are cut off in the product as decimals, in accordance with the rule, there being four decimals in the two factors.

2	Multiply 175.2 by 45.72.	*Ans.* 8010.144.
3	Multiply 15.75 by 1.05.	*Ans.* 16.5375.
4.	Multiply 37.99 by 25.77.	*Ans.* 979.0023.
5.	Multiply 100.00 by 0.01.	*Ans.* 1.0000.
6.	Multiply 3.45 by .16.	*Ans.* .5520.
7.	Multiply 25.238 by 12.17.	*Ans.* 307.14646.
8.	Multiply 27.56 by 12.22.	*Ans.* 336.7832.
9.	Multiply .01 by .01.	*Ans.* .0001.
10.	Multiply 7.02 by 5.27.	*Ans.* 36.9954.
11.	Multiply .001 by .001.	*Ans.* .000001.
12.	Multiply .25 cents by .25 cents.	*Ans.* .0625, or $6\frac{1}{4}$ cts.

NOTE.—To multiply a decimal by 10, 100, 1000, &c., it is necessary only to remove the decimal point as many places to the right as there are ciphers in the multiplier.

13.	Multiply 1.56 by 10.	*Ans.* 15.6.
14.	Multiply 36.541 by 100.	*Ans.* 3654.1.
15.	Multiply .42 by 100.	*Ans.* 42.
16.	Multiply 46.3789 by 1000.	*Ans.* 46378.9.

DIVISION OF DECIMALS.

§ **93.** We are now to reverse the preceding operation. In multiplying decimals, we were directed to point off as many decimal figures in the product as there were in both factors. In division, the dividend corresponds to the product in multiplication, and the divisor to one of the factors which produced that dividend, and we are required to obtain the other factor. Therefore, the decimals of the quotient and divisor united must equal those of the dividend

Rule.—*Divide as in simple numbers, and point off from the right of the quotient as many decimals as are equal to the excess of decimals in the dividend, over those in the divisor.*

NOTE 1.—If the decimal places in the divisor be more than those in the dividend, annex ciphers to the dividend to make them equal.

2. If, after dividing, there be a remainder, ciphers may be annexed to the remainder, and the division continued. The ciphers thus added are decimals.

3. If the decimals in the divisor and dividend are equal, and there is no remainder after dividing, the quotient will be a whole number.

4. If the figures in the quotient do not equal the excess of decimal places in the dividend over those of the divisor, supply the defect by prefixing ciphers.

5. To divide the decimal number by 10, 100, 1000, &c., it is necessary only to remove the point as many figures to the left as there are ciphers in the divisor.

Ex. 1. Divide 34.317 by 21.75.

PERFORMED.

```
21.75)34.317(1.577+
      21.75
      -----
      12567
      10875
      ------
       16920
       15225
       ------
        16950
        15225
        -----
        1725, Rem.
```

In the solution of this example, two ciphers have been added to the remainders of the dividend. By Note 2, the whole number of decimals in the dividend is five, and there are two in the divisor; three should, therefore, be cut off from the quotient. The plus sign in the quotient always implies a remainder.

2. Divide 30515.50 by 100. *Ans.* 305.1550.

For the solution of the preceding sum, see Note 5.

3. Divide 483.125 by 386.5. *Ans.* 1.25.
4. Divide 198.15625 by 186.5. *Ans.* 1.0625.
5. Divide .56 by 1.12. *Ans.* .5.
6. Divide 99.99 by 33.3. *Ans.* 3.0027 +.
7. Divide 1.00 by .12. *Ans.* 8.333 +.
8. Divide 14325.16 by 1.33. *Ans.* 10770.721 +.
9. Divide 36.5 by 10. *Ans.* 3.65.
10. Divide 36.5 by 100. *Ans.* .365.
11. Divide 981 by 1000. *Ans.* .981.
12. Divide 543.67 by 3.46. *Ans.* 157.13 +.

APPLICATION.

1. If 36.34 bushels of corn grow on an acre, how many acres will produce 674 bushels? *Ans.* 18.547 + acres.

2. If 6 yards of cloth cost $24.48, what was the price per yard? *Ans.* $4.08.

3. Bought 56.87 yards of cloth, at $2.31 per yard. What was the whole cost? *Ans.* $131.3697.

4. The first of three men possessed \$685.423; the second, \$746.03; and the third, \$10864.273. How much had they all? *Ans.* \$12295.726.

5. What cost 9.6 yards of cloth, at \$6.42 per yard? *Ans.* \$61.632.

6. If a man earn 2 dollars 2 mills per day, how much would he earn in 93.5 days? *Ans.* \$187.1870.

7. What cost .675 of a cord of wood, at \$3 a cord? *Ans.* \$2.025.

8. If a yard of cloth cost \$5.5625, how much will .25 of a yard cost? *Ans.* \$1.3906.

REDUCTION OF VULGAR AND DECIMAL FRACTIONS.

§ **94.** The value of a vulgar fraction is the quotient arising from dividing the numerator by the denominator. Therefore,

CASE I.

To reduce a Vulgar Fraction to a Decimal.

Rule. — *Annex ciphers to the numerator, and divide it by the denominator.*

Note. — If the given fraction be proper, the quotient will always be a decimal, and will consist of figures equal in number to the ciphers annexed; or, if the number of figures be less, ciphers must be prefixed to complete the number.

Ex. 1. Reduce $\frac{3}{4}$ to a decimal.

OPERATION.

$$4)\underline{3.00}$$

.75, the decimal required.

2. Reduce $\frac{1}{5}$ to a decimal.

OPERATION.

$$5)\underline{1.0}$$

.2, the decimal required.

3. Reduce $\frac{1}{3}$ to a decimal. *Ans.* .333 +.
4. Reduce $\frac{1}{8}$ to a decimal. *Ans.* .125.
5. Reduce $\frac{15}{16}$ to a decimal. *Ans.* .9375.
6. Reduce $\frac{1}{10}$ to a decimal. *Ans.* .1.

7. Reduce $\frac{12}{13}$ to a decimal. *Ans.* .923 +.
8. Reduce $\frac{9}{10}$ to a decimal. *Ans.* .9.
9. Reduce $\frac{2}{3}$ to a decimal. *Ans.* .666 +.

CASE II.

§ **95.** TO REDUCE A DECIMAL TO A VULGAR FRACTION.

Rule. — *Write down the given decimal as a numerator, and, for a denominator, write* 1, *with as many ciphers annexed as there are figures in the numerator, and then reduce the fraction to its lowest terms.* (See remarks introductory to Decimals.)

1. Reduce the decimal .125 to a vulgar fraction. Performed: $\frac{125}{1000} \div 5 = \frac{25}{200}$, and again, $\frac{25}{200} \div 25 = \frac{1}{8}$, *Ans.* (See Case I., Vulgar Fractions.)

2. Reduce .75 to a vulgar fraction. Performed: $\frac{75}{100} \div 25 = \frac{3}{4}$ *Ans.*

3. Reduce .9375 to a vulgar fraction. *Ans.* $\frac{15}{16}$.
4. Reduce .2 to a vulgar fraction. *Ans.* $\frac{1}{5}$.
5. Reduce .16 to a vulgar fraction. *Ans.* $\frac{4}{25}$.
6. Reduce .25 to a vulgar fraction. *Ans.* $\frac{1}{4}$.
7. Reduce .45 to a vulgar fraction. *Ans.* $\frac{9}{20}$.
8. Reduce .55 to a vulgar fraction. *Ans.* $\frac{11}{20}$.
9. Reduce .8 to a vulgar fraction. *Ans.* $\frac{4}{5}$.
10. Reduce .24 to a vulgar fraction. *Ans.* $\frac{6}{25}$.
11. Reduce .945 to a vulgar fraction. *Ans.* $\frac{189}{200}$.
12. Reduce .844 to a vulgar fraction. *Ans.* $\frac{211}{250}$.

CASE III.

§ **96.** TO REDUCE LOWER DENOMINATIONS TO DECIMALS OF A HIGHER DENOMINATION.

Rule 1. — *Write down the several denominations which are to be reduced to decimals of a higher denomination, one above another, with the lowest uppermost; then divide each denomination, commencing with the lowest, by that number which is required of each to make a unit of the next higher denomination; and at each division place the quotient, as a decimal, on the right of the next higher denomination. The number last obtained will be the required decimal.*

NOTE. — It will be obvious that the division of the lowest denomination must be effected by adding ciphers to that denomination. Ciphers must also be added to each of the higher denominations to reduce them, unless the decimal figures previously obtained be sufficient.

The reason of the above rule is readily shown. Suppose it is required to reduce 7 pence to the fraction of a shilling. The fraction would be $\frac{7}{12}$, because the shilling is divided into 12 equal parts, and 7 of these parts are taken, and this vulgar fraction is reduced to a decimal by adding ciphers to its numerator and dividing by its denominator. (See Case I. of Decimals.)

Ex. 1. Reduce 7 s. 6 d. to the decimal of a pound sterling. The statement would be, $\begin{array}{r|l} 12 & 6 \\ 20 & 7 \end{array}$ That is, the 6 d. is to be divided by 12, and the 7 s. by 20. This must be effected by adding ciphers.

OPERATION.

$$\begin{array}{r|l} 12 & 6.0 \\ 20 & 7.500 \\ \hline & .375 = \text{required decimal.} \end{array}$$

2. Reduce 9 d. 2 qr. to the decimal of a pound sterling.

OPERATION.

$$\begin{array}{r|l} 4 & 2.0 \\ \hline 12 & 9.500000 \\ \hline 20 & .791666 \\ \hline \textit{Ans.} & .0395833+. \end{array}$$

Rule 2. — *Reduce the given quantity to its lowest denomination, and divide it by a unit of the denomination of the required fraction, reduced to the same denomination.*

Ex. 3. Reduce 10 s. 9 d. 2 qr. to the decimal of a pound sterling.

10 s. 9 d. 2 qr. = 518 qr.; and, by the rule, 518 qr. are to be divided by 1 £. reduced to qr., viz., by 960 qr. Therefore, $\frac{518}{960}$ is the fractional answer, and 518 ÷ 960 = .539583 +, *Ans.*

To understand the above operation, the scholar should remember that 10 s. 9 d. 2 qr., or 518 qr., are to be divided into as many equal parts as there are farthings in 1 £., = 960 qr., and one of these parts = $\frac{518}{960}$, or the decimal .539583 +.

4. Reduce 9 s. 8 d. to the decimal of a pound sterling. *Ans.* .4833 +.

5. Reduce 3 qr. 16 lb. to the decimal of a cwt. *Ans.* .8928571 +.

6. Reduce 16 £. 12 s. 8 d. to a decimal expression. *Ans.* 16.633333 +.

7. Reduce 3 qr. 2 na. to the decimal of a yard. *Ans.* .875.

8. Reduce 2 roods and 20 rods to the decimal of an acre. *Ans.* .625.

9. Reduce 3 furlongs 16 rods to the decimal of a mile. *Ans.* .425.

10. Reduce 12 hours, 15 minutes, and 30 seconds, to the decimal of a day. *Ans.* .51076 +.

11. Reduce 2 cwt. 3 qr. 24 lb. to the decimal of a ton. *Ans.* .14821428.

CASE IV.

§ 97. To find the Value of a Decimal in Integers of lower Denominations.

Rule. — *Multiply the decimal by the number required to reduce it to the next lower denomination, and from the right of the product cut off as many figures as there are in the given decimal. The figures on the left of the point will be integers of the denomination next below that given. Proceed in the same way through all the denominations, and the figures on the left of the several points will be the answer required.*

This rule, being directly the reverse of the preceding, needs no explanation.

Ex. 1. What is the value of the decimal .5638 of a pound sterling?

OPERATION.

```
  .5638
     20
 -------
 11.2760 = the shillings and decimals of a shilling
     12    in .5638 of a pound sterling.
 -------
  3.3120 = the pence and decimals of a penny in
      4    .2760 of a shilling.
 -------
  1.2480 = the farthings and decimals of a farthing
           in .3120 of a penny.
```

The value of the above decimal in shillings, pence, &c., is 11 s. 3 d. 1.2480 qr.

Note. — The integers on the left of the points are the numbers which compose the answer.

2. What is the value of .75 of a pound sterling? *Ans.* 15 s.
3. What is the value of .53854 of a pound sterling? *Ans.* 10 s. 9 d. 1 qr., nearly.
4. What is the value of .625 of an acre? *Ans.* 2 roods, 20 rods.
5. What is the value of .148712678 of a ton? *Ans.* 2 cwt. 3 qr. 25 lb. 1 oz. +.
6. What is the value of .676 of a cwt.? *Ans.* 2 qr. 19 lb. 11 oz. +.
7. How many furlongs, &c., in .425 of a mile? *Ans.* 3 fur. 16 rods.
8. How many quarters, &c., in .66 of a yard? *Ans.* 2 qr. 2 nails, 1.26 inches.
9. How many roods, &c., in .321 of an acre? *Ans.* 1 rood, 11 rods, 10 yd. 8 feet.

10. What is the value of .875 of a hogshead of wine?
Ans. 55 gal. 1 pint.

11. What is the value of .875 of a yard?
Ans. 3 qr. 2 nails.

12. What is the value of .9 of an acre?
Ans. 3 roods, 24 rods.

To reduce Shillings, Pence, and Farthings, to the Decimal of a Pound BY INSPECTION.

1 s. $= \frac{1}{20}$ or .05 of a pound. 2 s. $= \frac{2}{20} = \frac{1}{10}$ or .1 of a pound. Hence, every 2 s., in any given number of shillings, makes a *tenth* of a pound. Again, 1 £. = 960 qr., or 1 qr. $= \frac{1}{960}$ £. If, then, there were 40 qr. more in a pound, one qr. would equal $\frac{1}{1000}$, or .001 £.; and since 40 is $\frac{1}{24}$ part of 960, (for 960 ÷ 40 = 24,) if any given value, in pence and farthings, be increased by $\frac{1}{24}$ part of itself, the result will be thousandths of a pound; that is, 24 qr. $= \frac{25}{1000}$, or .025 £. It is, however, sometimes difficult to obtain just $\frac{1}{24}$ part of a given number of farthings. To avoid perplexity from this source, it will be found sufficiently accurate for practical purposes, to increase the farthings in the given pence and farthings by 1, when they exceed 12, and by 2, when they exceed 36.

From the above we derive the following rule: —

Rule. — *Take half the greatest even number of shillings, and call them so many tenths of a pound; also call the odd shilling (whenever there is one) 5 hundredths of a pound. Reduce the pence and farthings to farthings, and, having increased the number by* 1, *when they exceed* 12, *and by* 2, *when they exceed* 36, *call them so many thousandths of a pound. The amount of these several decimals will be the decimal required.*

13. Find the decimal value of 19 s. 5 d. 3 qr.

The greatest even number in 19 is 18, and 18 ÷ 2 = .9 £., and the odd shilling = .05 £. By uniting these two decimals, we obtain .95 £. Again, 5 d. 3 qr. = 23 qr., which, when increased by 1, (see Rule,) = .024 £., and this, united with the preceding decimal, gives .974 £. as the decimal value of 19 s. 5 d. 3 qr.

By reversing the preceding, any decimal of a pound, consisting of not more than three figures, may readily be reduced to shillings, pence, and farthings.

Rule. — *Double the left-hand figure of the decimal, for shillings, and increase this number by* 1, *whenever the second figure is* 5 *or more than* 5. *After deducting the* 5 *from the second figure, consider the remainder of the decimal so many farthings, abating* 1, *when it exceeds* 12, *and* 2, *when it exceeds* 36.

14. Find the integral value of .478 £. in shillings, pence, &c.

.4 tenths of a pound, ...	=	8 shillings.
.05 hundredths of a pound, ...	=	1 do.
.028 thousandths (abating 1) = 27 qr. . . .	=	6 d. 3 qr.
.478 £.	=	9s. 6 d. 3 qr.

15. Find by inspection the decimal value of 9s. 9 d. 3 qr. *Ans.* .491 £.

16. Find by inspection the integral value of .666 £. *Ans.* 13 s. 3 d. 3 qr.

17. Find by inspection the decimal value of 17 s. 6 d. 1 qr. *Ans.* .876 £.

18. Find by inspection the integral value of .962 £. *Ans.* 19 s. 3d.

APPLICATION.

1. What is the sum of 17 and sixteen hundredths, 12 and five hundredths, 216 and one tenth, and 16 and one hundredth? *Ans.* 261.32.

2. How many pounds of tea are there in three chests, each weighing as follows, viz., the first, 72 and one hundredth pounds; the second, 66 and three hundredths pounds; and the third, 60 and nine thousandths pounds? *Ans.* 198.049 lb.

3. A merchant bought 5 parcels of cloth; the first of which contained 42 and five thousandths yards; the second, 63 and nine tenths yards; the third, 57 and three hundredths yards; the fourth, 60 and three millionths yards; and the fifth, 16 and seven tenths yards. How many yards did he buy? *Ans.* 239 yd. 2.540012 qr.

4. If one man mow 3.15 acres of grass in a day, how many acres will 16 men mow? *Ans.* 50.40 acres.

5. A has $726, and B has $89, 7 dimes, 3 cents, and 9 mills. How much has A more than B? *Ans.* $636.261.

6. Bought 26.39 yards, at $1.32 per yd. What was the whole cost? *Ans.* $34.8348.

7. Bought 7.3 yd. of cloth, for $38.69. What was the value of one yard? *Ans.* $5, three dimes.

8. What will 75 hundredths of a cord of wood cost, when a cord is worth $3? *Ans.* $2.25.

9. What is the value of .675 £.? *Ans.* 13 s. 6 d.

10. Find by inspection the value of .337 £. *Ans.* 6 s. 8 d. 3 qr.

11. What is the difference between .96 of a day and .42 of a day? *Ans.* 12 h. 57 m. 36 sec.

12. Find by inspection the decimal value of 15 s. 9 d. 3 qr. *Ans.* .791 £.

13. Reduce .595 to a vulgar fraction. *Ans.* $\frac{119}{200}$.

14. What is the value of .9628 of a year, allowing the year to contain just 365 days? *Ans.* 351 days, 10 h. 7 m. 40.8 sec.

15. A man, having a certain sum of money, gained, in six years, a sum equal to 87 hundredths of it; and, in 9 years more, he doubled the amount he had at the expiration of the 6 years. How many times his original sum of money had he at the expiration of the last-mentioned period? *Ans.* 3.74 times.

16. What is the value of 3.66 £.? *Ans.* 3 £. 13 s. 2 d. 1.6 qr.

17. A traveled 250 miles by rail-road, and C traveled .75 of that distance, in the same time, by steam-boat. How many miles did C travel? *Ans.* 187.5 miles.

18. What would be the cost of 17 cwt. 3 qr. of wool, if 1 cwt. be worth $35.75? *Ans.* $634.5625.

19. What would $4\frac{3}{4}$ bales of cotton cost, each weighing 3.56 cwt., at $12.50 per cwt.? *Ans.* $211.375.

20. What would be the value of 4 A. 2 R. of land, valued at $78.625 per acre? *Ans.* $$353.81\frac{1}{4}$.

QUESTIONS.—How is the unit divided in Vulgar Fractions? How is it divided in Decimal Fractions? In what ratio do the several denominations decrease? What are they called? What relation is there between integers and decimals? What is the only important consideration which has not already been explained? How many terms are required to express decimals? How may that term be regarded? Repeat the table of whole numbers and decimals. Of what does the denominator of decimal fractions always consist? How do ciphers, added to the right of a decimal, affect its value? Give the illustration. How do ciphers, placed on the left of a decimal, affect its value? Give the illustration. How do decimals and the denominations of Federal Money correspond? How do we reduce dollars to cents? How do we reduce them to mills? How do we reduce cents and mills to dollars? What is the rule for the addition of decimals? What is it for subtracting decimals? What is the rule for multiplying decimals? How does division stand related to multiplication? What is the rule for division of decimals? What is the rule for pointing off decimals in addition of decimals? What in subtraction? What in multiplication? And what in division? On what does the value of a fraction depend? What is the rule for reducing vulgar fractions to decimals? What is Case II.? What is the rule for it? What is Case III.? What is the rule? What note follows the rule? Explain the nature of the rule. What is the second rule? What is Case IV.? What is the rule for it? What note follows the rule?

REDUCTION OF CURRENCIES.

§ 98. The currency of the United States was originally pounds, shillings, and pence, the same as in England. This, however, was abolished by an act of Congress, in 1786, and Federal Money, consisting of dollars, cents, and mills, was adopted.

The following table gives the value of the dollar in the old currency of several states: —

TABLE OF CURRENCIES.

In English or Sterling Money, ... $\$1 = 4$ s. 6 d.
" Canada Currency, ... $\$1 = 5$ s., or $\$1 = \frac{1}{4}$ of 1 £.

" New England, " Virginia, " Kentucky, " Tennessee,	6 s. $= \$1$; hence, $\$1 = \frac{3}{10}$ of 1 £., and 1 £. $= \frac{10}{3}$ of $\$1$;	called *New England Currency.*
" New York, " Ohio, " N. Carolina,	8 s. $= \$1$; hence, $\$1 = \frac{2}{5}$ of 1 £., and 1 £. $= \frac{5}{2}$ of $\$1$;	called *New York Currency.*
" New Jersey, " Pennsylvania, " Delaware, " Maryland,	7 s. 6d. $= \$1$; hence, $\$1 = \frac{3}{8}$ of 1 £., and 1 £. $= \frac{8}{3}$ of $\$1$;	called *New Jersey Currency.*
" S. Carolina and " Georgia,	4 s. 8 d. $= \$1$; hence, $\$1 = \frac{7}{30}$ of 1 £., and 1 £. $= \frac{30}{7}$ of $\$1$;	called *Georgia Currency.*

That the scholar may understand why the dollar is composed of a different number of shillings and pence in the different states, it is necessary only to say, that these states (or colonies, as they were at first called) originally issued each their own money, in pounds, shillings, and pence, the value of which soon depreciated. This depreciation was greater in some states than in others; and hence, when Federal Money was adopted, more of the old currency was required in some states than in others, to equal the dollar of the new currency. The value of this dollar was 4 s. 6d. Sterling Money, or English currency; while 6 s. New England currency, 8 s. New York, 7 s. 6 d. New Jersey, and 4 s. 8 d. Georgia currency, were required to equal the same value.

To understand changing these several currencies to dollars, cents, and mills, the scholar needs carefully to examine the preceding table. He will there observe that, in New England currency, 1 £. $= \frac{10}{3}$ of $1. Therefore, if pounds of that currency be multiplied by 10, and the product divided by 3, they will be changed to Federal Money, or dollars, cents, and mills. A similar explanation is applicable to other currencies.

Note. — If the given money consist of pounds, shillings, and pence, reduce the shillings and pence to the decimal of a pound, (see Case III. of the Reduction of Vulgar and Decimal Fractions,) and then proceed according to the following rule: —

Rule. — *Notice, from the preceding table, what fraction of one dollar makes 1 £. of the given currency, and multiply the given sum by that fraction; that is, multiply it by the numerator, and divide the product by the denominator.*

Ex. 1. Reduce 72 £. New England currency to Federal Money. In N. E. currency the pound $= \frac{10}{3}$ of one dollar; therefore,

$$\begin{array}{r} 72 \\ \underline{10} \\ 3)\underline{720} \\ \$240, \textit{ Ans.} \end{array}$$

2. Reduce 75 £. 6 s. 8 d. N. E. currency to Federal Money.

$$\begin{array}{r|l} 12 & 8.0 \\ 20 & \underline{6.6666+} \\ & .3333 = \text{the decimal value of } 6\text{ s. }8\text{ d.; therefore,} \end{array}$$

75.3333 £. $\times 10 \div 3 = \$251.111$, *Ans.*

3. Reduce 15 £. 6 s. 6 d. New York currency to Federal Money. *Ans.* $38.3125.

To abbreviate the given operation by canceling, the following rule may be adopted: —

§ 99. **Rule for Canceling.** — *Place the given sum above a horizontal line, and, noticing as before what fraction of a dollar makes 1 £. of the given currency, write its numerator above the line, and its denominator below; then cancel, multiply, and divide.*

4. Reduce 48 £. 10 s. 6 d. New England currency to Federal Money.

$$\begin{array}{r|l} 12 & 6 \\ 20 & \underline{10.5} \\ & .525 = \text{decimal value of } 10\text{ s. }6\text{ d.} \end{array}$$

Therefore, $\dfrac{\overset{16.175}{\cancel{48.525}}.\ 10}{\cancel{3}}$, and $16.175 \times 10 = \$161.75$.

5. Reduce 240 £. New Jersey currency to Federal Money. *Ans.* $640.

6. Reduce 243 £. New Jersey currency to Federal Money. *Ans.* $648.

7. Reduce 150 £. South Carolina currency to Federal Money. *Ans.* $642.857+.

8. Reduce 27 £. New England currency to Federal Money. *Ans.* $90.

9. Reduce 304 £. 12 s. 9 d. New England currency to Federal Money. *Ans.* $1015.458.

10. Reduce 80 £. New York currency to Federal Money. *Ans.* \$200.

11. Reduce 84 £. 10 s. 6 d. New Jersey currency to Federal Money. *Ans.* \$225.40.

12. Reduce 100 £. New Jersey currency to Federal Money. *Ans.* \$266.666 +.

13. Reduce 16 £. Canada currency to Federal Money. *Ans.* \$64.

14. Reduce 96 £. Nova Scotia currency to Federal Money. *Ans.* \$384.

15. Reduce 112 £. Georgia currency to Federal Money. *Ans.* \$480.

16. Reduce 365 £. 10 s. 6 d. New York currency to Federal Money. *Ans.* \$913.812 +.

17. Reduce 29 £. 12 s. 8 d. Canada currency to Federal Money. *Ans.* \$118.533 +.

18. Reduce 276 £. 10 s. 3 d. South Carolina currency to Federal Money. *Ans.* \$1185.053 +.

19. Reduce 45 £. 12 s. New Jersey currency to Federal Money. *Ans.* \$121.60.

20. Reduce 42 £. 6 s. New England currency to Federal Money. *Ans.* \$141.

§ **100.** The scholar will now reverse the preceding, that is, he will change any sum of Federal Money to either of the preceding currencies. To do this, he must adopt the opposite mode of operation. Therefore,

Rule. — *Notice, from the preceding table, what fraction of 1 £. of the required currency makes \$1; then multiply the given sum by the numerator of that fraction, and divide the product by the denominator.*

Ex. 1. Reduce \$161.75 to pounds, shillings, and pence, New England currency.

PERFORMED.

$$\begin{array}{r} 161.75 \\ 3 \\ \hline 485.25 \div 10 = 48.525\text{ £.} \end{array}$$

Now, to find the value of the decimal .525, (see Case IV., Decimal Fractions,) $.525 \times 20 = 10.500$ s., and $.500 \times 12 = 6.000$ d.; therefore the required pounds, shillings, &c., are 48 £. 10 s. 6d.

2. Reduce \$38.3125 to pounds, shillings, and pence, New York currency. *Ans.* 15 £. 6 s. 6 d.

These sums may also be solved by canceling. Take the following rule: —

Rule FOR CANCELING. — *Place the given dollars, cents, and mills, above a horizontal line, and, noticing as before what fraction of 1 £. of the required currency makes $1, place the numerator of the fraction above the line, and the denominator below. Cancel, multiply, and divide for the answer.*

3. Reduce $95.75 to pounds, shillings, &c., in New England currency. *Ans.* 28 £. 14 s. 6 d.

$$\text{Statement: } \frac{95.75.\ \ 3}{10}.$$

4. Reduce $648 to pounds, New Jersey currency. *Ans.* 243 £.
5. Reduce $642.857 to pounds, South Carolina currency. *Ans.* 150 £.
6. Reduce $578 to pounds, &c., New York currency. *Ans.* 231 £. 4 s
7. Reduce $580 to pounds, &c., Canada currency. *Ans.* 145 £.
8. Reduce $742.50 to pounds, &c., New England currency. *Ans.* 222 £. 15 s.
9. Reduce $21.758 to pounds, &c., New England currency. *Ans.* 6 £. 10 s. 6 d. 2 qr.+.
10. Reduce $141 to pounds, &c., New England currency. *Ans.* 42 £. 6 s.
11. Reduce $250 to pounds, &c., Canada currency. *Ans.* 62 £. 10 s.
12. Reduce $121.60 to pounds, &c., New Jersey currency. *Ans.* 45 £. 12 s.
13. Reduce $475.75 to pounds, &c., New York currency. *Ans.* 190 £. 6 s.
14. Reduce $89.54 to South Carolina currency. *Ans.* 20 £. 17 s. 10 d.
15. Reduce $75 to pounds, &c., New England currency. *Ans.* 22 £. 10 s.
16. Reduce $384 to pounds, &c., Nova Scotia currency. *Ans.* 96 £.

QUESTIONS. — What was the original currency of the United States? When this was abolished, what currency was substituted? What are the denominations of Federal Money? What is the value of the dollar, New England currency? What fraction of a dollar does the pound of that currency equal? What fraction of a pound does the dollar equal? Similar questions should be asked relative to the currencies of the other states. The scholar may explain why it is that the dollar is composed of a different number of shillings and pence in the different states. What note precedes the rule? What is the rule for bringing pounds, &c., into dollars? How are the terms arranged for canceling? What is the rule for bringing dollars into pounds, &c.? What is the rule for canceling?

SIMPLE INTEREST.

§ **101.** Interest is an allowance made for the use of money. It is computed at a certain rate per cent.; that is, at a *certain number of dollars for the use of a hundred for one year.*

If the sum on interest be more or less than $100, or the time during which it draws interest, more or less than one year, the *sums paid as interest* must be in proportion both to the time and the sum lent.

For illustration; if $100 be borrowed for one year, and if the interest allowed be 6 per cent., the interest for one year will be $6. Now, if twice that sum, viz., $200, be borrowed for the same time, or if the same sum be borrowed for twice that time, viz., for two years, the interest will in either case be twice that sum, viz., $12. Again, if twice the sum be borrowed for twice the time, that is, if $200 be borrowed for two years, the sum to be paid as interest will be increased fourfold; or it would be $6 \times 4 =$ $24.

The scholar will carefully notice the following particulars:

1st. The *principal* is the money lent, or the sum on which interest is paid.

2d. The *interest* is the money paid for the use of the principal. *Legal interest* is that established by law. In the New England states, it is fixed at 6 per cent.; in New York, at 7, and in Louisiana, at 8 per cent.

3d. The *amount* is the sum obtained by adding the interest to the principal.

4th. The *rate per cent.* is always a decimal of two places, when expressed by cents, and of three, when expressed by cents and mills.

CASE I.

To cast Interest on any Sum for one or more Years.

Rule.—*Multiply the principal by the rate per cent. written as a decimal; the product will be the interest for one year; which, being repeated as many times as there are years given, will be the required interest.*

Note 1.—In all cases where no rate per cent. is mentioned, six per cent. is always implied.

Ex. 1. What is the interest of $215, for 1 year, at 6 per cent.?

It is obvious that the required interest will be 6 times 215 cents, for it is 6 cents on each dollar. Therefore,

```
  2 1 5
   . 0 6
 -------
1 2 9 0 cents = $12.90, Ans.
```

On a note of $215, which had been on interest two years, there would then be due $227.90; that is, $215, principal, + $12.90, interest, = $227.90, the amount. Therefore, the amount is found by adding the principal and interest together.

2. What is the interest of $47.86, for 3 years, at 6 per cent. per annum?

OPERATION.

```
 4 7 . 8 6
     . 0 6
 ---------
 2 . 8 7 1 6 = interest for one year.
           3
 ---------
$8 . 6 1 4 8 = interest for three years.
```

To understand why 4 figures are cut off in this sum, the scholar should remember that one cent is $\frac{1}{100}$ of a dollar; and consequently 6 cents are $\frac{6}{100}$ of a dollar, which is the same as .06. (See introductory remarks to Decimal Fractions.)

3. What is the interest of $72.72, for 4 years, at 6 per cent.? *Ans.* $17.45 +.

4. What is the amount of $456, for 2 years? *Ans.* $510.72.

5. What is the interest of $146.31, for 5 years, at 6 per cent.? *Ans.* $43.893.

6. What is the interest of $24.91, for 6 years, at 5 per cent.? *Ans.* $7.473.

7. What is the interest of $222.46, for 4 years, at 3 per cent.? *Ans.* $26.695 +.

8. What is the amount of $42, for 6 years, at 3 per cent.? *Ans.* $49.56.

9. What is the amount of $566.33, for 1 year, at 5 per cent.? *Ans.* $594.646 +.

10. What is the amount of $1567 for 9 years, at 2 per cent.? *Ans.* $1849.06.

CASE II.

When the Time consists of Years and Months.

Lawful interest in the New England states is 6 per cent. per annum; that is, it is 6 cents for 12 months, or $\frac{1}{2}$ cent per month on a single dollar. Therefore,

Rule.—*Reduce the given years to months, and add the given months; then, with half this number of months as a multiplier, multiply the given sum. The product will be the interest for the whole time.*

NOTE 2.—It is obvious that half the whole number of months in the given time, is the same as the number of cents on a dollar for the whole time, when the interest is at 6 per cent.; for, from the above remark, the interest of one dollar for one month is evidently $\frac{1}{2}$ per cent.; therefore, the whole number of months, divided by 2, must determine the number of cents on each dollar for the whole time. If, in taking half the number of months, there be an odd one, the interest for that odd month will be $\frac{1}{2}$ cent, or 5 mills.

Ex. 1. What is the interest of $220, for 2 years and 6 months, at 6 per cent.?

2 yr. 6 mo. = 30 months, and 30 ÷ 2 = 15. The interest on each dollar for the whole time is, therefore, 15 cents. Consequently, $33 is the interest of the given sum.

$$\begin{array}{r} 220 \\ .15 \\ \hline 1100 \\ 220 \\ \hline \$33.00 \end{array} \quad \textit{Ans.}$$

2. What is the interest of $756.20, for 1 year and 3 months, at 6 per cent.?

1 yr. 3 mo. = 15 months; therefore, $7\frac{1}{2}$ cents, or 7 cents 5 mills, is the interest on each dollar for the whole time. Therefore,

$$\begin{array}{r} 756.20 \\ .075 \\ \hline 378100 \\ 529340 \\ \hline 56.71500 \end{array} \quad \textit{Ans.}\ \$56.715.$$

NOTE 3.—If the interest be required at some other than 6 per cent., first cast it at 6 per cent. by the preceding rule. Then divide the interest so found by 6, and the quotient will be the interest of the given sum, at 1 per cent. for the time specified; and this, multiplied by the given per cent., will be the required interest.

3. What is the interest of $656, at 8 per cent. per annum, for 3 years and 4 months?

3 yr. and 4 mo. = 40 months, and 40 ÷ 2 = 20, the per cent. for the whole time. Therefore,

$$\begin{array}{rl} 656 & \\ .20 & \\ \hline 6\,)\,131.20 & = \text{interest at 6 per cent.} \\ \hline 21.8666 & = \text{interest at 1 per cent.} \\ 8 & \\ \hline 174.9328 & = \text{interest at 8 per cent., viz.} \\ & \quad \$174.932+. \end{array}$$

4. What is the interest of $37.50, for 3 years and 6 months, at 6 per cent.? *Ans.* $7.875.

5. What is the interest of $672, for 3 years and 8 months, at 6 per cent.? *Ans.* $147.84.

6. What is the interest of $372, for 2 years and 11 months, at 5 per cent.? *Ans.* $54.25.

7. What is the interest of $215.34, for 4 years and 6 months, at $3\frac{1}{2}$ per cent.? *Ans.* $33.916 +.

8. What is the interest of $350, for 2 years and 6 months, at 6 per cent. *Ans.* $52.50.

CASE III.

When the given Time consists of Years, Months, and Days.

It was shown, in the preceding case, that the interest of $1 for 1 month is $\frac{1}{2}$ cent = 5 mills, whenever the rate is 6 per cent. Now, since 30 days are always allowed for a month, in computing interest, 5 mills also equal the interest for 30 days; and $30 \div 5 = 6$, the number of days required for 1 dollar, at 6 per cent. per annum, to gain one mill. If, therefore, the number of days given be divided by 6, the quotient will be the number of mills to which the interest of one dollar will amount during that time. Therefore,

Rule. — *Find the interest of one dollar for the given time, by allowing half a cent for every month, and one mill for every six days; the sum thus obtained will be the per cent. for the whole time. Multiply the principal by this, and the product will be the interest.*

Note 4. — If the given days be less than 6, or if they be more, and, in taking a sixth part of them, a number remain, (which in all cases will be less than 6,) the interest for those days will be a fraction of a mill; that is, it will be as many sixths of a mill as there are days.

Note 5. — This rule always gives the interest at 6 per cent. per annum. If, therefore, any other per cent. be required, proceed as directed in Note 3.

Ex. 1. What is the interest of $150, for 1 year, 3 months, and 15 days, at 6 per cent. per annum?

Solution: 1 year and 3 months = 15 months; the interest for that time is 15 half cents = $7\frac{1}{2}$ cents, or 7 cents, 5 mills; and the interest for 15 days is $15 \div 6 = 2\frac{1}{2}$ mills; therefore, 7 cents, 5 mills + $2\frac{1}{2}$ mills = 7 cents, $7\frac{1}{2}$ mills, which is the interest of 1 dollar, at 6 per cent., for the whole time. Consequently,

$$\begin{array}{r} 150 \\ .077\frac{1}{2} \\ \hline 1050 \\ 1050 \\ 75 \\ \hline \$11.625 \end{array}$$

= *Ans.*, or interest required.

2. What is the interest of $320, for 3 years, 4 months, and 18 days, at 6 per cent. per annum? *Ans.* $64.96.

3. What is the interest of $75, for 1 year, 6 months, and 12 days, at 6 per cent. per annum? *Ans.* $6.90.

4. What is the interest of $162, for 1 year, 8 months, and 15 days, at 6 per cent.? *Ans.* $16.605.

5. What is the interest of $350, for 2 years, 3 months, and 5 days, at 6 per cent.? *Ans.* $47.54 +.

6. What is the interest of $275, for 3 years, 6 months, and 12 days, at 4 per cent.? *Ans.* $38.866 +.

7. What is the interest of $560, for 9 months and 20 days, at 6 per cent.? *Ans.* $27.066 +.

8. What is the interest of $420, for 2 years, 1 month, and 27 days, at 6 per cent.? *Ans.* $54.39.

§ **102.** NOTE 6.—If the interest of $100 for 1 year be $6, the interest of $200 for the same time is obviously $12. Again; if the interest of $100 for 1 year be $6, for 2 years it must be twice as much, viz., $12; and for 3 years, three times as much, viz., $18. In casting interest, we are therefore to regard not only the principal, but also the time during which it is on interest. This may help the scholar to understand the following rule for casting interest by canceling.

Rule FOR CANCELING; AND, FIRST, WHEN YEARS ONLY ARE GIVEN.—*Draw a horizontal line, and write the sum on which the interest is to be cast, above it. On the right of this, also above the line, place the given time in years, and the rate per cent. Then place $100 below the line. Proceed to cancel, &c.; the sum obtained will be the required interest.*

Ex. 1. What is the interest of $300, for 3 years, at 6 per cent.?

$$\text{Statement: } \frac{300.\ 3.\ 6}{100}.$$

It is obvious that the interest of $300, for 1 year, will be three times as much as the interest of $100 for the same time; and that for 3 years it will be three times as great as for 1 year.

The above statement canceled: $\frac{\overset{3}{\cancel{300}}.\ 3.\ 6}{\cancel{100}}$; and $3 \times 3 \times 6 =$ $54, *Ans.*

2. What is the interest of $350, for 4 years, at 5 per cent. per annum?

$$\text{Statement: } \frac{350.\ 4.\ 5}{100.}.$$

$$\text{Canceled: } \frac{35\cancel{0}.\ \overset{2}{\cancel{4}}.\ \cancel{5}}{\underset{2}{\cancel{100}}}; \text{ and } 35 \times 2 = \$70, \textit{Ans.}$$

3. What is the interest of $500, for 5 years, at 9 per cent. per annum? *Ans.* $225.

4. What is the interest of $240, for 6 years, at 4 per cent. per annum? *Ans.* $57.60.

AGAIN, WHEN THE GIVEN TIME CONSISTS OF YEARS AND MONTHS.

Rule.—*Having reduced the given time to months, arrange the terms as in the preceding rule, and in addition to the terms there introduced, place* 12, *the number of months in one year, below the line. Cancel, &c.*

NOTE 7.—If the interest of $100 for 1 year, be $6, for 2 years, it is $12, &c. (See the preceding Note.) Consequently, for 1 year and 6 months, it would be $\frac{3}{2}$ of $6 $=\frac{18}{12}$, and for 2 years and 9 months, $\frac{11}{4}=\frac{33}{12}$ of $6, &c. These fractional expressions are obtained by reducing the given time to months, for the numerator, and making 12, the number of months in a year, the denominator. Hence we see the reason of the above rule.

Ex. 5. What is the interest of $360, for 2 years and 6 months, at 6 per cent. per annum? Solution: 2 years, 6 months $=$ 30 months; therefore, $\frac{360.\ 30.\ 6}{100.\ 12}$. It will be observed that the whole time is $\frac{30}{12}$ of a year.

$$\text{Canceled: } \frac{\overset{3}{\cancel{360}}.\ \cancel{30}.\ 6}{\cancel{100}.\ \cancel{12}}; \text{ and } 3\times3\times6=\$54, \textit{ Ans.}$$

6. What is the interest of $440, for 2 years and 4 months, at 6 per cent. per annum?

The time $=\frac{28}{12}$ of a year; therefore, $\frac{440.\ 28.\ 6}{100.\ 12}$. *Ans.* $61.60.

7. What is the interest of $150, for 1 year and 6 months, at 6 per cent. per annum?

Statement: $\frac{150.\ 18.\ 6}{100.\ 12}$. *Ans.* $13. 50.

8. What is the interest of $25.32, for 9 months, at 4 per cent. per annum?

Statement: $\frac{25.32.\ 9.\ 4}{100.\ 12}$. *Ans.* $0.759+.

9. What is the interest of $375.75, for 4 years and 9 months, at 7 per cent.? *Ans.* $124.936+.

10. What is the interest of $1500, for 3 years and 4 months, at 6 per cent.? *Ans.* $300.

11. What is the interest of $175, for 4 years and 2 months, at 8 per cent.? *Ans.* $58.333+.

Lastly, when the Time consists of Years, Months, and Days.

Rule.—*Reduce the years and months to months, and the days to the fraction of a month; then, placing this fraction on the right of the months, reduce the whole to an improper fraction. This expression will represent the whole time in months; therefore, having arranged the other terms as before, write its numerator above the horizontal line, and its denominator below, and proceed as before directed.*

Note 8. — As the fractional expression of the whole time obtained by the preceding rule, represents that time in months, 12, the number of months in one year, must be placed below the line, as before directed, to reduce those months to years.

Ex. 1. What is the interest of $150, for 1 year, 3 months, and 15 days, at 6 per cent.?

1 year and 3 months = 15 months; and 15 days = $\frac{1}{2}$ of a month; therefore $15\frac{1}{2}$ months is the whole time given, and this, reduced to an improper fraction, equals $\frac{31}{2}$ of a month.

Agreeably to the rule, we then have the following

$$\text{Statement, viz.: } \frac{150.\ 31.\ 6}{100.\ 2.\ 12}.$$

$$\text{Canceled: } \frac{\overset{3}{\cancel{150}}.\ 31.\ \cancel{6}}{\underset{2}{\cancel{100}}.\ 2.\ \underset{2}{\cancel{12}}};\ \text{then, } 31 \times 3 = 93, \text{ and } 2 \times 2 \times 2 = 8.$$

Therefore, $93 \div 8 =$ $11.625, *Ans*

2. What is the interest of $320, for 3 years, 4 months, and 18 days, at 6 per cent. per annum?

3 years and 4 months = 40 months, and 18 days = $\frac{18}{30} = \frac{3}{5}$ of a month. The whole time is therefore $40\frac{3}{5} = \frac{203}{5}$ of one month.

$$\frac{320.\ 203.\ 6}{100.\ 5.\ 12} = \$64.96.$$

3. What is the interest of $75, for 1 year, 6 months, and 10 days, at 6 per cent.? *Ans.* $6.875.

The time = $\frac{55}{3}$ of a month.

Note 9. — In interest, 30 days are always allowed for a month.

In solving the following sums, the scholar can apply either of the preceding rules.

4. What is the interest of $420, for 2 years, 1 month, and 27 days, at 6 per cent.? *Ans.* $54.39.

5. What is the interest of $80, for 1 year, 5 months, and 12 days, at 6 per cent.? *Ans.* $6.96.

6. What is the amount of $275 for 3 years, 6 months, and 12 days, at 4 per cent.? *Ans.* 313.866 +.

7. What is the amount of $560, for 9 months and 20 days, at 6 per cent.? *Ans.* $587.066 +.

8. What is the interest of $50.11, for 1 year and 21 days, at 6 per cent.? *Ans.* $3.18 +.

9. What is the interest of $340.50, for 3 months and 1 day, at 6 per cent.? *Ans.* $5.16 +.

10. What is the interest of $90, for 1 year, 2 months, and 6 days, at 6 per cent.? *Ans.* $6.39.

11. What is the interest of $4119.20, for 1 year and 5 days, at 6 per cent.? *Ans.* $250.584 +.

12. What is the interest of $23.08, for 3 years, 6 months, and 22 days, at 6 per cent.? *Ans.* $4.93 +.

13. What is the interest of $439.50, for 1 year, 11 months, and 9 days, at 6 per cent.? *Ans.* $51.20.

14. What is the amount of $28, for 9 years, 8 months, and 3 days, at 6 per cent.? *Ans.* $44.254.

15. What is the amount of $42, for 5 years, 5 months, and 9 days, at 5 per cent.? *Ans.* $53.427.

16. What is the amount of $50.50, for 1 year and 3 months, at 3 per cent.? *Ans.* $52.393 +.

17. What is the amount of $300, for 16 years and 8 months, at 6 per cent.? *Ans.* $600.

18. What is the interest of $375.75, for 4 years and 9 months, at 7 per cent.? *Ans.* $124.936 +.

19. What is the interest of $3.75, for 12 years, at 6 per cent. per annum? *Ans.* $2.70.

20. What is the amount of $75, for 6 years and 3 months, at 6 per cent.? *Ans.* $103.125.

21. What is the interest of $63, for 1 year and 3 months, at 6 per cent.? *Ans.* $4.725.

22. What is the interest of $156, for 2 years and 4 months, at 8 per cent.? *Ans.* $29.12.

23. What is the amount of $650, for 3 years and 2 months, at 6 per cent.? *Ans.* $773.50.

24. What is the amount of $128.30, for 1 year and 9 months, at 9 per cent.? *Ans.* $148.507.

25. What is the interest of $33.50, for 2 years and 6 months, at 5 per cent.? *Ans.* $4.1875.

26. What is the interest of $150, for 2 years, 7 months, and 7 days, at 6 per cent.? *Ans.* $23.425.

27. What is the interest of $730, for 5 years, 9 months, and 12 days, at 6 per cent.? *Ans.* $253.31.

28. What is the interest of $875.49, for 5 years, 8 months, and 20 days, at 6 per cent.? *Ans.* $300.58 +.

29. What is the amount of $630, for 8 months, at 6 per cent.? *Ans.* $655.20.

30. What is the amount of $7342, for 1 year and 4 months, at 6 per cent.? *Ans.* $7929.36.

31. What is the amount of $750, for 9 months, at 7 per cent.? *Ans.* $789.375.

32. What is the amount of $375, for 5 months and 15 days, at 6 per cent.? *Ans.* $385.31.

33. What is the amount of $460.50, for 4 months, at 6 per cent.? *Ans.* $469.71.

34. What is the amount of $230.25, for 8 months, at 7 per cent.? *Ans.* $240.995.

35. What is the amount of $764.50, for 3 years and 10 months, at 6 per cent.? *Ans.* $940.335.

CASE IV.

§ 103. To find the Interest, at Six per Cent., for any Number of Days.

In computing interest, the month is reckoned at 30 days, and the interest of one dollar for that time at 6 per cent., as has already been shown, is $\frac{1}{2}$ cent, or 5 mills; hence, for twice that time, or 60 days, it would be just 1 cent for every dollar on interest. We therefore have the following rule: —

Rule. — *Consider the number of dollars given so many cents, and reduce these cents to dollars again by removing the point of separation two places to the left; the result will be the interest of the given sum for 60 days. Then, if the given days be more or less than 60, add to, or subtract from, the interest of 60 days, such parts of itself as the given days require.*

Ex. 1. What is the interest of $450.82, for 93 days, at 6 per cent. per annum?

SOLUTION.

4.5082 = interest for 60 days. (See the Rule.)
2.2541 = interest for 30 days, being half the interest for 60 days.
.22541 = interest for 3 days, being $\frac{1}{10}$ of the interest for 30 days.

$6.98771 = interest for 93 days.

2. What is the interest of $4562, for 45 days, at 6 per cent. per annum?

SOLUTION.

45.62 = interest for 60 days.

22.81 = interest of 30 days, or one half the interest of 60 days.
11.405 = interest of 15 days, or one half the interest of 30 days.

$34.215 = interest of 30 + 15 = 45 days.

In computing by this rule, 12 months, of only 30 days each, are allowed for the year, equal to 360 days. It consequently gives $\frac{1}{73}$ part more than 6 per cent. interest. On small sums, and for short intervals, however, the difference is trifling.

3. What is the interest of \$420.72, for 120 days, at 6 per cent. per annum? *Ans.* \$8.414 +.

4. What is the interest of \$56.74, for 25 days, at 6 per cent. per annum? *Ans.* \$0.236 +.

5. What is the interest of \$156.36, for 96 days, at 6 per cent. per annum? *Ans.* \$2.50 +.

6. What is the interest of \$1000, for 29 days, at 6 per cent. per annum? *Ans.* \$4.833 +.

7. What is the interest of \$204, for 40 days, at 6 per cent. per annum? *Ans.* \$1.36.

8. What is the interest of \$472, for 18 days, at 6 per cent. per annum? *Ans.* \$1.416.

§ 104. Banking. — When a promissory note is presented at a banking institution, if properly endorsed, or otherwise secured, it is received by the officers of the bank as security, and so much money, in their own notes, is given in return as is equal to the face of the note after the interest is deducted for 3 days more than the whole time till payment is promised. Hence, if the time specified be 60 days, the interest on the face of the note for 63 days is deducted, and the balance drawn from the bank; then, at the expiration of the 63 days, the whole face of the note is due. The 3 days added to the specified time of payment are called "*days of grace.*"

The preceding rule is, therefore, a convenient one for banking institutions.

Note. — In solving the following sums, the scholar will allow "3 *days of grace;*" that is, he will find *bank discount* for 3 days more than are specified in the sum.

Ex. 1. What is the bank discount on \$256, for 30 days, and grace?

SOLUTION.

2.56	= discount for 60 days.
1.28	= discount for 30 days.
.128	= discount for 3 days' grace.
\$1.408	= required discount.

Therefore, \$256 — \$1.408 = \$254.592, the sum to be drawn from the bank.

2. How much money would be drawn from the bank on a note of \$650, payable in 90 days?

$$
\begin{array}{rl}
6.50 & = \text{discount for 60 days.} \\
3.25 & = \text{discount for 30 days.} \\
.325 & = \text{discount for 3 days.} \\
\hline
\$10.075 & = \text{discount for 93 days.}
\end{array}
$$

Therefore, \$650 — \$10.075 = \$639.925, the money to be drawn.

3. What is the bank discount on \$1056.29, for 30 days, and grace? *Ans.* \$5.81.
4. What is the bank discount on \$756, for 90 days, and grace? *Ans.* \$11.718.
5. What is the bank discount on \$676.19, for 25 days, and grace? *Ans.* \$3.155 +.
6. How much money may be drawn on a note of \$692, payable 70 days after date, if discounted at a bank? *Ans.* \$683.581.
7. How much money may be drawn on a note of \$1567.89, payable 120 days from date? *Ans.* \$1535.748.
8. What is the bank discount on \$542.78, for 90 days, and grace? *Ans.* \$8.413 +.
9. What is the bank discount on \$195.77, for 60 days, and grace? *Ans.* \$2.055 +.
10. How much money may be drawn on a note of \$726, payable 80 days from date? *Ans.* \$715.957.

CASE V.

§ 105. WHEN THE MONEY ON WHICH THE INTEREST IS TO BE CAST, IS IN POUNDS, SHILLINGS, AND PENCE.

Rule.—*Reduce the shillings and pence to the decimal of a pound, (see Case III., Decimal Fractions,) and cast the interest in the same manner as when the principal consists of dollars, cents, and mills. The decimal part of the answer may then be reduced to shillings and pence by Case IV., Decimal Fractions.*

Ex. 1. What is the interest on 42 £. 16 s. 6 d., for 1 year and 6 months, at 6 per cent. per annum?

SOLUTION.

1 yr. 6 mo. = 18 months, and 18 ÷ 2 = 9, the per cent. for the whole time.

$$
\begin{array}{r|l}
12 & 6 \\
20 & 16.5 \\
\hline
 & .825 = \text{the decimal value of 16 s. 6 d.}
\end{array}
$$

Therefore,

$$\begin{array}{r} 42.825£. \\ .09 \\ \hline 3.85425£. \end{array} = 3£.\ 17\text{s. }1\text{d.}$$ (See Case IV., Reduction of Decimal Fractions.)

2. What is the interest of 55 £. 15 s. 6 d. for 2 years, 6 months, at 6 per cent.? *Ans.* 8 £. 7 s. 3 d. 3 qr.
3. What is the interest of 21 £. 18 s. 4 d., for 3 years and 4 months, at 6 per cent.? *Ans.* 4 £. 7 s. 8 d.
4. What is the amount of 156 £. 9 s. 3 d., for 1 year and 9 months, at 6 per cent.? *Ans.* 172 £. 17 s. 9 d. 3 qr. +.
5. What is the amount of 27 £. 2 s. 6 d. 3 qr., for 1 year and 10 months, at 6 per cent.? *Ans.* 30 £. 2 s. 2 d. 3 qr. +.
6. What is the interest of 36 £. 15 s., for 2 years and 3 months, at 6 per cent.? *Ans.* 4 £. 19 s. 2 d. 2.8 qr.
7. What is the interest of 45 £. 10 s., for 8 months, at 6 per cent.? *Ans.* 1 £. 16 s. 4 d. 3.2 qr.
8. What is the interest of 9 £. 12 s. 6 d., for 2 years, 4 months, and 12 days, at 6 per cent.? *Ans.* 1 £. 7 s. 4 d. +.

CASE VI.

§ 106. WHEN INTEREST IS REQUIRED ON NOTES OR BONDS ON WHICH PARTIAL PAYMENTS HAVE BEEN MADE.

Rule. — *Cast the interest on the principal, at the given rate per cent., up to the time of the first payment; then, if the payment exceed the interest, deduct the excess from the principal; but if it be less, set both payment and interest aside, and cast the interest on the same principal to the time of the next payment, or to the time of some payment, which, when added to the preceding payments, will exceed the sum of interest then due, and deduct the sum of these payments from the amount of the principal. The remainder will form a new principal, with which proceed as before, till the time of settlement.*

1. For value received, I promise to pay A. B. & Co., or order, fifteen hundred dollars, on demand, with interest.
Jan. 1, 1825. JOHN JAMES.

On this note are the following endorsements: — Oct. 1, 1825, three hundred dollars; July 1, 1827, four hundred and fifty dollars; Sept. 1, 1828, six hundred and fifty dollars. What was due on settlement, July 1, 1830?

The principal, on interest from Jan. 1, 1825, ...	$1500.00
Interest from Jan. 1, 1825, to Oct. 1, of the same year, 9 mo. .	67.50
	1567.50
The payment being more than the interest, deduct it, . .	300.00
Remainder, forming a new principal, ...	1267.50
The interest from Oct. 1, 1825, to July 1, 1827, 1 yr. and 9 mo.	133.87
	1400.587
Deduct second payment, because it is more than the interest,	450.000
Remainder, forming a new principal, ...	950.587
The interest from July 1, 1827, to Sept. 1, 1828, 1 yr. and 2 mo.	66.541
	1017.128
Deduct the third payment, ...	650.000
Remainder, forming a new principal, ...	367.128
The interest from Sept. 1, 1828, to July 1, 1830, the time of settlement, ...	40.384
Ans.	$407.512

SOLUTION BY CANCELING.

yr.	m.	d.
1825	10	1
1825	1	1
	9	0

= time till first payment.

Therefore, $\dfrac{1500.\ 9.\ 6}{100.\ 12}$. Canceled: $\dfrac{15\cancel{00}.\ 9.\ \cancel{6}}{1\cancel{00}.\ \underset{2}{\cancel{12}}}$;

and $15 \times 9 \div 2 = \$67.50$, the interest till first payment; and $1500 + 67.50 = \$1567.50$, amount; and $1567.50 - 300 = \$1267.50$, the new principal.

yr.	m.	d.
1827	7	1
1825	10	1
1	9	0

= time from 1st to 2d payment.

Statement: $\dfrac{1267.50.\ 21.\ 6}{100.\ \ 12}$. Canceled: $\dfrac{1267.50.\ 21\ \cancel{6}}{100.\ \ \underset{2}{\cancel{12}}}$;

$1267.50 \times 21 \div 200 = 133.087$, the interest till 2d payment; then, $1267.50 + 133.087 = 1400.587$; and $1400.587 - 450 = 950.587$, the new principal.

yr.	m.	d.
1828	9	1
1827	7	1
1	2	0

= time from 2d to 3d payment.

Statement: $\dfrac{950.587.\ 14.\ 6}{100.\ \ 12}$. Canceled: $\dfrac{950.587.\ \overset{7}{\cancel{14}}.\ \cancel{6}}{100.\ \ \underset{\cancel{2}}{\cancel{12}}}$;

and $950.587 \times 7 \div 100 = 66.541$; interest till the third payment, and $950.587 + 66.541 = 1017.128$; and $1017.128 - 650 = 367.128$, the new principal.

yr.	m.	d.
1830	7	1
1828	9	1
1	10	0 = time from 3d payment to settlement.

Statement: $\dfrac{367.128.\ 22.\ 6}{100.\ \ 12}$. Canceled: $\dfrac{367.128.\ \overset{11}{\cancel{22}}.\ \cancel{6}}{100.\ \ \underset{2}{\cancel{12}}}$;

and $376.128 \times 11 \div 100 = 40.384$, interest till time of settlement; and $367.128 + 40.384 = \$407.512$, answer, or sum due on settlement.

2. I have in my possession a note, dated April 15, 1833, for \$2150.25, on which are the following endorsements: — Nov. 8, 1834, \$500.00; Sept. 1, 1835, \$723.64; January 1, 1837, \$378.295; and Oct. 29, 1837, \$850.00. What amount was due on this note, April 15, 1838? *Ans.* \$138.337.

3. On a note of \$767.95, given Dec. 25, 1827, and drawing interest after 90 days, were the following endorsements: — Jan. 1, 1830, \$75.00; March 25, 1831, \$565.25. What was due Jan. 1, 1833? *Ans.* \$294.118.

BOSTON, *Jan.* 13, 1809.

4. On demand, I promise to pay J. Anderson, or order, one thousand five hundred eighty-five and 33/100 dollars, with interest, for value received.

Received, May 5, 1812, \$863.12.
Received, May 7, 1814, \$221.
Received, July 21, 1815, \$1009.03.

Will the scholar determine whether the note is fully paid?

5. What was due on a note of \$2100, dated June 15, 1820, on settlement, June 15, 1830, the following sums being endorsed on the back of it, viz., June 30, 1824, \$750, and Sept. 30, 1828, \$1200, on interest at 6 per cent.? *Ans.* \$1249.527.

6. For value received of A. B., I promise to pay him, or order, seven hundred and fifty dollars, with interest at 6 per cent.?

Jan. 1, 1824. M. S.

On the above were the following payments endorsed: — April 1, 1826, one hundred and fifty dollars; July 1, 1829, four hundred and fifty dollars. What was due on settlement, Sept. 1, 1832? *Ans.* \$461.71 +.

ANNUAL INTEREST.

It is often required, by those loaning money for a number of years, that the interest thereof be paid at the expiration of each successive year. The interest thus accruing is called *annual interest.*

When not paid according to agreement, that is, annually, the *interest* on the principal, for each year, draws interest from the time it became due, till paid, or till the time of settlement.

For this method of computing interest the following is the rule: —

Rule. — *Find what is the yearly interest of the principal, and cast the interest on each year's interest, from the time it became due, till the time of settlement. The amounts of the several years' interest will be the total amount of interest due.*

Ex. 1. What is the annual interest, and also the amount of $1500, at 6 per cent., the interest being 3 years in arrears?

The interest on $1500 one year is $90.00, and the first year's interest will obviously draw interest two years.

The amount of the first year's interest is therefore, .	$100.80
The amount of the second year's interest is	95.40
The interest of the third year, not being due till the expiration of the year, will draw no interest, and is therefore only	$90.00
Annual interest =	286.20
Principal . . . =	1500.00
Sum due . . . =	$1786.20

2. What is the amount of a note of $900, for 5 years, at 6 per cent., interest annually? *Ans.* $1202.40.

3. What is the interest on $1300, at 6 per cent., for 4 years, interest computed annually? *Ans.* $340.08.

4. What is the amount of $786, for 3 years and 6 months, the interest being annual and at 6 per cent.? *Ans.* $963.78.

5. What is the difference of interest on $300, for 6 years, at 6 per cent., whether it be simple or annual interest? *Ans.* $16.20.

6. A man loaned $800, at 6 per cent., for 5 years, on condition that the interest be paid annually. This condition not

being complied with, how much was due at the expiration of the time? *Ans.* $1068.80.

7. What is the amount of $1200, at 5 per cent., annual interest, for 7 years? *Ans.* $1683.

COMPOUND INTEREST.

§ **107.** Compound Interest is that which is computed annually, and immediately added to the principal. The amount of each year is made the principal for the succeeding year.

Rule. — *Cast the interest, at the given rate per cent., for the first year, by multiplying by that rate per cent., and make the amount the principal for the second year. Make the amount of the second year principal for the third, and the amount of the third, the principal for the fourth, &c., through the whole number of years. From the amount thus obtained subtract the principal; the remainder will be the compound interest.*

Ex. 1. What is the compound interest of $256, for 3 years, at 6 per cent.?

256	
.06	
15.36	interest.
256.00	principal, added.
271.36	principal for 2d year.
.06	
16.2816	= interest of 2d year.
271.36	= principal for 2d year, added.
287.6416	principal for 3d year.
.06	
17.258496	interest of 3d year.
287.6416	principal.
304.900096	amount of 3d year.
256.00	the original principal, subtracted.
48.90	compound interest on $256, for 3 years.

2. What is the compound interest of $450, for 3 years, at 6 per cent.? *Ans.* $85.957 +.

3. What is the compound interest of $50, for 3 years, at 5 per cent.? *Ans.* $7.881 +.

4. What is the compound interest of $400, at 6 per cent., for 4 years? *Ans.* $104.99 +.

5. What will $675 amount to, at compound interest, in 3 years and 6 months, at 6 per cent. per annum? *Ans.* $828.053 +.

6. What is the amount of $40.20, at 6 per cent. compound interest, for 4 years? *Ans.* $50.75 +.

7. What is the amount of $63, at 6 per cent. compound interest, for 2 years? *Ans.* $70.786 +.

8. What is the compound interest of $127.85, for 3 years, at 6 per cent.? *Ans.* $24.421 +.

COMMISSION.

§ **108.** Commission is an allowance made by merchants and others to an agent for buying and selling goods. This allowance is usually a certain per cent. on the amount of money received for the sales effected, or on that expended in making purchases. The only respect in which it differs from interest, is, that, in computing commission, no regard is paid to *time;* hence,

Rule.—*Multiply by the rate per cent.*

Ex. 1. An agent sold for his employer goods to the value of $1800, for which he received 5 per cent. What was the amount of his commission? *Ans.* $90.

2. What is the amount of my commission for selling goods to the value of $975, it being 8 per cent.? *Ans.* $78.

3. My agent sends me word that he has purchased goods on my account to the value of $2768. What will his commission amount to, at 6 per cent.? *Ans.* $166.08.

4. The commission of $1250, at 10 per cent., is required. What is it? *Ans.* $125.

5. An agent sells 750 bales of cotton, at $52 per bale, and is to receive $2\frac{1}{2}$ per cent. commission. How much money will he receive? and how much will he pay over to his employer?

Ans. He will receive $975, and pay over $38025.

6. What will my commission amount to, at 3 per cent., in purchasing goods to the value of $7846.90?

Ans. $235.407.

INSURANCE.

§ **109.** Insurance against the loss of buildings and goods by fire, and also of ships and their cargoes by storm, is obtained by paying a certain per cent. on the estimated value of the property insured.

The instrument which binds the contracting parties is called the *policy*, and the sum paid by the party insured to the insuring party, is called the *premium*.

Rule.—*Multiply the estimated value of the property insured by the per cent.*

Ex. 1. What is the premium for the insurance of buildings and appurtenances, valued at $3758.50, at $\frac{1}{2}$ per cent.?
Ans. $18.79 +.

2. What is the premium for insuring property valued at $3600, against loss by fire, at $\frac{3}{4}$ per cent.? *Ans.* $27.

3. What is the premium for insuring property valued at $845, at $\frac{1}{5}$ per cent.? *Ans.* $1.69.

4. What is the premium for the insurance of a ship and cargo valued at $20500, at $\frac{1}{3}$ per cent.? *Ans.* $68.333 +.

5. What would be the premium for insuring a ship and cargo valued at $18000, at $\frac{3}{8}$ per cent.? *Ans.* $67.50.

6. Insured my house and out-buildings, valued at $21560.38, at $\frac{2}{5}$ per cent. What was the amount of the premium?
Ans. $86.24 +.

QUESTIONS.—What is Interest? How is it computed? Suppose the sum on interest be more or less than $100, or the time more or less than one year, how must the sum paid as interest compare? Give the illustration. What is the principal? What is interest? What is legal interest? What is the legal rate per cent. in New England? What in New York? What in Louisiana? What do you understand by the *amount*? The rate per cent. is a decimal of how many places, when expressed by cents? and when expressed by mills? What is Case I.? What is the rule for it? What is Note 1? What is Case II.? What is the rule? Give the reason of the rule. What is Note 2? Note 3? What is Case III.? What is the rule? Give the explanation which precedes the rule. What is Note 4? What is Note 5? What is Note 6? What is the rule for canceling? What is the rule for canceling, when the time consists of years and months? What is Note 7? What is the rule for canceling, when the time consists of years, months, and days? What is Note 8? What is Note 9? What is Case IV.? What is the rule? What explanation precedes the rule? What is said relative to banking institutions? What is Case V.? What is the rule for it? What is Case III. of Decimal Fractions? What is Case IV. of Decimal Fractions? What is Case VI.? What is the rule for it? Will the scholar now inform me why it is correct to multiply by one half of the even number of months in the given time, in casting interest at 6 per cent.? What is Compound Interest? What is the rule for it? What is Commission? In what respect does it differ from Simple Interest? What is the rule? How is Insurance obtained? What is to be understood by the policy? What by the premium? What is the rule?

RATIO.

§ **110.** Ratio is the relation which one quantity or number has to another quantity or number of the same kind.

The former of the two numbers, between which the ratio exists, is called the antecedent, and the latter, the consequent.

The direct ratio of any two numbers is obtained by dividing the consequent of any couplet by the antecedent, and the inverse ratio, by dividing the antecedent by the consequent.

Thus, the direct ratio of 2 to 4, is 2, because the antecedent, 2, is contained in the consequent, 4, two times; and the inverse ratio is $\frac{2}{4}$, because 4, the consequent, is contained in 2, the antecedent, $\frac{2}{4}=\frac{1}{2}$ of a time. Both these expressions establish the same general fact, viz., that one of the given numbers is twice as large as the other.

From the above, it is obvious that ratio can exist *only* between quantities of the same kind. It would be absurd to inquire how many times 3 acres must be repeated to equal 12 tons.

Any simple ratio is expressed by two dots placed between the antecedent and the consequent; thus, 2 : 8, or 5 : 10.

The following four propositions require to be carefully studied: —

1. *The ratio of any couplet is not affected either by multiplying or by dividing its antecedent and consequent by the same number;* for the ratio of 9 : 18 is 2; and the ratio of $\overline{9:18}\times 2 = 18:36$, is also 2. The same result is obtained by dividing these terms by any number whatever; thus, $\overline{9:18}\div 3 = 3:6$, and this equals 2.

2. *The ratio of any couplet is multiplied by any number, by multiplying the consequent, or by dividing the antecedent, by that number.* The ratio of 12 to 36, is 3; if, however, the consequent be multiplied by 3, the ratio, 3, will be multiplied by the same number; thus, $12:36\times 3=12:108=9$. The same result is obtained by dividing the antecedent by the same number; thus, 12, the antecedent, divided by 3, equals 4, and the ratio of 4 to 36 is 9, the same as before.

3. *The ratio of any couplet is divided by any number, by dividing the consequent, or by multiplying the antecedent, by that number.* Take the ratio 12 : 36, as before, and let it be divided by 2. If the consequent be divided, we obtain the ratio $12:18=1\frac{1}{2}$; but if the antecedent be multiplied, we obtain the ratio $24:36=1\frac{1}{2}$ also.

4. *If two or more ratios be multiplied together, that is, if the antecedents be multiplied into the antecedents, and the consequents into the consequents, the resulting ratio is called* COMPOUND RATIO; *and is equal to the product of the simple ratios.* The ratio compounded of the simple ratios, 2 : 4, 3 : 6, and 4 : 8, is the ratio, 24 : 192 = 8; but the ratio 2 : 4 = 2, also 3 : 6 = 2, and 4 : 8 = 2; and $2 \times 2 \times 2 = 8$.

Ex. 1. What is the ratio of 6 to 36? *Ans.* 6.

2. What is the inverse ratio of 7 to 12? *Ans.* $\frac{7}{12}$.

3. What is the ratio of 7 to 42?

4. What is the inverse ratio of 5 to 20?

5. What is the ratio of 1 £. sterling to 7 s. 6 d? It is obvious that the terms here given must be reduced to the same denomination, in order to compare them; therefore, 1 £. = 240 d., and 7 s. 6 d. = 90 d. The direct ratio, therefore, is $\frac{90}{240} = \frac{3}{8}$.

6. What is the ratio of 16 lb. to 3 cwt.? *Ans.* 21.

7. Multiply the terms 3 : 7 by 9, and see how the given ratio is affected.

8. Divide the terms 9 : 15 by 3, and compare ratios. (See proposition 1.)

9. Multiply the ratio 11 : 16 by 5. *Ans.* 11 : 80.

10. Multiply the ratio 9 : 11 by 3. *Ans.* 3 : 11. (See proposition 2.)

11. Divide the ratio 3 : 12 by 6. *Ans.* 3 : 2, or $\frac{2}{3}$.

12. Divide the ratio 3 : 19 by 4. *Ans.* 12 : 19. (See proposition 3.)

13. What is the ratio compounded of the ratios 2 : 3, 3 : 4, and 4 : 5? *Ans.* 24 : 60, or 2 : 5.

NOTE.—Whenever the antecedent of any couplet is the same as the consequent of any other couplet, the several ratios may be reduced to one, by rejecting such antecedents and consequents. Hence, the preceding simple ratios may be reduced to a compound one, by rejecting their similar terms. The ratio thus obtained is 2 : 5.

14. What is the ratio compounded of the simple ratios 3 : 5, 4 : 5, and 2 : 3? (See proposition 4.)
Ans. 24 : 75, or 8 : 25.

15. Multiply the ratio 5 : 9 by 4. *Ans.* 5 : 36.

16. Divide the same ratio by 3. *Ans.* 5 : 3.

17. Multiply the ratio 12 : 21 by 6. *Ans.* 2 : 21.

18. Divide the same by 7. *Ans.* 84 : 21, or 12 : 3.

19. Reduce the ratios 4 : 7, 3 : 1, and 6 : 2, to a compound ratio. *Ans.* 36 : 7.

PROPORTION.

§ **111.** Equality of ratios constitutes proportion.

Each simple statement in proportion requires at least two equal ratios, or four terms, the first and second of which are of one kind, and constitute one of the ratios; and the third and fourth are of another kind, and constitute the other ratio.

If 6 men earn 12 dollars in a given time, 36 men will earn 72 dollars in the same time. In this proposition, the two ratios required to constitute a proportion, are, the ratio of 6 men to 36 men, and of 12 dollars to 72 dollars; and the proposition can be true only on the condition that these two ratios are equal; and that is actually the case, for $36 \div 6 = 6$, and $72 \div 12 = 6$. Therefore, 6 is the ratio of each couplet.

Proportion is usually expressed by dots, thus, $6 : 36 :: 12 : 72$; and thus read, 6 is to 36 as 12 is to 72, or 6 men are to 36 men as 12 dollars are to 72 dollars.

When any four numbers are proportionals, the first and fourth are called *extremes*, and the second and third, the *means;* and the product of the extremes must always equal the product of the means. This is true with regard to the statement made above; for $6 \times 72 = 432$, and $12 \times 36 = 432$.

Since, therefore, these products are always equal, if any three terms are given, two of which bear a given ratio to each other, a fourth term may be found, to which the third given term shall bear the same ratio; for the required term must be either one of the extremes or one of the means. If it be one of the extremes, the product of the means divided by the given extreme, will give the required extreme; and if it be one of the means, the product of the extremes divided by the given mean will give the required mean.

Suppose, in the proposition stated above, it be required to find how many dollars 36 men would earn in a given time, if 6 men earn 12 dollars in the same time.

Let the three given numbers stand as before, thus; 6 men : 36 men : : 12 dollars : what number of dollars?

Now, since 6, the left-hand term, multiplied by the required term, must produce the same number as 36 multiplied by 12, it follows that 36 multiplied by 12, and the product divided by 6, will give the required number. Thus, $36 \times 12 = 432$, and $432 \div 6 = 72$, the fourth term in the preceding proposition. Or, suppose it required to find what number bears the same ratio to 36 that 12 does to 72. The statement would be, What number : 36 : : 12 : 72. The other extreme is here required; hence, $36 \times 12 = 432$, and $432 \div 72 = 6$, the required extreme.

Again; let one of the means be required, thus; 6 is to what number as 12 is to 72? The statement may stand thus; 6 : — : : 12 : 72. One of the means is here required, and the extremes are given; therefore, $72 \times 6 = 432$, and $432 \div 12 = 36$, the required mean. Lastly, let the other mean be required. We then have the statement as follows; 6 : 36 : : — : 72. The third term of the statement is here wanting, or the other mean. Therefore, $72 \times 6 = 432$, and $432 \div 36 = 12$, the required mean.

From the preceding remarks we learn that operations in simple proportion consist in having three of the terms of a proportion given and a fourth term required. For finding this fourth term, we have the following rule:—

Rule.—*Notice which of the three terms given is of the same name or kind as the required term or answer, and give it the right-hand place. Notice, again, whether the term required must be greater or less than this; and, if it is to be greater, place the greater of the two remaining terms next it, on the left, for the second term of the proportion, and the less number for the first; but if it is to be less, place the less of the two remaining numbers for the second term, and the greater for the first term; then multiply the second and third terms together for a dividend, and divide their product by the first; the quotient will be the fourth term, or answer, and of the same denomination as the third term.*

NOTE 1.—If the third term consists of different denominations, it must be reduced to the lowest mentioned before stating, and the fourth term will be of the same denomination; but if either the first or second terms be of different denominations, they must *both* be reduced to the lowest mentioned, before stating.

Ex. 1. If 8 yards of cloth cost \$3.20, what will 96 yards cost?

Since 8 yards cost money, 96 yards will cost money; the required term must, therefore, be money; and must be a number to which \$3.20 will bear the same ratio that 8 yards bears to 96 yards. \$3.20 is, therefore, to be made the right-hand term.

The next inquiry is, which will cost most, 8 yards, or 96 yards? The answer to this inquiry places (in accordance with the rule) the 96 in the second place, and the 8 in the first, thus; 8 : 96 : : 3.20 : what number of dollars?

PERFORMED.

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8 : 96 : : 3.20
             96
           ----
           1920
          2880
         ------
       8)307.20
         ------
        $38.40 Ans.
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Therefore, 8 yards : 96 yards : : \$3.20 : \$38.40.

2. If 9 men earn 72 dollars in a given time, how much will 24 men earn in the same time?

Statement; 9 men : 24 men : : \$72 : *Ans.* \$192.00.

§ **112.** Sums of this description may be solved with peculiar ease by canceling.

Rule for Canceling. — *Notice which of the given terms is of the same kind or name as the required answer, and place it above a horizontal line, towards the left. Notice again whether the required term must be greater or less than this; and, if greater, place the greater of the two remaining terms on the right of the preceding term, and also above the line, and the less of the two terms below the line; but if less, place the less of the remaining terms above the line, and the greater below it; then cancel, multiply, and divide, as before directed.*

Note 2. — In arranging the terms as directed for canceling, the number placed first above the line, is the third term of the proportion, and that standing on the same side, on the right of this, is the second, and the number standing below the line, the first.

3. If 12 horses consume 42 bushels of oats in a given time, how much will 20 horses consume in the same time?

$$\text{Statement; } \frac{42.\ 20}{12}$$

The answer required is obviously the oats that 20 horses would consume. It is also evident that 20 horses would consume more than 12. Hence the reason of the above statement.

$$\text{The same canceled; } \frac{\overset{7.}{\cancel{42}}.\ \overset{10}{\cancel{20}}}{\cancel{12}.\ \cancel{2}};$$

and $7 \times 10 = 70$, the bushels required.

It will be recollected that the numbers above the horizontal line form a dividend, and those below the line, a divisor. Hence, 42, the number of bushels consumed by 12 horses, may be regarded as divided by 12. This division would give the quantity of oats which 1 horse would consume in the given time; and this quantity, multiplied by 20, would give the quantity 20 horses would consume in the same time.

4. If 72 yards of cambric cost \$119.44, what will 9 yards cost?

If \$119.44 be divided by 72, the quotient will be the price of 1 yard, and this price multiplied by 9, will be the required answer.

$$\text{Therefore, } \frac{119.44.\ 9}{72},$$

$$\text{Canceled; } \frac{119.44.\ \cancel{9}}{8.\ \cancel{72}}; \text{ and } 119.44 \div 8 = 14.93, \textit{Ans.}$$

5. If 10 shillings pay for 20 pounds of beef, how many pounds may be bought for 5 shillings? *Ans.* 10.

6. If 3 lb. of sugar cost 4 s. how much will 6 lb. cost? *Ans.* 8 s.

7. If 12 bushels of wheat are worth $16, what is the value of 48 bushels? *Ans.* $64.

8. Sold 12 yards of cloth for $72. What is the value of 5 yards, at the same rate? *Ans.* $30.

9. What is the value of 16 cords of wood, if 48 cords are worth $120? *Ans.* $40.

10. If 16 cords of wood are worth $40, what is the value of 48 cords? *Ans.* $120.

11. What is the value of 4 gallons of wine, if 108 gallons of the same kind are worth $324? *Ans.* $12.

12. If 8 yards of cloth cost $3.20, how many yards of the same kind may be bought for $38.40? *Ans.* 96 yards.

13. Paid 75 cents for 7 lb. of sugar. How many pounds of the same kind may be bought for $6? *Ans.* 56 pounds.

14. If 7 men can accomplish a piece of work in 12 days, how many men are required to accomplish the same in 3 days? *Ans.* 28 men.

15. If a ship sail 24 miles in 4 hours, in how many hours will she sail 150 miles, if she continue at the same rate? *Ans.* 25 hours.

16. If 17 men perform a piece of work in 25 days, in how many days would 5 men perform the same? *Ans.* 85 days.

17. A staff, 4 feet long, casts a shadow 6 feet. Another staff, placed in the same situation, casts a shadow 58 feet; what is its length? *Ans.* $38\frac{2}{3}$ feet.

18. A garrison has provision for 8 months, at the rate of 15 ounces per day. What must be each man's daily allowance, in order that the same provision may last them 11 months? *Ans.* $10\frac{10}{11}$ ounces.

19. When a quarter of wheat affords 60 tenpenny loaves, how many eight-penny loaves may be made from the same? *Ans.* 75.

20. If $10 worth of provision serve 7 men 4 days, how many days will the same provision serve 9 men? *Ans.* $3\frac{1}{9}$ days.

21. If 12 gallons of wine cost $30, what is the value of 56 gallons, at the same price per gallon? *Ans.* $140.

22. If 15 pounds of sugar cost $1.20, how many pounds may be bought for $38? *Ans.* 475 pounds.

23. If a staff, 4 feet long, casts a shadow 7 feet long, what is the height of a steeple, whose shadow, at the same time, measures 198 feet? *Ans.* $113\frac{1}{7}$ feet.

24. If a pole, 6 feet long, casts a shadow 10 feet on level

ground, what would be the length of a shadow from a steeple 72 feet high, at the same time? *Ans.* 120 feet.

25. If 12 men build a house in 48 days, in how many days can 36 men do the same work? *Ans.* 16 days.

26. If 100 men can finish a piece of work in 12 days, how many men will be required to do the same in 4 days? *Ans.* 300 men.

27. How many men must be employed to complete in 15 days what 5 men can do in 24 days? *Ans.* 8 men.

28. If a man perform a journey in 3 days, when the days are 16 hours long, how many days, of 12 hours each, will he require to perform the same, if he continue to travel at the same rate? *Ans.* 4 days.

29. If 48 men can build a wall in 24 days, how many men can do the same in 192 days? *Ans.* 6 men.

30. In how many days will 8 men finish a piece of work which 5 men can do in 24 days? *Ans.* 15 days.

31. In what time will $600 gain $50 interest, if $80 gain it in 15 years? *Ans.* 2 years.

32. When $100 principal will gain $6 in 12 months, what principal will gain the same in 8 months? *Ans.* $150.

33. How many yards of cloth, 3 qr. wide, are equal to 30 yards, 5 qr. wide? *Ans.* 50 yards.

34. How many yards of paper, $1\frac{1}{4}$ yards wide, will be sufficient to hang a room 20 yards square, and 4 yards high? *Ans.* 256 yards.

35. If a board be 9 inches wide, how much in length will make a square foot? *Ans.* 16 inches.

36. What quantity of shalloon, 3 qr. wide, will line $7\frac{1}{2}$ yards of cloth, $1\frac{1}{2}$ yards wide? *Ans.* 15 yards.

37. Lent a friend $100 for 6 months. Afterwards he lent me $75. How long ought I to retain it to balance my favor, allowing to each the same rate per cent. of interest? *Ans.* 8 months.

38. In what time will $858 gain as much as $286 will gain in 12 months? *Ans.* 4 months.

39. If 375 cwt. may be carried 660 miles for a given sum, how many cwt. may be carried 60 miles for the same money? *Ans.* 4125.

40. If 10 s. worth of wine will suffice for 46 men, when a tun is worth $240, for how many will the same 10 s. suffice, when a tun costs $160? *Ans.* 69 men.

41. If $5\frac{1}{2}$ yards of muslin, that is $1\frac{1}{2}$ yards wide, will make a dress, how many yards of lining will be required, that is but 3 qr. wide? *Ans.* 11 yards.

42. If 40 rods in length and 4 in breadth make an acre, what

is the width of a piece of ground containing the same quantity, that is 24 rods in length? *Ans.* 6 rods, 3 yards, 2 feet.

43. An insurance company, consisting of 82 persons, sustains a loss, of which each man's share was $12. What would their shares have been, had the company consisted of only 32 persons? *Ans.* $30.75.

44. If a hogshead of wine, which cost $180, afford a handsome profit, when retailed at $4 per gallon, how must another be retailed, which cost $196, to gain the same per cent.?

Ans. $4.355 +.

45. A lot of ground was walled in by 16 men in 6 days. The same, being demolished, is required to be rebuilt in 4 days. How many men must be employed? *Ans.* 24 men.

46. A person, by traveling 12 hours per day, performs a journey of 800 miles in 32 days. How many days will he require to perform the same journey, if he travel 15 hours per day?

Ans. $25\frac{3}{5}$ days.

47. If 1800 cwt. may be carried 64 miles for a given sum, how far may 225 cwt. be carried for the same money?

Ans. 512 miles.

48. If 50 gallons of water fall into a cistern of sufficient capacity to contain 230 gallons, in one hour, and by a pipe 35 gallons be drawn off in the same time, how long will it take to fill the cistern? *Ans.* 15 hours, 20 minutes.

49. If 150 lb. of soap cost $15.60, what would 15 lb. cost?

Ans. $1.56.

50. How many yards of cloth, 3 qr. wide, are equal to 39 yards, 5 qr. wide? *Ans.* 65.

51. A cistern has a pipe by which it may be emptied in 10 hours. How many pipes, of the same capacity, will empty it in 30 minutes? *Ans.* 20 pipes.

§ **113.** It has already been said, that, if the given terms (see Note 1) are of different denominations, they must be reduced before stating the sum. This labor is, for the most part, saved, whenever the question is solved by canceling. Take the following sums, in which it is required to find the value of a quantity in one denomination, the price of some other denomination being given.

Rule.—*Write the quantity the price of which is required, above a horizontal line; then, (if the price of a lower denomination be given,) on the right of this, also above the line, place the numbers required to reduce the given quantity to that denomination, together with the price of the same denomination; then, below the line, write such numbers as will reduce the given price to*

the required denomination. But if the price of a higher denomination be given, and that of a lower denomination be required, place the quantity the price of which is required, as before, and write the numbers necessary to reduce that quantity to the denomination of which the price is given, below the line; then, lastly, place the quantity the price of which is given below, and its price above the line. Solve the statement by canceling.)

NOTE 3. — If either the given quantity or price be of different denominations, they may be reduced to the lowest given, before stating; or, if preferred, the lower denominations may be reduced to a decimal.

Ex. 1. How many pounds sterling will 3 cwt. of sugar cost, at 20 pence per pound?

The price of 3 cwt. is required, and that of 1 pound is given. The given price is also in pence, and pounds sterling are required. Hence, $\frac{3.\ 4.\ 28.\ 20}{12.\ 20}$ is the required statement. The numbers 4 and 28 above the line, are required to reduce the 3 cwt. to pounds, and these pounds, multiplied by 20, will give the price of the whole quantity in pence. If, then, these pence be divided by 12 and 20, they will be reduced to pounds sterling.

Statement solved: $\frac{\cancel{3}.\ \cancel{4}.\ 28.\ \cancel{20}}{\cancel{12}.\ \cancel{20}}$.

Therefore, 28 £. is the required answer.

2. How many pounds sterling will 3 pipes of wine cost, at 10 s. a gallon?

Statement: $\frac{3.\ 2.\ 63.\ 10}{20} = 189$ £.

In this statement, the 2 and 63 above the line are the numbers required to reduce pipes to gallons; then, the gallons, multiplied by 10, will give the cost in shillings; and, lastly, the shillings, divided by 20, (the number below the line,) will be reduced to pounds.

3. How many dollars, New York currency, will 12 cwt. of sugar cost, at 10 d. a pound? *Ans.* \$140.

4. How many dollars, New England currency, will 2 hogsheads of wine cost, at 6 d. a pint?

Statement: $\frac{2.\ 63.\ 4.\ 2.\ 6}{12.\ 6}$. *Ans.* \$84. (6 s. = \$1 New England currency.)

5. At 15 pence a pound, what will 1 cwt. of loaf sugar cost, in dollars, New England currency? *Ans.* \$23.333 +.

If it is preferred to solve sums of this kind without canceling, it may be done by the following rule: —

Rule. — *Reduce the given quantity to that denomination, the*

price of which is given, and multiply it by the price; then divide by such numbers as are required to reduce the value obtained to the required denomination.

6. How many dollars, New York currency, will 6 cwt. of sugar cost, at 10 d. a pound?

SOLUTION.

$$\begin{array}{rl} 6 & \\ \underline{4} & = \text{qr. in cwt.} \\ 24 & \\ \underline{28} & = \text{lbs. in qr.} \\ 192 & \\ \underline{48} & \\ 672 & \\ \underline{10} & = \text{price of 1 lb.} \\ 6720 & = \text{cost in pence.} \end{array} \qquad \begin{array}{rl} 12\,)\,\underline{6720} & \\ 8\,)\,\underline{560} & = \text{cost in shillings.} \\ 70 & = \text{dollars, } \textit{Ans.} \end{array}$$

$$\text{Canceled: } \frac{\cancel{6}.\ \cancel{4}.\ \overset{7}{\cancel{28}}.\ 10}{\underset{\cancel{2}}{\cancel{12}}.\ \underset{2}{8}}.\ \textit{Ans. } 10 \times 7 = 70.$$

7. How many dollars will 53 ells English cost, at 8 s., New York currency, per yard? *Ans.* \$66.25.

8. If I purchase melasses at 1 s. 3 d. per quart, how much in pounds, shillings, and pence, will 12 hogsheads of the same kind cost? *Ans.* 189 £.

9. If I purchase 16 cwt. of steel for \$156, what will 1 qr. of a cwt. cost, at the same rate?

$$\text{Statement: } \frac{1.\ 156}{4.\ 16}.\quad \text{Canceled: } \frac{1.\ \overset{39}{\cancel{156}}}{\cancel{4}.\ 16}.\quad 39 \div 16 = \$2.437 +.$$

10. What cost 9 cwt. of sugar, at 10 pence per pound? *Ans.* 42 £.

11. What cost 12 cwt. of sugar, at 9 pence per pound? *Ans.* 50 £. 8 s.

12. What cost 42 cwt. of sugar, at 3 s. 8 d. per pound? *Ans.* 862 £. 8 s.

13. What would 480 yards cost, in Federal Money, at 2 pence, New York currency, per yard?

$$\text{Statement: } \frac{480.\ 2.}{12.\ 8}.\quad \textit{Ans. } \$10 +.$$

For the solution of the following sums, see the Table of Currencies, given in Reduction of Currencies.

14. What would 862 yards cost, in Federal Money, at 3 pence per yard, New England currency?

Statement: $\frac{862.\ 3.}{12.\ 6}$. *Ans.* \$35.916+.

15. What would 920 yards cost, in Federal Money, at $4\frac{1}{2}$ pence per yard, New Jersey currency? *Ans.* \$46.

16. How much will 672 yards cost, in Federal Money, at 6 s. 6 d., New York currency, per yard? *Ans.* \$546.

17. How many dollars will 123 yards of cloth cost, at 10 s., New Jersey currency, per yard? *Ans.* \$164.

18. C has a journey of 75 leagues to perform. In what time will he complete it, if he travel 30 miles a day?

Ans. $7\frac{1}{2}$ days.

19. If 9 lb. of coffee cost 27 s., how many dollars, at 6 s. each, will 45 lb. cost? *Ans.* \$22.50.

20. If I buy 20 pieces of cloth, each 20 ells, for 12 s. 6 d. per ell, how many dollars, at 8 s., will pay for the same?

Ans. \$625.

21. A vessel at sea discharges a cannon, the report of which reaches me in 1 minute, 30 seconds. How far distant is she, allowing sound to travel 1142 feet in a second?

Ans. 19 miles, 3 furlongs, and $29\frac{1}{11}$ rods.

22. How many yards of cloth may be bought for \$37.62, if $\frac{3}{4}$ of a yard cost 66 cents? *Ans.* 42 yd. 3 qr.

QUESTIONS. — What is Ratio? What is the former of the two numbers, between which the ratio exists, called? What is the latter? How is the direct ratio of any two numbers obtained? How is the inverse ratio obtained? Between what quantities only does ratio exist? How is simple ratio expressed? How is the ratio of any couplet affected by multiplying or dividing both the antecedent and the consequent by the same number? In what two ways may the ratio of any two numbers be multiplied? In what two ways is the ratio of any couplet divided by any number? When two or more ratios are multiplied together, what is the resulting ratio called? What does it equal? What is Note 1? What constitutes Proportion? How many equal ratios are required for a statement of proportion? What terms must be of the same kind? When any four terms are proportional, what are the first and fourth called? And what are the second and third called? How does the product of the extremes compare with the product of the means? When any of the four terms are wanting, explain how it may be found. In what do operations in Simple Proportion consist? What is the rule? What note follows the rule? What is the rule for canceling? What is Note 2? What is the rule, when it is required to find the value of a quantity in one denomination, the price of some other denomination being given? What is Note 3? What is the rule for solving sums of this kind without canceling?

COMPOUND PROPORTION.

§ **114.** Simple Proportion consists of two equal ratios. Compound Proportion is that in which the relation of one of the given quantities to a required quantity of the same name, is traced through two or more simple proportions.

The smallest number of terms, of which a statement in compound proportion can consist, is five. Of these terms, one is always of the same name as the answer required; and the others are always two of a kind. The following sum will serve as an illustration:—

If 3 men, in 4 days, spend \$5, how many dollars will 6 men spend in 12 days?

In the above sum, there are five terms given, viz., two of men, two of days, and one of dollars; and dollars are also required for the answer; so that, when the sixth term is found, the sum may be resolved into three simple ratios, the third of which is a compound of the preceding two. These ratios are 3 men : 6 men, and 4 days : 12 days, and \$5: the required number of dollars. Now, it is obvious that the ratio of \$5 to the required number of dollars, is not the same as the ratio of 3 men to 6 men; for then no regard would be paid to the time, and the solution would be effected on the supposition that one man or a number of men would spend as much in one day, as in any given number of days. Nor is it the same as the ratio of 4 days to 12 days; for then the supposition would be, that 3 men would spend as much as 6, or any number of men. But it is a ratio compounded of the two ratios, viz., the ratios of 3 men : 6 men, and of 4 days : 12 days. The ratio of $3:6=2$, and $\$5\times2=\$10$. This would double the given quantity of money. Again, the ratio of $4:12$, is 3; this would treble the sum last obtained, viz., \$10, and would give $10\times3=\$30$, which is the answer to the above question. Now, it will be observed that the \$5 in the above operation was multiplied by the product of the two simple ratios; for $3:6=2$, and $4:12=3$; therefore, $2\times3=6$, the product of the simple ratios, and $\$5\times6=\$30$, *Ans.*

The same result would have been obtained by multiplying the \$5 by the consequents, or latter terms of each ratio, and dividing their product by the product of the antecedents, or former terms of the same ratios. Thus, 3 : 6, and 4 : 12, are the given ratios of which 6 and 12 are consequents; therefore, $6\times12\times5=360$, and $3\times4=12$, the product of the ante-

cedents; hence, $360 \div 12 = 30$ dollars, the same answer as before. Hence we have the following rule:—

Rule.—*Write the number which is of the same kind as the answer required, for the third term. Of the remaining terms, take any two of the same name, and arrange them as directed in Simple Proportion; then any other two of a kind, and so on till the terms are all taken. Lastly, multiply the product of the second terms by the third term, and divide the last product by the product of the first terms, and the quotient will be the required term or answer.*

Ex. 1. If 4 men build a wall 10 feet long, 3 feet high, and 2 feet thick, in 6 days, in what time will 12 men build one 100 feet long, 4 feet high, and 3 feet thick?

The question asked is, in what *time* will the work be done? Therefore, by the rule, 6 days is the third term.

$$\left.\begin{array}{lcl} 12 \text{ men} & : & 4 \text{ men} \\ 10 \text{ feet length} & : & 100 \text{ feet length} \\ 3 \text{ feet high} & : & 4 \text{ feet high} \\ 2 \text{ feet thick} & : & 3 \text{ feet thick} \end{array}\right\} :: 6 \text{ days.}$$

It is obvious that 12 men would require less time to execute a given piece of work than 4 men, and also that a wall 100 feet long, 4 feet high, and 3 feet thick, would require more time than a wall 10 feet long, 3 feet high, and 2 feet thick.

The same solved:—

$$\left.\begin{array}{r} 12 : 4 \\ 10 : 100 \\ 3 : 4 \\ 2 : 3 \end{array}\right\} :: 6;\ 4 \times 100 \times 4 \times 3 \times 6 = 28800; \text{ and } 12 \times 10 \times 3 \times 2 = 720; \text{ and } 28800 \div 720 = 40 \text{ days, the number required.}$$

The work of this sum may be much abbreviated by canceling.

The statement would be, $\dfrac{6.\ 4.\ 100.\ 4.\ 3}{12.\ 10.\ 3.\ 2} = 40$ days.

For the preceding mode of operation, we have the following rule:—

Rule for Canceling.—*Write the number which is of the same name as the answer required, above a horizontal line, towards the left. Then take each two terms of the same name, and compare them with this number, and notice whether more or less be required, as in Simple Proportion. If more be required, place the greater of the two numbers above the line, and the less below; but, if less be required, place the less of the two above the line, and the greater below. Cancel, &c.*

Ex. 2. If 4 men, in 12 days, mow 48 acres of grass, how many acres will 8 men mow in 16 days?

Common statement; $\left.\begin{matrix} 4 : 8 \\ 12 : 16 \end{matrix}\right\} : : 48.$

Statement for canceling; $\frac{48.\ 8.\ 16}{4.\ 12} = 128$ acres.

NOTE 1.—If the sixth term, or answer, be placed below the horizontal line, the other terms remaining as before, the product of the terms standing above the line, will just equal the product of those standing below, thus:

$$\frac{48 \times 8 \times 16 = 6144}{128 \times 4 \times 12 = 6144}.$$

3. If 12 horses eat 20 bushels of oats in 16 days, how many bushels will 24 horses eat in 48 days?

Statement; $\frac{20.\ 24.\ 48}{12.\ 16}$. *Ans.* 120 bushels.

4. If 18 men, in 32 days, consume 128 lb. of bread, how many pounds will 12 men consume in 64 days, their daily allowance being the same? *Ans.* 170$\frac{2}{3}$.

5. If 8 men receive 4 dollars for 3 days' work, how many days must 20 men work, to earn 40 dollars, if they receive the same daily allowance? *Ans.* 12.

6. If 4 men receive $3.20 for 3 days' work, how many men, if they receive the same per day, will earn $12.80 in 16 days? *Ans.* 3 men.

7. How much ought 60 men to receive for 25 days' work, if 12 men, under the same circumstances, receive 50 dollars for 4 days' work? *Ans.* $1562.50.

8. If 16 men cut 112 cords of wood in 7 days, how many cords will 24 men cut in 19 days? *Ans.* 456 cords.

9. If the transportation of 12 cwt. 150 miles, cost 75 shillings, for how many dollars, at 8 s. each, may 6 cwt. be transported 45 miles? *Ans.* $1.406 +.

10. If the freight of 9 hhds. of sugar, each weighing 12 cwt., 20 leagues, cost $38.40, what will be the expense of transporting 50 casks of the same, each weighing 2 cwt. 2 qr., 100 leagues? *Ans.* $222.22 +.

11. If 100 dollars, in 1 year, gain 6 dollars interest, how much will 200 dollars gain in 26 weeks? *Ans.* 6 dollars.

12. If 350 dollars, in 9 months, gain 15 dollars, what principal will gain 6 dollars in 12 months? *Ans.* 105 dollars.

13. If 8 men can build a wall, 20 feet long, 6 feet high, and 4 feet thick, in 12 days, in what time can 24 men build one, 200 feet long, 8 feet high, and 6 feet thick? *Ans.* 80 days.

14. A wall, 32 feet high, and 40 feet long, was built in 8 days, by 145 men. In how many days would 68 men build

another wall, 28 feet high, and of the same length, allowing each man to perform an equal portion of labor, in the same time?

Ans. $14\frac{63}{68}$ days.

15. What must be paid for the transportation of 56 bags of coffee, each weighing 3 qr. 16 lb., 66 miles, if 14 bags, weighing each 125 lb., be carried 6 miles for \$6.25?

Ans. 220 dollars.

16. If 6 persons can earn 120£. in 21 weeks, how much will 14 persons earn, in 46 weeks, if they receive the same per week? *Ans.* 613£. 6 s. 8 d.

17. If, when the days are 14 hours long, a person perform a journey of 276 miles in 16 days, in what time will he travel 860 miles, when the days are 12 hours long? *Ans.* $58\frac{34}{207}$ days.

18. If 960 dollars defray the expenses of 20 men 88 weeks, for how many weeks will \$1440 defray the expenses of 48 men, if they spend at the same rate? *Ans.* 55 weeks.

19. If 100£. gain 6£. in 12 months, what principal, at the same rate per cent., will gain 3£. 7 s. 6 d. in 9 months?

Ans. 75£.

20. If a man travel 240 miles in 12 days, when the days are 12 hours long, in how many days, of 16 hours, will he travel 720 miles, if he travel at the same rate? *Ans.* 27 days.

21. What is the interest of \$650 for 36 weeks, at 6 per cent. per annum? *Ans.* \$27.

22. If \$150, in 12 months, gain \$8 interest, what will \$400 gain in 4 months? *Ans.* \$7.11 +.

23. If 3 men lay 144 square yards of pavement in 7 days, how many square yards will 12 men lay in 49 days?

Ans. 4032 square yards.

24. If 200 lb. be carried 40 miles for 40 cents, how far may 20200 lb. be carried for \$60.60? *Ans.* 60 miles.

25. In what time will 200£. gain 6£., if 100£. gain 6£. in 52 weeks? *Ans.* 26 weeks.

26. In what time will 400£. gain 96£. interest, if 350£ gain 10£. 10 s. interest in 6 months? *Ans.* 4 years.

27. If 4 men mow 48 acres of grass in 12 days, in what time will 8 men mow 128 acres? *Ans.* 16 days.

28. If I receive 88£. 17 s. 4 d. for the principal and interest of 86£ for 8 months, what is the rate per cent. of interest?

Ans. 5 per cent.

29. What will the transportation of 7 cwt. 2 qr. 25 lb. 64 miles cost, if 5 cwt. 3 qr. be carried 150 miles for \$24.58?

Ans. \$14.086 +.

30. If I pay \$50.25 for the tuition of 2 boys, 3 quarters each, what will the tuition of 100 boys amount to, in $7\frac{1}{2}$ years, at the same rate? *Ans.* \$25125.

31. Purchased goods to the amount of 750£., and sold the same, six months after, for 825£. What did I gain per cent.?
Ans. 20 per cent.

32. If 56 lb. of bread be sufficient for 7 men 14 days, how long will 36 lb. suffice for 21 men? *Ans.* 3 days.

33. A person, having engaged to carry 8000 cwt. a certain distance in 9 days, removed 4500 cwt., with 18 horses, in 6 days. How many horses were required to remove the remainder in the 3 remaining days? *Ans.* 28 horses.

34. If 3 men perform a piece of work in 20 days, how many men will accomplish 4 times as much work, in 4 days?
Ans. 60 men.

35. If 4 men, in 5 days, eat 6 lb. of bread, how many pounds of the same will be sufficient for 16 men, 15 days?
Ans. 72 lb.

36. If it take 15 men 20 days to make 300 pairs of shoes, how many men will be required to make 1200 pairs in 60 days? *Ans.* 20 men.

37. A wall, which was to be raised to the height of 27 feet, was raised 9 feet, by 12 men, in 6 days. How many men were required to complete the work in 4 days, allowing each man to do the same amount of work per day? *Ans.* 36 men.

38. If 6 men, in 21 weeks, earn $120, how much will 14 men earn in 46 weeks? *Ans.* $613.33+.

39. If 48 bushels of corn produce 576 bushels in one year, what will be the product of 240 bushels in 6 successive years?
Ans. 17280 bushels.

40. In how many days will 25 men reap 200 acres of grain, if 12 men reap 80 acres in 6 days? *Ans.* $7\frac{1}{5}$.

41. If 20 men mow 208 acres, 1 rood, and 24 rods of grass in 24 days, how many men will cut down 8 times as much grass in twice the time? *Ans.* 80.

QUESTIONS.—Of what does Simple Proportion consist? What is Compound Proportion? What is the smallest number of terms of which a statement in compound proportion can consist? How must these terms compare with each other, in kind? How many simple ratios will there be in the example given for illustration, when the answer is found? Of what is the third ratio a compound? What are the three ratios in the illustration given? Give the entire illustration. What is the rule first given? What is to be the denomination of the third term? How are the other terms to be arranged? What is the rule for canceling? What term is to be placed first above the line? How are the other terms to be arranged? What is the given note?

SOLUTION OF ARITHMETICAL PROBLEMS BY ANALYSIS.

§ **115.** An arithmetical question is solved analytically, when the operation is guided entirely by the conditions embraced in the question itself.

Take the following illustration: —

Ex. 1. If 3 men perform a piece of work in 6 days, in what time will 9 men perform the same labor?

It is obvious that, if it take 3 men 6 days to perform the proposed labor, it will take 1 man 3 times 6, or 18 days, to perform the same. But 9 men, operating together, will perform 9 days' work in 1 day; consequently they will do the whole in 2 days; for $18 \div 9 = 2$, *Ans.*

2. If 21 men earn 63 dollars in a given time, how much will 42 men earn in the same time?

$63 \div 21 = 3$, the number of dollars 1 man will earn in the given time. Therefore, $42 \times \$3 = \$126$, the answer required. Or, the ratio of 21 men to 42 men is 2; and $\$63 \times 2 = \$126$, the same as before.

Solutions of this kind may therefore be effected by the following general principle: — *Find the ratio of the two given terms which are of the same kind, and by this ratio multiply the term corresponding in kind with the one required.*

3. If 42 men can make 3 rods of wall in a given time, how much can 8 men make in the same time?

The ratio of 42 men to 8 men is $42 : 8 = \frac{8}{42} = \frac{4}{21}$; therefore, $\frac{4}{21} \times 3 = \frac{12}{21} = \frac{4}{7}$ of 1 rod, which is the distance required.

4. If a staff, 4 feet long, cast a shadow 6 feet, how high is that steeple, whose shadow measures 75 feet?

The ratio of $6 : 75 = 12\frac{1}{2}$, and $12\frac{1}{2} \times 4 = 50$ feet, *Ans.* Or, the shadow is $1\frac{1}{2} = \frac{3}{2}$ as long as the staff; hence, $75 \div 3 = 25$, and $25 \times 2 = 50$ feet, the same as before.

5. An express, traveling at the rate of 60 miles per day, had been absent 5 days, when a second express was despatched on the same route, traveling 75 miles per day. How many miles must the second travel to overtake the first?

$60 \times 5 = 300$, the whole number of miles traveled by the first express before the second started, and consequently the number of miles the second had to gain; but the first traveled 60, and the second 75, miles per day; hence $75 - 60 = 15$, the number of miles gained daily by the second express. 15 miles are therefore gained in traveling 75 miles; consequently, 1 mile is gained in traveling 5 miles; and since 300 miles are to be gained, $300 \times 5 = 1500$ miles, *Ans.*

6. If 6 men, in 14 days, earn 84 dollars, how much will 9 men earn in 11 days? *Ans.* \$99.

7. If 6 persons spend \$300 in 8 months, how much will be sufficient for a family of 15 persons 20 months? *Ans.* \$1875.

8. If 12 men build 36 feet of wall in 9 days, how many men would build 108 feet in 16 days?

$36 \div 12 = 3$, and $3 \div 9 = \frac{1}{3}$, the distance built by 1 man in 1 day; and $\frac{1}{3} \times 16 = \frac{16}{3} = 5\frac{1}{3}$, the distance 1 man would build in 16 days; therefore, $108 \div 5\frac{1}{3} = 20\frac{1}{4}$, the number of men required.

9. A merchant, owning $\frac{4}{5}$ of a vessel, sold $\frac{2}{3}$ of his share for \$1456. What was the value of the whole vessel?

$\frac{4}{5}$ of $\frac{2}{3} = \frac{8}{15}$. If, then, $\frac{8}{15}$ cost \$1456, $1456 \div 8 =$ \$182, the value of $\frac{1}{15}$ of the vessel; hence, $182 \times 15 =$ \$2730, *Ans.*

10. If $\frac{3}{5}$ of a yard cost $\frac{7}{8}$ of a dollar, what will 40 yards cost?

If $\frac{3}{5}$ of a yard cost \$$\frac{7}{8}$, $\frac{1}{5}$ of a yard will cost $\frac{1}{3}$ of that sum, or $\frac{1}{3}$ of \$$\frac{7}{8}$ $=$ \$$\frac{7}{24}$; and 1 yard, or $\frac{5}{5}$, will cost 5 times that sum, or \$$\frac{35}{24}$; therefore, \$$\frac{35}{24} \times 40 =$ \$$\frac{1400}{24} =$ \$58.333 +, *Ans.*

11. If 240 men perform a piece of work in 8 months, how many men must be employed to finish the same work in 2 months?

The ratio of $2 : 8 = 4$, and $240 \times 4 = 960$ men, *Ans.*

§ 116. Application of Canceling to Analytical Solutions.

12. If 8 pounds of tea cost \$12, what will 32 pounds cost?

$$\text{Statement: } \frac{12.\ 32}{8}.$$

The two terms of the same name, here given, are 8 and 32, and their ratio is 4, and is obtained by

$$\text{Canceling: } \frac{12.\ \overset{4}{\cancel{32}}}{\cancel{8}}.$$

Therefore, $12 \times 4 =$ \$48, *Ans.* The third term is here multiplied by the ratio of the first and second, as required for analytical solution. The terms are also canceled and multiplied as directed by the rule for canceling.

13. If 16 horses consume 84 bushels of grain in 24 days, how many bushels will suffice 32 horses 48 days? *Ans.* 336.

In the preceding sum, it is evident that the given quantity of grain is to be increased by the ratios of 16 horses to 32 horses, and of 24 days to 48 days.

$\frac{84.\ 32.\ 48}{16.\ 24}$ is the statement expressing those ratios.

We therefore see, by the above example, that the effect of the operation is to increase the quantity of the same name as the required quantity, by all the given ratios. The same is true in all cases, that is, every statement for canceling is a complete analysis of the question under consideration.

14. If 8 men build 9 feet of wall in 12 days, how many men must be employed to build 36 feet in 4 days?

$$\text{Statement: } \frac{8.\ 36.\ 12}{9.\ \ 4} = 96 \text{ men.}$$

The number of men required will obviously depend on the ratios 9 : 36, and 4 : 12, the former of which is 4, and the latter 3. Therefore, 8 men $\times 4 \times 3 = 96$ men, the number required.

Hence we perceive, that, in a correct solution of any sum by canceling, a complete analysis of that sum is given.

15. If 10 men make 300 pairs of boots in 20 days, how many men must be employed to make 450 pairs in 30 days?

Ans. 10 men.

If 10 men make 300 pairs in 20 days, they would make 15 pairs in one day; and if 10 men make 15 pairs in one day, 1 man would make one and a half pairs per day, and in 30 days he would make 45 pairs; therefore, $450 \div 45 = 10$, *Ans.*

16. If $\frac{1}{2}$ of a yard cost $\frac{3}{5}$ of a pound sterling, what will $\frac{5}{8}$ of a yard of the same cloth cost?

If $\frac{1}{2}$ yard cost $\frac{3}{5}$ of a pound, the whole yard would cost $\frac{6}{5}$ of a pound, and $\frac{1}{8}$ of the same would cost $\frac{1}{8}$ of $\frac{6}{5}$ of a pound $= \frac{6}{40}$ of a pound; consequently, $\frac{5}{8}$ would cost 5 times that sum, or $\frac{30}{40} = \frac{3}{4}$ of a pound, or 15 s., *Ans.*

17. If $\frac{7}{16}$ of a house cost 49 pounds, what will be the value of $\frac{3}{32}$ of the same? *Ans.* 10£. 10 s.

18. A merchant bought a number of bales of velvet, each containing $129\frac{17}{27}$ yards, at the rate of \$7 for 5 yards, and sold the same at the rate of \$11 for 7 yards, and gained \$200 by the transaction. How many bales were there?

He paid $\frac{7}{5}$ of a dollar per yard, and received $\frac{11}{7}$ of a dollar for the same. Hence, $\frac{11}{7} - \frac{7}{5} = \frac{55}{35} - \frac{49}{35} = \frac{6}{35}$ of 1 dollar, the amount gained on 1 yard. Therefore, $\$200 \div \frac{6}{35} = \frac{7000}{6}$, the whole number of yards; and $\frac{7000}{6} \div 129\frac{17}{27} = \frac{7000}{6} \div \frac{3500}{27} = \frac{31500}{27} \div \frac{3500}{27} = 9$ bales, *Ans.*

19. If 7 horses consume $2\frac{3}{4}$ tons of hay in 6 weeks, how many tons will 12 horses consume in 8 weeks? *Ans.* $6\frac{2}{7}$ tons.

20. If 14 men finish a piece of work in 42 days, how long will it take 21 men to do it? *Ans.* 28 days.

21. If $\frac{5}{9}$ of a farm be valued at \$895, what is the whole farm worth? *Ans.* \$1611.

22. If 7 horses consume 29 bushels of oats in 5 weeks, how many will 12 horses consume in 6 weeks? *Ans.* $59\frac{23}{35}$ bushels.

23. A merchant, owning $\frac{7}{8}$ of a vessel, sold $\frac{5}{6}$ of his share for \$1200. What was the value of the whole vessel, at the same rate? *Ans.* \$1645.714 +.

24. There is a pole, $\frac{1}{4}$ in the mud, $\frac{3}{5}$ in the water, and 8 feet out of the water. What is its length? *Ans.* $53\frac{1}{3}$ feet.

25. In a certain orchard, $\frac{1}{2}$ of the trees bear apples, $\frac{1}{4}$ pears, $\frac{1}{6}$ plums, 30 of them peaches, and 20 cherries. How many trees does the orchard contain? *Ans.* 600.

26. A certain school is classified as follows: $\frac{1}{16}$ study grammar, $\frac{3}{8}$ study geography, $\frac{3}{10}$ arithmetic, $\frac{3}{20}$ write, and 9 learn to read. How many are there in all, and how many in each study?

Ans. Whole number, 80; — in grammar, 5; geography, 30; arithmetic, 24; writing, 12; and 9 read.

SIMPLE AND COMPOUND PROPORTION IN FRACTIONS.

§ **117.** In stating such sums in Simple or Compound Proportion as consist of fractions, *it is necessary only to compare terms as already directed, and then,* if they are solved without canceling, *having inverted the divisor, to divide the product of the numerators by the product of the denominators.* If, however, they are to be solved by canceling, *arrange the numerators of the several fractions as directed to arrange whole numbers, when whole numbers only are given, and place each denominator opposite its own numerator.*

NOTE. — Before stating the sum, mixed numbers, if any are given, must be reduced to improper fractions.

Ex. 1. If $\frac{3}{5}$ of a yard cost $\frac{7}{15}$ of a pound, what will $\frac{3}{14}$ of a yard cost?

$$\text{Statement: } \frac{7.\ 5.\ 3}{15.\ 3.\ 14} = \tfrac{1}{6} \text{ of } 1\,£. = 3\text{ s. } 4\text{ d.}$$

The $\frac{3}{5}$ is inverted, that its numerator may stand below the line, as the same term would stand if it were a whole number.

2. If $\frac{3}{4}$ of a pound of sugar cost $\frac{8}{9}$ of a shilling, what will $\frac{9}{10}$ of a pound cost?

$$\text{Statement: } \frac{8.\ 4.\ 9}{9.\ 3.\ 10}. \quad \textit{Ans. } 1\text{ s. } 0\text{ d. } 3\tfrac{1}{5}\text{ qr.}$$

3. A person, who owned $\frac{3}{5}$ of a vessel, sold $\frac{5}{8}$ of his share for 375£. What was the value of the whole vessel, at the same rate? *Ans.* 1000£.

These sums may all be solved analytically, if preferred. The following is the solution of the last: $\frac{3}{5}$ of $\frac{5}{8} = \frac{15}{40}$, and 375 £. ÷ 15 = 25£., or $\frac{1}{40}$ of the whole value; therefore, 25£. × 40 = 1000£. *Ans.*

4. If $\frac{3}{8}$ of a ship be worth 3740£., what is the value of the whole? *Ans.* 9973£. 6 s. 8 d.

5. If $1\frac{1}{4}$ yards cost 9 shillings, what is the value of $16\frac{1}{4}$ yards? *Ans.* 5£. 17 s.

6. What is the value of $\frac{8}{9}$ of a pound of lard, if $1\frac{5}{10}$ of a pound cost $\frac{1}{3}$ of a shilling? *Ans.* $2\frac{10}{27}$ pence.

7. A person, who owned $\frac{3}{8}$ of a lot of land, sold $\frac{2}{3}$ of his share for \$3024. What was the value of the whole lot, at the same rate? *Ans.* \$12096.

8. A certain vessel is valued at \$1562.50. What is the value of $\frac{2}{5}$ of $\frac{4}{5}$ of the same? *Ans.* \$500.

9. I owned $\frac{2}{5}$ of a ship, and sold $\frac{5}{8}$ of my share for \$780. What was the value of the whole, at the same rate? *Ans.* \$3120.

10. A merchant bought $5\frac{2}{5}$ pieces of cloth, each containing $24\frac{2}{3}$ yards, for $6\frac{2}{3}$ shillings per yard. How many dollars did the whole cost, in New York currency? *Ans.* \$111.

11. A merchant had $4\frac{3}{8}$ cwt. of sugar, at $6\frac{1}{4}$ pence per lb., which he exchanged for tea, at $8\frac{3}{4}$ shillings per pound. How many pounds of tea did he receive? *Ans.* $29\frac{1}{6}$.

12. How many pounds sterling will 150 yards of cloth cost, at $1\frac{1}{5}$ shilling per yard? 150 6 *Ans.* 9£.

13. If $3\frac{1}{2}$ times $3\frac{1}{2}$ yards cost $1\frac{1}{2}$ times $1\frac{1}{2}$ pounds sterling, how many shillings and pence will $\frac{1}{2}$ of $\frac{1}{3}$ of $12\frac{1}{4}$ yards cost? *Ans.* 7 s. 6 d.

14. What is the value of $\frac{3}{4}$ of an ounce of silver, if 2 oz. be valued at $12\frac{2}{3}$ shillings? *Ans.* 4 s. 9 d.

15. What quantity of shalloon, that is $\frac{3}{4}$ of a yard wide, will be sufficient to line $7\frac{1}{2}$ yards of cloth, $1\frac{1}{2}$ yards wide? *Ans.* 15 yards.

16. If $2\frac{1}{2}$ yards, $1\frac{1}{7}$ yard wide, be sufficient to make a coat, how much will it require of cloth that is $\frac{3}{5}$ of a yard wide, to make the same kind of garment? *Ans.* 4 yd. $3\frac{1}{21}$ qr.

17. How many pieces of cloth, at \$$18\frac{2}{3}$ per piece, are equal in value to $224\frac{2}{5}$ pieces, at \$$12\frac{1}{2}$ per piece? *Ans.* $150\frac{15}{56}$.

18. A merchant exchanged $7\frac{7}{8}$ cwt. of sugar, at $7\frac{2}{3}$ pence per pound, for tea at $9\frac{1}{4}$ shillings per pound; how many pounds of tea did he receive? *Ans.* $60\frac{34}{37}$ lb.

19. If 8 men can perform a piece of work in $6\frac{2}{3}$ hours, in what time will 20 men do the same? *Ans.* 2 hours, 40 minutes.

20. How many yards of cloth, $\frac{5}{4}$ of a yard wide, will line 20 yards, $\frac{3}{4}$ of a yard wide? *Ans.* 12 yards.

21. How many pieces of cloth, at $18\frac{1}{8}$ shillings per yard, are equal in value to $350\frac{4}{5}$ pieces, at $12\frac{1}{2}$ shillings per yard? *Ans.* $241\frac{27}{29}$ pieces.

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22. Lent a friend \$72$\frac{1}{5}$ for 8$\frac{3}{4}$ months. What sum must he lend me for 2$\frac{1}{2}$ years, to balance the favor? *Ans.* \$21.233+.

The following sums properly belong to Compound Proportion. They may be solved either by canceling, by analysis, or by the common rule of Compound Proportion.

Ex. 23. If $\frac{3}{4}$ of a yard of cloth, which is $\frac{7}{8}$ of a yard wide, cost $\frac{2}{5}$ of a pound sterling, what is the value of $\frac{5}{8}$ of a yard that is 1$\frac{3}{4}$ yard wide?

Analysis: $\frac{3}{4} \times \frac{7}{8} = \frac{21}{32}$, the fraction of a square yard purchased, which cost $\frac{2}{5}$ of a pound sterling. Therefore, $\frac{2}{5} \div 21 = \frac{2}{105}$, the value of $\frac{1}{32}$ part of a square yard, and $\frac{2}{105} \times 32 = \frac{64}{105}$, the price of 1 yard. $\frac{5}{8} \times \frac{7}{4} = \frac{35}{32}$, the quantity of which the price is required. Therefore, $\frac{64}{105} \times \frac{35}{32} = \frac{2240}{3360} = \frac{2}{3}$ of 1 £.

Ans. 13s. 4d.

The same canceled: $\frac{2.\ 5.\ 7.\ 4.\ 8}{5.\ 8.\ 4.\ 3.\ 7} = 13\text{s. } 4\text{d.}$

For the inversion of the fractions $\frac{3}{4}$ and $\frac{7}{8}$, see § 72, Vulgar Fractions.

24. If 9 men spend 12$\frac{1}{2}$ £. in 27 days, what sum will 25 men spend in 40 days?

Analysis: 12$\frac{1}{2}$ £. $= \frac{25}{2}$ £., and $\frac{25}{2} \div 9 = \frac{25}{18}$ £., the money 1 man spends in 27 days; and $\frac{25}{18}$£. $\div 27 = \frac{25}{486}$, the money spent by 1 man daily. Therefore, $\frac{25}{486}$ £. $\times 25 = \frac{625}{486}$ £., the money 25 men spend daily; and $\frac{625}{486}$ £. $\times 40 = \frac{25000}{486}$£., the sum of money required, which, reduced, gives 51 £. 8 s. 9$\frac{55}{81}$ pence, *Ans.*

The same stated for canceling: $\frac{25.\ 25.\ 40}{2.\ \ 9.\ 27} = 51$ £. 8 s. 9$\frac{55}{81}$ d.

25. If 18 persons consume $\frac{27}{16}$ lb. of tea in 1 month, how much will 8 persons consume in 6 months? *Ans.* 4$\frac{1}{2}$ lb.

26. If the tuition of 2 boys for $\frac{3}{4}$ of a year be \$56$\frac{1}{4}$, how much will be the tuition of 3 boys for 5$\frac{1}{3}$ years? *Ans.* \$600.

27. If 90 cwt. be carried 30 miles for \$29, how many cwt. may be carried 45 miles for \$5$\frac{4}{5}$? *Ans.* 12 cwt.

28. If 10 persons drink 15$\frac{3}{5}$ gallons of wine in 1 week, how much will 16 persons drink in 43 weeks?

Ans. 1073$\frac{7}{25}$ gallons.

29. If $\frac{1}{2}$ cwt. be carried 600$\frac{5}{6}$ miles for \$12$\frac{1}{2}$, how far may $\frac{3}{4}$ of a cwt. be carried for \$30$\frac{5}{6}$? *Ans.* 988$\frac{1}{27}$ miles.

QUESTIONS.—When is an arithmetical question solved analytically? What is the general principle by which sums may be solved analytically? How are sums in Simple or Compound Proportion solved without canceling? How are they solved by canceling? What is the note?

CONJOINED PROPORTION.

§ **118.** Conjoined Proportion consists of a comparison instituted between a series of terms bearing a certain relation to each other, as the coins, weights, and measures of different countries.

The principle involved in this rule is the same as in Simple and Compound Proportion. No further explanation is therefore needed.

Rule. — *Above a horizontal line, near the left, place the demanding term; then, below the line, place the term of the same name as the demand, with the term which it equals in value, directly above it. Next, seek another term of the same name as the one last placed, and set it also below the line, with the one it equals in value, above it. Thus proceed to arrange the terms, making each term standing below the line of the same name as the preceding term standing above it. The product of the numbers standing above the line divided by the product of those standing below it, will give the required number.*

NOTE. — The numbers may, of course, be canceled as far as practicable, before multiplying and dividing.

Ex. 1. If 100 lb. English make 90 lb. Flemish, and 22 lb. Flemish make 28 lb. Bologna, how many pounds English are equal to 56 lb. Bologna?

The demand obviously lies on the 56 lb. Bologna; therefore,

$$\frac{56.\ 22.\ 100}{28.\ 90}.$$

Canceled: $\dfrac{\overset{2}{\cancel{56}}.\ 22.\ 10\cancel{0}}{\cancel{28}.\ \ 9\cancel{0}}$. $2 \times 22 \times 10 = 440 \div 9 = 48\frac{8}{9}$ lb. English, *Ans.*

2. If 40 lb. at New York make 48 lb. at Antwerp, and 30 lb. at Antwerp make 36 at Leghorn, how many pounds at New York are equal to 144 at Leghorn?

Statement: $\dfrac{144.\ 30.\ 40}{36.\ 48}.$

Canceled: $\dfrac{\overset{4}{\cancel{144}}.\ \overset{5}{\cancel{30}}.\ \overset{20}{\cancel{40}}}{\cancel{36}.\ \cancel{48}.\ \cancel{12}.\ \cancel{6}}$, and $5 \times 20 = 100$ lb. New York, *Ans.*

3. If 70 braces at Venice make 84 braces at Leghorn, and 12 at Leghorn make 7 American yards, how many braces at Venice are equal to 96 American yards? *Ans.* $137\frac{1}{7}$.

4. If 24 lb. at New London make 20 lb. at Amsterdam, and 50 lb. at Amsterdam make 60 lb. at Paris, how many lb. at Paris are equal to 40 lb. New London? *Ans.* 40 lb.

5. If 50 lb. at New York make 45 lb. at Amsterdam, and 80 lb. at Amsterdam, 103 lb. at Dantzic, how many lb. at Dantzic are equal to 240 lb. New York? *Ans.* 278$\frac{1}{10}$.

6. If 24 braces at Leghorn be equal to 15 vares at Lisbon, and 45 vares at Lisbon be equal to 90 braces at Lucca, how many braces at Lucca are equal to 120 braces at Leghorn? *Ans.* 150 braces.

QUESTIONS.—In what does Conjoined Proportion consist? How does the principle involved compare with Simple and Compound Proportion? What is the rule for Conjoined Proportion?

DISCOUNT.

§ **119.** Discount is an allowance made for the payment of money before it becomes due.

The present worth of any sum of money, payable at some future time without interest, *is that sum which, if put at interest, would, in the given time and at the rate per cent., amount to the whole debt.*

Discount is not, therefore, a deduction of the given per cent. from a hundred cents or a hundred dollars. If I have a claim upon an individual for \$100, payable a year hence, and propose to allow him 6 per cent. discount for present payment, I must receive more than \$100 — \$6 = \$94; since \$94 put on interest, at 6 per cent., will not amount to \$100 in the given time. The interest on \$94 one year at 6 per cent. is \$5.64; and \$94 + \$5.64 = \$99.64, which is 36 cents less than the required sum, or \$100. If, however, a person owe me \$106, payable in one year, without interest, and I propose to allow him the same discount for immediate payment, he must obviously pay me \$100, since \$100 in one year, at 6 per cent., will amount to precisely \$106.

Hence, we learn that the ratio which any sum, due a year hence, without interest, bears to its present worth, is as 106 to 100; or, what is the same thing, as \$1.06 to \$1.00, whenever the discount is at 6 per cent. If the rate per cent. be any other than 6, or the time more or less than 1 year, the ratio varies accordingly. Therefore, *as the amount of \$1, for the given time and rate per cent., is to \$1, so is the given sum to its present worth.*

Ex. 1. What is the present worth of $450, due 2 years hence, 6 per cent. discount being allowed?

The interest of $1, for 2 years, at 6 per cent., is 12 cents, and, consequently, the amount of $1, for the same time, is $1.12. Therefore, 1.12 : 1 : : 450 : the required sum. And, since nothing is effected by multiplying by 1, the required sum is obtained by dividing $450 by $1.12. Hence, $450.00 ÷ $1.12 = $401.785 +, *Ans.*

From the above we derive the following rule: —

Rule. — *Divide the sum on which the discount is to be made, by the amount of one dollar for the given time and rate per cent.*

2. What is the present worth of $700, due three years hence, at 5 per cent. discount?

The amount of $1, for 3 years, at 5 per cent., is $1.15. Therefore, $700.00 ÷ 1.15 = $608.695 +, *Ans.*

3. Sold goods to the amount of $1200, on 6 months' credit. What is the present worth, allowing 8 per cent. discount? *Ans.* $1153.846 +.

4. What is the present value of a legacy of $2000, due 2 years hence, discounting at 5 per cent. per annum? *Ans.* $1818.18 +.

5. What is the difference between the interest and discount on $600, for 12 years, at 5 per cent.?

Ans. Interest, $360; discount, $225; difference, $135.

Note. — To obtain the discount, *subtract the present value from the sum due.*

6. What is the discount on $300, for 60 days, at 6 per cent. per annum? *Ans.* $2.97.

7. What is the present value of $750, due 3½ years hence, discounting at 4 per cent. per annum? *Ans.* $657.894 +.

8. What is the discount on $500, for 2 years, at 9 per cent. per annum? *Ans.* $76.272 +.

9. What is the present value of 350 £., due 4 years hence, discounting at 4 per cent. per annum?

Ans. 301 £. 14 s. 5 d. $3\frac{1}{26}$ qr.

10. What is the present worth of $672, due 2 years hence, discounting at the rate of 6 per cent. per annum? *Ans.* $600.

11. Bought goods to the amount of $820, on 6 months' credit. What ought I to have paid, if I had advanced the money on the receipt of the goods, and had been allowed 4 per cent. discount? *Ans.* $803.92 +.

12. Sold goods to the amount of $1200, one half of which is to be paid in 6 months, and the other half in 8 months. What is the discount for the present payment of the whole, discounting at 6 per cent. per annum? *Ans.* $40.553.

13. A person, having a legacy of $1450 left him, payable in 6 years, requests present payment, and proposes to allow 6 per cent. discount. What must he receive? *Ans.* $1066.176+.

14. What is the discount of $458, for 8 months, discounting at 8 per cent. per annum? *Ans.* $23.188+.

QUESTIONS.—What is Discount? What is the present worth of any sum of money, payable at some future time, without interest? Is discount a deduction of a given per cent. from a hundred cents, or a hundred dollars? Why? What numbers express the ratio which any sum due a year hence, at 6 per cent., bears to its present worth? What is the rule for discount? How is the discount obtained? How is discount proved? Ans. *Cast the interest on the present worth, for the time and rate per cent. of discount, and add it to the present worth.*

PROFIT AND LOSS.

§ **120.** Profit and Loss is the rule by which merchants and others engaged in trade, determine how much is gained or lost by any transaction. It also enables them so to regulate the price of their goods as to gain or lose a certain per cent. on the first cost.

I. To find how much is gained or lost on a Quantity of Goods sold at Retail, the Purchase Price of the whole Quantity being given.

Rule.—*Find the value of the whole quantity, at the retail price; then, if there be a gain, subtract the purchase price from the same, and the remainder will be the sum gained; but if there be a loss, subtract the amount received from the purchase price, and the remainder will be the sum lost.*

Ex. 1. Bought 40 yards of cloth, for $160, and sold the same for $5.20 per yard? How much did I gain? $\$5.20 \times 40 = \$208.00$; and $\$208.00 - \$160 = \$48.00$, *Ans.*

2. Bought a hogshead of melasses, for $25, and sold the same for 8 cents a pint. How much did I gain? *Ans.* $15.32.

3. Bought 12 cwt. of sugar, at 8 d. per pound, and sold it at 3£., New York currency, per cwt. Did I gain, or lose? and how much? *Ans.* Lost $22.

4. Purchased 2 hogsheads of wine, for $94.50, and retailed the same at 2 s., New York currency, per quart. How much did I gain? *Ans.* $31.50.

5. Paid $57 for 456 yards of cloth, and sold the same at the

rate of 4 s. 6 d., New York currency, for 3 yards. What did I gain? *Ans.* \$28.50.

6. Bought 12 rolls of ribbon, each containing 50 yards, for \$18.75, and sold the same at 6 d., New York currency, per yard. How much did I gain by the operation? *Ans.* \$18.75.

7. Bought 44 lb. of tea, for \$16.50, and sold it for 3 s. 6 d., New England currency, per pound. What did I gain? *Ans.* \$9.166 +.

8. What do I gain on 15 cwt. of rice, which cost me \$50, by retailing the same at 4 d., New York currency, per pound? *Ans.* \$20.

§ 121. II. To find what is gained or lost per cent.

Rule.—*Find the whole gain or loss by the preceding rule, and, having multiplied it by* 100, *divide the product by the first cost. Or say, As the first cost is to the whole gain or loss, so is* \$100 *or* 100 £. *to the gain per cent.*

Ex. 1. If I buy broadcloth at \$5.50 per yard, and sell the same for \$6.00 per yard, what do I gain per cent.?

\$6.00 — \$5.50 = \$0.50, the gain on \$5.50. Therefore, .50 × 100 = 50.00, and 50.00 ÷ \$5.50 = 9.09 +, the gain on \$100. Or, \$5.50 : .50 : : 100 : the gain on \$100.

Or, the operation may be canceled, by placing *first, above the horizontal line, the whole gain or loss found by subtraction, and* \$100 *or* 100 £. *at the right of this, on the same side, and the whole cost below the same. Cancel, &c.*

The above sum thus stated; $\frac{\cancel{.50}\ \ 100}{\underset{11}{\cancel{5.50}}}$, and 100 ÷ 11 = \$9.09 +.

2. What do I gain per cent., if I buy wheat at 12 s. a bushel, and sell the same for 15 s. a bushel? *Ans.* 25 per cent.

3. Purchased pepper for 8 d. per pound, and sold the same for 9 d. per pound. What per cent. did I gain? *Ans.* 12½.

4. Bought 650 lb. of sugar, for 10 cents per lb., and sold the same for 12 cents per lb. What was my whole gain, and what my gain per cent.?
Ans. Whole gain, \$13.00; gain per cent., \$20.

5. Bought goods to the amount of \$325, and sold the same for \$370. What was the per cent. gained? *Ans.* \$13.846 +.

6. If I lose \$2 on \$25, at what rate per cent. do I lose? *Ans.* 8 per cent.

7. Purchased a hogshead of wine, for \$50, and sold the same

for $75, on 6 months' credit. What was my gain per cent., allowing 4 per cent. discount for the 6 months' credit?

604)75 = 72,115 − 50 = 22,115 *Ans.* $44.23.

8. Bought 6 cwt. of cheese, for $48; but, it being damaged, I am willing to sell it for the same on a year's credit. What is my loss per cent., discounting at 6 per cent. per annum? *Ans.* $5.66 +.

1,06)48 = 45.28 − 48.00 = 2.72

§ 122. III. TO FIND HOW A COMMODITY MUST BE SOLD TO GAIN OR LOSE A CERTAIN PER CENT. ON THE WHOLE COST.

Rule. — *If the purchase price of the quantity for which the retail price is required, be not given, it must first be found; then, if a* GAIN *per cent. be required, multiply that price by $100 increased by that per cent., and that product divided by $100, will give the answer. But if a* LOSS *per cent. be required, the purchase price must be multiplied by $100, and the product* DIVIDED *by $100 increased by the per cent. to be lost.*

Ex. 1. Bought 300 yards of cloth, for $550. How must I sell the same per yard, to gain 25 per cent.?

$\$550 \div 300 = \$1.83\frac{1}{3}$, the price of 1 yard; and $\$1.83\frac{1}{3} \times 125 = \$229.16$; and $\$229.16 \div 100 = \$2.29+$, *Ans.* Or, $100 : 125 :: 1.83\frac{1}{3} :$ *Ans.*, or $2.29 +.

Or the operation may be canceled by the following rule: —

Rule. — *Write the given price above a horizontal line, and the quantity which cost that price directly below it. Then, when a* GAIN *per cent. is required, place $100 increased by the per cent. to be gained,* ABOVE *the same, and $100 below. But if a* LOSS *per cent. is required, place $100 increased by the per cent. to be lost,* BELOW *the line, and $100 above. Cancel, &c.*

The above sum stated; $\dfrac{550.\ 125}{300.\ 100}$.

Canceled; $\dfrac{\cancel{550}.\ \overset{5}{\cancel{125}}}{\underset{6}{\cancel{300}}.\ \underset{4}{\cancel{100}}}$; and $55 \div 6 \times 4 = \$2.29+$.

2. How must I retail melasses by the gallon, for which I paid $30 per hogshead, to gain $12\frac{1}{2}$ per cent.? *Ans.* $0.535 +.

3. Purchased 100 gallons of wine, for $130, but by accident 15 gallons leaked out. How must I sell the remainder, per gallon, to gain 15 per cent.? *Ans.* $1.758 +.

4. Paid $1.10 per gallon for melasses. How must I sell the same, per quart, to gain 30 per cent.? *Ans.* $35\frac{3}{4}$ cents.

5. Received from Lisbon 180 casks of raisins, which cost me \$2.13 per cask. How shall I sell them, per cask, to gain 25 per cent.? *Ans.* \$2.66.

Statement: $\frac{2.13.\ 125}{100}$.

6. Bought 2 cwt. of pepper, at 1 s., New York currency, per pound, but it being damaged, I am willing to lose 10 per cent. How must I sell it by the lb.? *Ans.* 11 cents 3 + mills.

7. Bought one ton of steel for \$184. How must I sell the same, per pound, to gain 5 per cent.? *Ans.* 8 cents, 6 mills.

8. A merchant bought 160 yards of cloth, for \$240. How must he sell the same, per yard, to gain 12 per cent.? *Ans.* \$1.68.

9. Bought 8 pieces of cloth, each containing 15 yards, at 3s., New England currency, per yard. How must I retail the same, to gain 8 per cent.? and how much must I receive for the whole? *Ans.* 54 cents, retail price; whole value, \$64.80.

10. If I buy 6 cwt. of sugar, at 10 d., New York currency, per pound, and am allowed 4 per cent. discount for ready money, and sell the same so as to gain 15 per cent. on the money advanced, how much money do I receive? *Ans.* \$77.40+.

11. Bought 12 chests of tea, each weighing 56 pounds, at 4 s. 6 d., New England currency, per lb. For ready money, was allowed 2 per cent. discount. How much must I receive for the whole, to realize a profit of 10 per cent. on the money paid out? *Ans.* \$543.53, nearly.

Statement: $\frac{12.\ 56.\ 54.\ 100.\ 110}{12.\ \ 6.\ 102.\ 100}$.

12. Bought 700 yards of ribbon, at 6 d., New York currency, per yard. How must I sell the same, to gain $12\frac{1}{2}$ per cent.? and what shall I receive for the whole quantity? *Ans.* $6\frac{3}{4}$ d. per yard; and for the whole, \$49.218+.

13. How must cloth, which costs 13 s. 4 d., be sold to gain 20 per cent.? *Ans.* At 16 s. per yard.

QUESTIONS. — What is Profit and Loss? What is Case I.? What is the rule? What is Case II.? What is the rule? How is the statement made for canceling? What is Case III.? What is the rule? What is the rule for canceling?

Q

BARTER.

§ **123.** Barter is a rule by which those engaged in trade exchange commodities so that neither party suffer loss.

Take the following illustration: —

Ex. 1. How much sugar, at 10 d. per lb., must be given in exchange for 12 cwt. of butter, at 15 d. per pound?

It is obvious that the 12 cwt. of butter bears the same ratio to the number of cwt. of sugar required, as 10 d. bears to 15 d.

Therefore, as 10 : 15 : : 12 : the *Ans.*, viz., 18 cwt. Or, $\frac{12.\ 15}{10}$ is the statement for canceling:

$$\frac{\overset{6}{\cancel{12}}.\ \overset{3}{\cancel{15}}}{\underset{2}{\cancel{10}}},\ 6 \times 3 = 18,\ \textit{Ans.}$$

NOTE.—The scholar will perceive that the principle here involved is the same as in the rule of Proportion, which has already been fully explained.

2. How much tea, at 8 s. per lb., must be given for 2 cwt. of chocolate, at 4 s. 6 d. per lb.? *Ans.* 1 cwt. 14 lb.

Statement; 8 s. : 4 s. 6 d. : : 2 cwt. : *Ans.*

For canceling; $\frac{2.\ 54}{12.\ 8}$.

In this statement, the price of the chocolate is reduced to pence. The 12 below the line reduces the pence to shillings.

3. How much rice, at 32 s. per cwt., must be given for 4 cwt. 2 qr. of raisins, at 6 d. per pound? *Ans.* 7 cwt. 3 qr. 14 lb.

4. B has 90 yards of linen, worth 4 s. 6 d. per yard, which he wishes to exchange with C for muslin at 2 s. per yard. How many yards of muslin must B receive? *Ans.* 202½ yards.

5. Bought 75 yards of broadcloth, at $5 per yard, and paid for it in muslin, at 1 s. 3 d., New York currency, per yard. How many yards of muslin did it take? *Ans.* 1440 yards.

6. Bought 7 tuns of wine, at 10 d. a pint, and paid for it with melasses at 3 s. per gallon. How many hogsheads of melasses did it take? *Ans.* 62⅔ hhd.

7. Sold 5 cwt. 1 qr. of sugar, at $8.50 per cwt., and received in pay 24 yards of cloth. What was the value of the cloth per yard? *Ans.* $1.859.

8. How many gallons of melasses, at 8 cents a quart, must be given for 24 cwt. of rice, at $8 per cwt.? *Ans.* 600 gallons.

9. I have nuts, worth $4 per bushel; and B has sugar, worth

10 cents per pound. Now, if I charge him \$4.50 per bushel for my nuts, what ought he to charge me for his sugar? 450–10 4
Ans. $11\frac{1}{4}$ cents.

10. What quantity of butter, worth $12\frac{1}{2}$ cents a pound, must be given in exchange for 12 lb. of indigo, worth \$2.25 per pound? 12–225 12½
Ans. 216 lb.

11. Sold 5 cwt. of cheese, at 8 d. per pound, and received in pay muslin, worth 1 s. 6 d. per yard. How many yards did I receive? 5–8–4–28 18
Ans. $248\frac{8}{9}$ yds.

12. Bought 90 bushels of wheat, at 15 s., and gave in pay 115 yards of Irish linen, worth 6 s. per yard, and the remainder in cash. How many dollars did I pay, the currency being New York? 15×90=1350. 115×6=690–1350=660–8= *Ans.* \$82.50.

13. Received of B a quantity of broadcloth, which he had sold at \$4.50 per yard, but charged me \$5.00. In pay, I let him have a quantity of wheat, which I had retailed at \$1.50 per bushel. What ought I to have charged him, to meet the advance he made on his broadcloth? 500 150 450
Ans. $\$1.66\frac{2}{3}$.

14. Bought 12 gross of penknives, at 9 dollars per gross, and paid for them in cloth, at 3 s. 6 d., New York currency, per yard. How many yards were required to pay for the knives? 12–9–8–12 42
Ans. $246\frac{6}{7}$ yards.

15. Sold 48 cwt. of hops, at 4 d. per pound, and received pay in prunes, at 6 d. per pound. How many cwt. did I receive? 4–28–4–4 6
Ans. 32 cwt.

16. Received 460 yards of linen, worth 6 s. 6 d., New York currency, per yard, in exchange for 84 yards of broadcloth. What was the value of the broadcloth, per yard? 460–78 84–12–8
Ans. \$4.449 +.

17. Exchanged 320 lb. of chocolate, valued at 4 s. 6 d., New York currency, per pound, for a quantity of cotton, at 8 d. per lb.; but, there not being cotton enough to pay me, I received the balance, viz., 50 dollars, in cash. How many lb. of cotton did I receive?
Ans. 1560 lb.

18. How many cwt. of sugar, valued at 10 d. per lb., New England currency, must be given in exchange for 24 cwt. of pepper, at 15 dollars per cwt.? *Ans.* $23\frac{1}{4}$ cwt.

19. What deduction ought to be made on wheat, which has been valued at 9 s. per bushel, in exchange for cloth, when cloth, valued at 14 s. per yard, is sold for 12 s. per yard?
Ans. 1 s. $3\frac{3}{7}$ d. per bushel.

20. How many pounds of cinnamon, at 10 s. per lb., must be given in exchange for 5 cwt. of salæratus, at 9 pence per lb.?
5–4–28–9 12–10
Ans. 42 lb.

QUESTIONS.—What is Barter? What is the principle involved in this rule?

PARTNERSHIP.

§ **124.** Partnership is the rule by which individuals trading with a joint stock, determine their several shares of the profit or loss realized.

The individuals thus trading are called *stockholders.* The sum of money advanced by the company forms a *capital stock*; and the money from time to time received on the several shares is called *dividend.* When the whole stock owned by the company is employed for the same period of time, the gain or loss which results from the adventure, is divided among the individuals composing the company, in proportion to their several shares of the stock.

We have, then, the following rule: —

Rule. — *As the whole stock is to each man's share of the stock, so is the whole gain or loss to each man's share of the gain or loss.*

Ex. 1. Three persons traded in company. A's stock was \$1200; B's stock was \$4800, and C's, \$2000. They gained \$800. What was each man's share of the gain?

\$1200 + \$4800 + \$2000 = 8000, amount of the stock. Therefore, 8000 : 1200 : : 800 : A's share, viz., \$120. Then, to obtain B's share, 8000 : 4800 : : 800 : B's share, or \$480. And, lastly, for C's share, 8000 : 2000 : : 800 : C's share, or \$200.

To state sums for canceling: —

Rule. — *Write the whole gain or loss above the horizontal line, with the stock of one of the company standing at the right of it; then place the whole stock below the line, and proceed to cancel, &c.*

The above sum thus stated: $\frac{800.\ 1200}{8000} = \$120$, A's share; $\frac{800.\ 4800}{8000} = \$480$, B's share; and $\frac{800.\ 2000}{8000} = \$200$, C's share.

Note. — The operations in this rule are proved by adding together the several shares. Their sum must equal the whole gain. \$120 + \$480 + \$200 = \$800.

2. Three men, A, B, and C, traded in company. A put in \$560; B, \$700; and C, \$640. At the expiration of their partnership, they found that they had gained only \$420. What was each man's share?

Ans. A's, \$123.789; B's, \$154.737; and C's, \$141.473.

3. Three merchants, trading in company, suffered a loss of $600. Their several stocks were $800, $1000, and $1200. What was each man's loss? *Ans.* $160, $200, and $240.

4. A, B, and C, freighted a ship. A put on board 36 tons; B, 40 tons; and C, 60 tons. In a storm, 30 tons were thrown overboard. What was each man's share of the loss?

Ans. A's, $7\frac{16}{17}$ tons; B's, $8\frac{14}{17}$ tons; and C's, $13\frac{4}{17}$ tons.

5. A bankrupt, who has property valued at $2500, owes A $900; B, $1000; C, $800; and D, $500. How much will each man receive, if the whole property be given up to them?

Ans. A, $703.125; B, $781.25; C, $625; and D, $390.625.

6. Three individuals unite in purchasing 600 acres of land. A pays $3000; B, $5000; and C, $7000. What are their several proportions of the whole?

Ans. A has 120 acres; B, 200; and C, 280.

7. Three individuals, L, M, and N, freighted a ship with 324 tons burden. L put on board 72 tons; M, 108 tons; and N, 144 tons. They cleared $1200 by the adventure. What was each man's share of the gain?

Ans. L's, $266.666 +; M's, $400; and N's, $533.333 +.

8. Three persons agreed to pay $150 for the use of a pasture, and that each should pay in proportion to the number of creatures he put in. What would be their several shares, provided the first put in 100 oxen; the second, 150 oxen; and the third, 200 oxen? *Ans.* $33.333 +; $50; and $66.666 +.

9. A person, at his death, owes A 70 £.; E, 400 £.; and H, 140 £.; and his property is valued at only 400 £. How much ought each to receive?

Ans. A, 45 £. $18\frac{2}{61}$ s.; E, 262 £. $5\frac{55}{61}$ s.; and H, 91 £. $16\frac{4}{61}$ s.

10. Four individuals purchased a horse, for $96. A paid $20; B, $30; C, $25; D, $21. They sold the horse for $150. What was each man's share of the gain?

Ans. A's, $11.25; B's, $16.875; C's, 14.06\frac{1}{4}$; and D's, 11.81\frac{1}{4}$.

11. A man, whose property was worth only $500, owed G $150; H, $180; and M, $300. He therefore concluded to give his property up to his creditors. What did each receive?

Ans. G, $119.047 +; H, $142.857; M, $238.095.

12. L has a farm of 90 acres; M has one of 120 acres; and N, one of 150 acres. Their farms all joining, they agree to unite them for the purpose of renting. This they do, and receive for the whole the annual rent of $820. What is each man's share of the rent?

Ans. L's, $205; M's, $273.333 +; N's, $341.666 +.

§ **125.** Whenever the different shares are not continued through the same period of time, it is obvious that each man's share of the profit and loss will depend not entirely on his portion of the whole stock, but also on the time during which his stock was invested. Take the following sum as an illustration: —

13. Three merchants traded in company. A put in \$120, for 10 months; B, \$100, for 18 months; and C, \$150, for 5 months. They gained \$100. What was each man's share?

A's, \$120, for 10 months, $=$ \$120 $\times$ 10 $=$ \$1200, for 1 month; B's, \$100, for 18 months, $=$ \$100 $\times$ 18 $=$ \$1800, for 1 month; and C's, \$150, for 5 months, $=$ \$150 $\times$ 5 $=$ \$750, for 1 month. Therefore, \$1200 + \$1800 + \$750 $=$ \$3750; and 3750 : 1200 : : 100 : A's share, or \$32; 3750 : 1800 : : 100 : B's share, or \$48; and 3750 : 750 : : 100 : C's share, or \$20.

Hence, when the time of investment of the several shares differs,

Rule. — *Multiply each man's stock by the time during which he continued in trade, and use the several products as directed in the preceding rule to use the several shares of the whole stock.*

14. A, B, and C, hold a pasture in common, for which they pay 30 £. per annum. In this pasture, A has 40 oxen, for 75 days; B, 45 oxen, for 50 days; and C, 50 oxen, for 90 days. What part of the 30 £. ought each to pay?

Ans. A, 9 £. 4 s. $7\frac{5}{13}$ d.; B, 6 £. 18 s. $5\frac{7}{13}$ d.; and C, 13 £. 16 s. $11\frac{1}{13}$ d.

15. Three men enter into partnership on the following terms: — A invests \$1500, for 5 months; B, \$1800, for 6 months; and C, \$2000, for 8 months. During the continuance of their partnership, they sustain a loss of \$1000. What is each man's share of the loss?

Ans. A's loss is \$218.658; B's, \$314.868; and C's, \$466.472.

16. E and S enter into partnership for 1 year. E at first advances \$480, and S puts in his share 3 months after. How much must he advance to be entitled to an equal share of the gain at the expiration of 1 year? *Ans.* \$640.

17. Two merchants, trading in company, gain \$200. A's stock was \$220, for 6 months, and B's, \$380, for 9 months. How ought they to share the gain?

Ans. A's portion is \$55.696; B's, \$144.304.

18. Two men commenced trading in company, on Jan. 1, 1825. A advanced \$1000, at the time specified; but B, finding it inconvenient, did not advance his share till the 1st of

May following. At the end of the year, they shared the profits equally. What capital did B advance? *Ans.* $1500.

19. A and B traded in company. A put in $1200; B advanced his share 3 months after. What sum was it necessary for him to advance so as to be entitled to one half of the profit at the expiration of 1 year? *Ans.* $1600.

20. L, M, and N, entered into partnership. L advanced $300, for 3 years; 6 months after, M put in $450; and 6 months after M put in his share, N put in $520. At the expiration of 3 years, they found they had cleared $900. What was each man's share?

Ans. L's, $264.274; M's share, $330.342; and N's, $305.383.

QUESTIONS.—What is Partnership? What are the individuals trading in company called? What does the money advanced by the company form? What is the money they receive on their several shares called? What is the rule for operating when each man's stock is employed for the same period of time? What is the rule for canceling? When all the shares are not continued for the same period of time, on what will each man's share of the profit and loss depend? What is the rule?

COMMERCIAL EXCHANGE.

§ **126.** Under this rule are included the operations of purchasing goods in one country, in the currency of that country, and selling them, at wholesale or retail, in the currency of another country, so as to gain or lose some required per cent.

TABLE OF GOLD COINS.

NAMES OF COINS.	VALUE.	NAMES OF COINS.	VALUE.
Austrian Dominions.		*Brazil.*	
Souverein,	$3.37 7	Johannes, (half in prop.,)	$17.06 4
Double Ducat,	4.58 9	Dobraon,	32.70 6
Hungarian Ducat,	2.29 6	Dobra,	17.30 1
		Moidore, (half in proportion,)	6.55 7
Bavaria.		Crusade,	63 5
Carolin,	4.95 7	*Brunswick.*	
Max d'Or, or Maximilian,	3.31 8	Pistole, (double in prop.,)	4.54 8
Ducat,	2.27 5	Ducat,	2.23
Berne.		*Cologne.*	
		Ducat,	2.26 7
Ducat, (double in proportion,)	1.98 6	*Colombia.*	
Pistole,	5.54 2	Doubloon,	15.53 5

NAMES OF COINS.	VALUE.
Denmark.	
Ducat, Current,	$1.81 2
Ducat, Specie,	2.26 7
Christian d'Or,	4.02 1
East India.	
Rupee, Bombay,	7.09 6
Rupee, Madras,	7.11
Pagoda, Star,	1.79 8
England.	
Guinea, (half in proportion,)	5.07 5
Sovereign, (half in prop.,)	4.84 6
Seven-Shilling Piece,	1.69 8
France.	
Louis, coined before 1786,	4.84 6
Double Louis, before 1786,	9.69 7
Louis, coined since 1786,	4.57 6
Double Louis, since 1786,	9.15 3
Napoleon, or 20 francs,	3.85 1
Double Napoleon, or 40 francs,	7.70 2
Frankfort on the Maine.	
Ducat,	2.27 9
Geneva.	
Pistole, old,	3.98 5
Pistole, new,	3.44 4
Hamburg.	
Ducat, (double in prop.,)	2.27 9
Genoa.	
Sequin,	2.30 2
Hanover.	
Double George d'Or,	7.87 9
Ducat,	2.29 6
Gold Florin, (double in proportion,)	1.67
Holland.	
Double Ryder,	12.20 5
Ryder,	6.04 3
Ducat,	2.27 5
Ten-Guilder Piece,	4.03 4
Malta.	
Double Louis,	9.27 8
Louis,	4.65 2
Demi-Louis,	2.33 6
Mexico.	
Doubloon,	15.53 5
Milan.	
Sequin,	2.29
Doppia, or Pistole,	$3.80 7
Forty-Livre Piece,	7.74 2
Naples.	
Six-Ducat Piece, 1783,	5.24 9
Two do., or Sequin, 1762,	1.59 1
Three do., or Oncetta, 1818,	2.49
Netherlands.	
Gold Lion, or Fourteen-Florin Piece,	5.04 6
Ten-Florin Piece,	4.01 9
Parma.	
Quadruple Pistole,	16.62 8
Pistole, or Dappia, 1787,	4.19 4
do. do., 1796,	4.13 5
Maria Theresa, 1818,	3.86 1
Piedmont.	
Pistole, coined since 1785,	5.41 1
Sequin, (half in proportion,)	2.28
Carlino, coined since 1785,	27.34
Piece of 20 Francs, or Marengo,	3.56 4
Poland.	
Ducat,	2.27 5
Portugal.	
Dobraon,	32.70 8
Dobra,	17.30 1
Johannes,	17.06 4
Moidore, (half in proportion,)	6.55 7
Piece of 16 Testoons, 1600 Rees,	2.12 1
Old Crusado, or 400 Rees,	58 8
New Crusado, or 480 Rees,	63 5
Milree, coined in 1755,	73
Prussia.	
Ducat, 1748,	2.27 9
Ducat, 1787,	2.26 7
Frederick, double, 1769,	7.97 5
Frederick, double, 1800,	7.95 1
Frederick, single, 1778,	3.99 7
Frederick, single, 1800,	3.97 5
Rome.	
Sequin, coined since 1760,	2.25 1
Scudo of Republic,	15.81 1
Russia.	
Ducat of 1796,	2.29 7
Ducat of 1763,	2.26 7
Gold Ruble of 1756,	96 7
Gold Ruble of 1799,	73 7
Gold Polten of 1777,	35 5
Imperial of 1801,	7.82 9

NAMES OF COINS.	VALUE.
Half Imperial of 1801, . . .	$3.91 8
Half Imperial of 1818, . . .	3.93 3
Sardinia.	
Carlino, (half in proportion,)	9.47 2
Saxony.	
Ducat of 1784, ...	2.26 7
Ducat of 1797, ...	2.27 9
Augustus of 1754, ...	3.92 5
Augustus of 1784, ...	3.97 4
Sicily.	
Ounce of 1751, ...	2.50 4
Double Ounce, ...	5.04 4
Spain.	
Doubloon of 1772, (parts in proportion,)	16.02 8
Doubloon, ...	15.53 5
Pistole, ...	3.88 4
Coronilla, Gold Dollar, or Vintern, 1801, ...	98 3
Sweden.	
Ducat, ...	2.23 5
Switzerland.	
Pistole of the Helvetic Republic, ...	4.56
Treves.	
Ducat, ...	$2.26 7
Turkey.	
Sequin fonducli of Constantinople, ...	1.86 8
Sequin fonducli of Constantinople, 1789, ...	1.84 8
Half Misseir, 1818, ...	52 1
Sequin fonducli, ...	1.83
Yeermeeblekblek, ...	3.02 8
Tuscany.	
Zechino, or Sequin, ...	2.31 8
Ruspone of the Kingdom of Etruria, ...	6.93 8
Venice.	
Zechino, or Sequin, (shares in proportion,)	2.31
Wirtemberg.	
Carolin, ...	4.89 8
Ducat, ...	2.23 5
Zurich.	
Ducat, (double and half in proportion,)	2.26 7

§ 127. TABLE OF UNCOINED AND SILVER MONEY.

English Currency.	
1 £. sterling, before 1832 = $4.444. Since 1832 =	$4.80
Or, prior to 1832, 9 £. = $40. Since 1832, 5 £. =	24.00
1 English Crown, ...	1.10
Or 10 Crowns . . . . . =	11.00
1 English Shilling, ...	22 2
1 Pistareen, ...	20
Or 5 Pistareens . . . . =	1.00
1 English Penny, ...	0185
Irish Currency.	
1 £. Irish, ...	4.10
Or 10 £. . . . . =	41.00
1 Shilling, ...	20 5
1 Penny, ...	01 7
French Currency.	
1 French Crown, ...	1.10
Or 10 Crowns . . . . . =	11.00
1 Five-Franc Piece, ...	93
1 Franc	18 6
1 Decimes, ...	0186
Spanish Currency.	
1 Spanish Dollar, ...	1.00
1 Real, new Plate, ...	10
1 Real Vellon, ...	05
Currencies of other Nations.	
1 Milree of Portugal, . . .	$1.24
1 Russian Silver Ruble, . . .	75
1 Rix Dollar of Sweden, . .	1.00
1 Russian Rix Dollar, ...	66 6
Or 3 Rix Dollars . . . . . =	2.00
1 Danish Rigsbank Dollar, . .	50
1 Silver Ducat of Naples, . . .	80
Or 5 Ducats . . . . =	4.00
1 Scudo of Sicily, ...	96
1 Oncia of Sicily, ...	2.40
1 Pezza of Leghorn, ...	90
1 Pezza of Genoa, ...	89
1 Florin, of Trieste, ...	48
1 Rix Dollar of Trieste, ...	96
1 Roman Crown	1.00
1 Gold Crown of Rome, . .	1.53
1 Maltese Scudo, ...	40
1 Rupee of Bengal, ...	55 5
1 Rupee of Bombay, ...	50
1 Pagoda of Madras, ...	1.80
1 Tale of Canton, ...	1.48
1 Japanese Tale, ...	75
1 Dollar of Sumatra, ...	1.10
1 Tale of Sumatra, ...	4.16
1 Florin of Java, ...	40
1 Mark Banco of Hamburg, .	33 3
1 Guilder of Amsterdam, . . .	40

§ **128.** To reduce a foreign currency to Federal Money, *multiply the given money of that currency by the value of its unit in Federal Money, as given in the preceding tables,*

If, however, it be required to find how goods, the value of which is given in a foreign currency, must be sold in Federal Money, to gain or lose a given per cent. (*first change the foreign currency to Federal Money, and calculate the gain or loss on this by the rules already given.*) The retail price of any denomination, as cwt., lb., yd., gal., &c., *may then be found by division.*

Ex. 1. Purchased in London 360 yards of broadcloth, which cost me, including transportation, 300£. sterling. How must I sell the same, per yard, in Federal Money, to make a profit of 20 per cent.?

Solution: 300£. × 40 = 12000; and 12000 ÷ 9 = \$1333$\frac{1}{3}$, the value of 300£. in Federal Money.

Then, \$1333$\frac{1}{3}$ × 1.20 = \$1600, the value in Federal Money increased by the gain per cent.; therefore, \$1600 ÷ 360 = \$4.444, + the required price of one yard.

If the wholesale price only be required, it is only necessary to omit the last step. Thus the \$1600 is the wholesale price of the cloth given in the preceding sum.

To solve sums like the preceding by canceling, —

Rule. — *Place the whole cost in the given currency first above, and* (if the retail price be required) *the number expressing the quantity procured for that price, first below, a horizontal line. Write next above the line the value of a unit of the given currency, in Federal Money. And, lastly, to increase the price by the required per cent., place* 100 *below the line, and* 100 *increased by the per cent. to be gained, above the same. If a loss per cent. be required, reverse the last two terms.*

Note 1. — If the wholesale price be required, the number expressing the whole quantity (by the preceding rule, placed below the line) must be rejected.

Note 2. — Whenever, in the preceding tables, the value of a *number of units* of foreign currency is given in Federal Money, place *that number below the line, and its Federal value above the same.* (See £'s sterling.)

The preceding sum stated for canceling:

$$\frac{300.\ 40.\ 120}{360.\ 9.\ 100} = \$4.444 +.$$

Note. — To understand the reason of the middle terms, see Note 2, and £'s sterling in the preceding tables.

2. Purchased in London 350 yards of sheeting, for 75£., and paid 12£. for its transportation to New York city. How must I retail the same, in Federal Money, to gain 15 per cent. on the first cost? *Ans.* \$1.27 +.

Statement : $\dfrac{87.\ 40.\ 115}{350.\ \ 9.\ 100}$. The 87 in the statement $=$ 75£. + 12£.

3. Received from London 470 yards of dimity, which, including transportation, cost me 65 £. Sold the same by the yard, so as to gain 30 per cent. on the first cost. How did I sell it? *Ans.* \$0.799 + per yard.

4. Received from Dublin 600 yards of Irish linen, the whole cost of which was 75 £. Irish currency. How must I retail the same, in Federal Money, to gain $12\frac{1}{2}$ per cent.? *Ans.* \$0.576 + per yard.

5. My agent in Dublin has forwarded to me 900 yards of linen; whole cost, 60 £. Irish currency. How must I retail the same, in Federal Money, to gain 15 per cent? *Ans.* \$0.314 +.

6. I have in my store 120 yards of broadcloth, forwarded me by my agent in Paris, which cost me, including transportation, 325 crowns. How must I sell the same, in Federal Money, to gain 16 per cent.? *Ans.* \$3.455 + per yard.

7. Received 680 yards of silk from Paris, for which I paid 560 crowns; expenses of transportation, 12 crowns. How must I sell the same, in Federal Money, to gain 30 per cent.? *Ans.* \$1.20 + per yard.

8. Received from Madrid 6 hogsheads of wine, each containing 63 gallons, for which my agent paid 188 Spanish dollars. How must I sell the same, per gallon, to gain $12\frac{1}{2}$ per cent.? *Ans.* \$0.559 +.

9. I have on hand a bale of silk, containing 174 yards, which I received from Cadiz, at a cost, including transportation, of 140 piasters or Spanish dollars. How must I sell the same, per yard, to gain 5 per cent. *Ans.* \$0.844 +.

10. Received from Oporto 3 hogsheads of port wine, containing 63 gallons each; cost, including transportation, 30 milrees per hogshead. How must I retail the same, by the gallon, to gain 25 per cent.? *Ans.* \$0.738 +.

11. How must I sell broadcloth, by the yard, in Federal Money, of which 3 pieces, each containing 35 yards, cost me 135 £. sterling, to gain 30 per cent.? *Ans.* \$7.428 +.

12. Received from A B, Dublin, 560 yards of linen; whole cost, 90 £. Irish currency. What must be the retail price, in Federal Money, to gain 15 per cent.? *Ans.* \$0.757 + per yard.

13. Received from the same 600 yards of muslin, worth 56 £. How must I sell the whole quantity, in Federal Money, to gain 5 per cent. *Ans.* \$241.08.

14. Consigned to my agent, G Jones, of London, 300 barrels of flour, for which I paid \$1500. How many pounds sterling ought he to receive for the same, to gain 10 per cent., the expense of transportation being \$50? *Ans.* \$383 £. 12 s. 6 d.

15. Received of my agent in London, J. Jones, 2510 gallons of Madeira wine, which cost me, per invoice, 1640£. sterling; but, it being of an inferior quality, I am willing to lose 5 per cent. on the cost. What must be the price, per gallon, in Federal Money? *Ans.* \$2.765+.

16. Three men, trading in company, received from France 1200 bottles of champagne, for which they paid 600 French guineas, each \$4.60. How must they sell the same, per bottle, in Federal Money, to gain 40 per cent.? and what will be each man's gain per bottle?

Ans. \$3.22 per bottle; each man's gain per bottle, \$0.306+.

17. Received 300 ells of cloth from Hamburg, which cost me 1500 mark bancos. How must the same be sold, in Federal Money, by the yard, to gain 12½ per cent., the ell Hamburg being 2½ qr.? *Ans.* \$3.00.

18. New York, Jan. 6, 1838. This day received from Amsterdam, 600 yards of carpeting; whole cost, 2400 guilders. Required the retail price in Federal Money, to gain 20 per cent. *Ans.* \$1.92 per yard.

19. Shipped to London 380 barrels of flour, which cost me, including transportation, \$6 per barrel. How many English crowns must I receive for the whole quantity, to gain 10 per cent.? *Ans.* 2280 crowns.

20. Shipped to Dublin 3000 bushels of flax-seed, which cost me \$2500. How many pounds, Irish currency, must I receive for the whole quantity, to gain 5 per cent.?

Ans. 640£. 4s. 10d. 2qr.+

21. Boston, Jan. 16, 1835. This day received from my agent at Lisbon, 16 hogsheads of wine, each 65 gallons; whole cost, 640 milrees. The whole being of an inferior quality, I am willing to lose 6 per cent. on the cost. How much, in Federal Money, must I charge per gallon? *Ans.* \$0.72, nearly.

22. New York, Sept. 6, 1835. This day received from A B, London, 1200 yards of superfine broadcloth; whole cost, 1500£. How must I sell the same, in Federal Money, at wholesale, to realize a profit of 20 per cent.? *Ans.* \$8000.

23. Received from Russia a quantity of fur; whole cost, 900 silver rubles. What must be the wholesale price, in Federal Money, to make an advance of 15 per cent. on the cost?

Ans. \$776.25.

24. Boston, Feb. 26, 1836. This day received from my agent in Paris, 24 hogsheads French wine, each 60 gallons; whole cost, 288 guineas. How must the same be retailed by the quart, in Federal Money, to gain 12½ per cent.?

Ans. \$0.258+.

25. Shipped to my agent in London, 650 barrels of flour;

whole cost, \$3600. How many pounds sterling must I receive for the same, to gain 10 per cent. on the cost? *Ans.* 891 £.

QUESTIONS. — What operations are included under this rule? How is a foreign currency reduced to Federal Money? How is the operation performed, when it is required to find how goods, the value of which is given in a foreign currency, must be sold to gain or lose a certain per cent. in Federal Money? What is the rule for canceling? What is Note 1? What is Note 2?

TARE AND TRET.

§ **129.** We come now to consider the allowances to be made in the purchase of goods by weight. The following particulars require to be first noticed: —

Gross weight is the whole weight of the goods purchased, including that of the box, barrel, bag, &c., containing them.

Draft is a deduction from the gross weight, made in favor of the buyer.

Tare is an allowance made for cask, box, barrel, &c., containing the goods, and may be either a certain deduction from the whole quantity, or so much per box, &c.

Tret is an allowance of 4 lb. for every 104 lb., made for the dust, &c.

Suttle is what remains after *some* of the preceding allowances have been made.

Net weight is what remains after all the deductions have been made.

Ex. 1. Bought a hogshead of sugar, weighing 7 cwt. 2 qr. 26 lb.; tare on the whole, 3 qr. 24 lb. What is the net weight?

cwt.	*qr.*	*lb.*	
7	2	26	
	3	24	
6	3	2	net weight.

2. What is the net weight of 12 casks of raisins, each weighing 2 cwt. 2 qr. 14 lb.; tare per cask, 12 lb.?

2 cwt. 2 qr. 14 lb. × 12 = 31 cwt. 2 qr., the gross weight. 12 × 12 = 144 lb. = 1 cwt. 1 qr. 4 lb.; and 31 cwt. 2 qr. — 1 cwt. 1 qr. 4 lb. = 30 cwt. 0 qr. 24 lb., *Ans.*

3. What is the net weight of 6 casks of prunes, each weighing 3 cwt. 2 qr. 10 lb.; tare 20 lb. per cask?

Ans. 20 cwt. 1 qr. 24 lb.

4. What is the net weight of 44 cwt. gross, if 14 lb. per cwt. be allowed for tare? 44 × 14 = 616 lb. = 5 cwt. 2 qr.; and 44 cwt. — 5 cwt. 2 qr. = 38 cwt. 2 qr., *Ans.*

Or, the solution may be effected by canceling, by the following rule: —

Rule. —*Place the whole gross weight first above a horizontal line. Then place* 112 *lb. below the line, with* 112 *diminished by the tare per cwt., standing directly above it. Cancel, &c*

The above sum solved by this rule: —

$$112 - 14 = 98. \quad \frac{44.\ 98}{112} = 38 \text{ cwt. } 2 \text{ qr.}$$

5. Bought 84 cwt. of sugar. What is the net weight, if 20 lb. per cwt. be allowed for tare? *Ans.* 69 cwt.

6. Bought 9 hogsheads of sugar, each weighing 8 cwt. 2 qr. From this, a deduction of 16 lb. per cwt. was made for tare. What was the net weight? *Ans.* 65 cwt. 2 qr. 8 lb.

Note 1. — When the price per cwt. or per lb. is given, the reduction for tare and tret may be made, *and the whole cost ascertained by a single statement, as may be seen from the following example:* —

7. What is the value of 8 hogsheads of sugar, each weighing 12 cwt. gross, tare 12 lb. per cwt., at $8.50 per cwt.?

$$\text{Statement: } \frac{8.\ 12.\ 100.\ 8.50}{112.} = \$728.57, \textit{ Ans.}$$

8. Bought 32 casks of figs, each weighing 2 cwt. 2 qr., at a deduction of 18 lb. per cwt. for tare. What did the whole cost me, at $4 per cwt. net weight? *Ans.* $268.57.

9. Bought 15 cwt. of sugar, at $6.50 per cwt. net weight. Reduction for tare, 12 lb. per cwt.; tret, 4 lb. per 104 lb. How must I sell the whole, to gain 20 per cent. on the first cost? and how must I retail it, to gain the same per cent.?

Ans. Wholesale price, $100.446; retail price, $0.059, nearly.

To effect all the reductions and the gain per cent. of the preceding sum, a single statement only is required; thus: —

$$\frac{15.\ 6.50.\ 100.\ 100.\ 120}{112.\ 104.\ 100}$$

10. Bought 32 chests of tea, each weighing 4 cwt. 2 qr., at $49 per cwt. net weight; tare, 12 lb. per cwt.; tret, 4 lb. per 104 lb. How must I sell the whole quantity, to gain 20 per cent.? and how must the same be retailed, to gain the same?

Ans. Wholesale price, $7269.23; retail price, $0.525.

11. Purchased 5 cwt. of sugar; tare allowed, 8 lb. per cwt. For the net weight I paid 6 d., New York currency, per lb. How must I sell the whole quantity, to gain 20 per cent? and how must the same be retailed, to gain the same per cent.?

Ans. Wholesale price, $39; retail price, $0.07½ per pound.

12. Purchased 12 bags of coffee, each weighing 96 lb.; tare,

per bag, 6 lb. What was the whole cost, at 30 cents per lb.? and the retail price, to gain 25 per cent.?

Ans. \$324, whole cost; \$0.37½, retail price.

13. How much will 8 hogsheads of sugar, each weighing 8 cwt. 3 qr., cost, at \$9 per cwt., if a deduction of 12 lb. per cwt. be allowed for tare? and what will be received for the whole, if it be sold at an advance of 30 per cent.?

Ans. \$562.50, cost; and \$731.25, received.

14. What is the net weight of 3 tierces of rice, each weighing 4 cwt. 3 qr. gross; the tare allowed, 16 lb. per cwt.; tret, 4 lb. per 104 lb.? *Ans.* 11 cwt. 2 qr. 27 lb. +.

15. What is the cost of 15 chests of tea, each containing 140 lb. gross, at 5 s., New York currency, per lb.; tare, 16 lb. per cwt.? *Ans.* \$1125.

16. Bought 12 hogsheads of sugar, each weighing 10 cwt. 2 qr., at \$9 per cwt. net weight; deduction for tare, 12 lb. per cwt. How much must I receive for the whole quantity, to gain 10 per cent. on the cost? *Ans.* \$1113.75.

17. Bought 16 firkins of butter, each weighing 108 lb.; reduction for tare, 8 lb. per cwt. Paid 15 pence, New England currency, per pound. What did it cost me? and what must be the wholesale, and what the retail, price, to gain 20 per cent. on the first cost?

Ans. Whole cost, \$334.285 +; wholesale price, \$401.142; retail price, \$0.25.

18. Bought 18 cwt. of sugar, at \$12 per cwt. net weight; tare, 16 lb. per cwt. How must I sell the same, per lb., to gain 12 per cent. on the first cost? *Ans.* ~~\$0.14.~~

19. Purchased in London, 16 cwt. of tea, at 28£. sterling per cwt. net weight; tare, 12 lb. per cwt. How much must I receive, in Federal Money, for the whole quantity, to realize a profit of 12 per cent.? and what retail price will allow the same profit? *Ans.* Wholesale price, \$1991.11; retail price, \$1.24.

QUESTIONS.—What is Tare and Tret? What is gross weight? What is draft? What is tare? What is tret? What is suttle? What is net weight? What is the rule? What is the note?

EQUATION OF PAYMENTS.

§ **130.** Equation of Payments is the method of finding a mean time for the payment of several debts due at different periods of time.

Ex. 1. I owe a friend \$380, to be paid as follows, viz., \$100 in 6 months; \$120 in 7 months; and \$160 in 10 months. If I pay the whole at once, at what time must the payment be made, so that neither I nor my friend shall lose interest?

SOLUTION.

The interest of \$100 for	6 mo.	= the interest of \$1 for	600 mo.
The interest of \$120 for	7 mo.	= the interest of \$1 for	840 mo.
The interest of \$160 for	10 mo.	= the interest of \$1 for	1600 mo.
Amt. of payments, \$380			3040 mo.

3040 = the months requisite for \$1 to gain as much interest as \$380 would gain in the required time. Therefore, \$380 : \$1 :: 3040 months : to the required time, viz., 8 months.

The 600, 840, and 1600 months, are obviously obtained by multiplying the several payments by the time which must elapse before they severally become due.

We, therefore, have the following rule: —

Rule. — *Multiply each payment by the time which must elapse before it becomes due, and divide the sum of the products by the sum of the payments.*

2. A owes me \$50, payable in 4 months; \$100, payable in 10 months; and \$150, payable in 16 months. In what time must he pay the whole, so that neither shall lose interest?
Ans. 12 months.

3. What is the equated time of payment for the three following sums, viz., \$500, payable in 3 years; \$400, payable in 4 years; and \$600, payable in 5 years? *Ans.* $4\frac{1}{15}$ years.

4. What is the equated time of payment for the three following sums, viz., \$50, payable in 4 months; \$75, payable in 6 months; and \$100, payable in 7 months? *Ans.* 6 months.

5. A owes B \$400, of which \$80 is payable in 6 months, \$120 in 10 months, and the remainder in 1 year and 8 months. What is the equated time of payment?
Ans. 1 year, 2 months, and 6 days.

6. A merchant has a certain sum of money due him, of which $\frac{1}{3}$ is payable in 2 months, $\frac{1}{4}$ in 4 months, and the remainder in 6 months. What is the equated time for the payment of the whole? *Ans.* $4\frac{1}{6}$ months.

7. I owe four sums of money, payable as follows, viz., \$60, payable in 9 months; \$80, in 10 months; \$50, in 11 months; and \$60, in 12 months. At what time may I pay the whole, without loss? *Ans.* $10\frac{11}{25}$ months.

8. A person owes a debt of \$2000, payable in 7 months, of which he proposes to pay \$600 down, on condition that the remainder be allowed to remain unpaid an adequate term of time. In what time ought it to be paid? *Ans.* 10 months.

§ **131.** When bills are contracted at different periods, and have each a specified time to run, the equated time of payment may be found by the following rule ;—

Rule.—*Observe at what date each payment becomes due; — also, how many days must elapse after the first becomes payable, before each of the others is due; — then, omitting the first payment, multiply the others by the days belonging to each. The sum of these products, divided by the sum of all the payments, will give the number of days intervening between the time of the first payment and the equated time of all the payments.*

9. If I purchase goods as follows, what will be the equated time of payment? viz.

1840.

Jan. 1,	on 4 months' credit,	a bill of goods	amounting to	$580.
April 12,	" "	" " " "	"	" $450.
June 15,	" "	" " " "	"	" $600.
July 20,	" "	" " " "	"	" $700.

The 1st bill becomes due on May 1, 1840.
" 2d " " " " Aug. 12, "
" 3d " " " " Oct. 15, "
" 4th " " " " Nov. 20, "

From May 1st to Aug. 12th = 103 days; — from May 1st to Oct. 15th = 167 days; — from May 1st to Nov. 20th = 203 days.

			580
450 × 103 =	46350		450
600 × 167 =	100200	and	600
700 × 203 =	142100		700
	288650		2330

Then, 288650 ÷ 2330 = 124, nearly; that is, the equated time extends into, but not through, the 124th day after May 1, which is Sept. 2, *Ans.*

10. What will be the equated time of payment, if I purchase goods as follows? viz.

March 12, 1840,	a bill of goods,	on 3 months' credit,	$630.
May 16,	" " " " "	" 4 months' "	$350.
June 24,	" " " " "	" 4 months' "	$470.
July 10,	" " " " "	" 3 months' "	$512.
Aug. 18,	" " " " "	" 5 months' "	$460.

Ans. Sept. 28th, 1840.

QUESTIONS. — What is Equation of Payments? What is the rule? What is the rule, when bills are contracted at different periods?

DUODECIMALS.

§ **132.** *Quodecimals are fractions of a foot.* The unit or foot is first supposed to be divided into 12 equal parts, called *inches* or *primes*, and marked $'$. Each inch or prime is then divided into 12 equal parts, called *seconds* and marked $''$. Each second is again divided in like manner, and each part obtained by this division is called a *third*, and marked $'''$. A foot is therefore divided duodecimally, when it is separated into 12 equal parts, or when the several divisions sustain a twelve-fold relation to each other. This relation may be thus illustrated: —

$1'$ inch or prime is . . . . $\frac{1}{12}$ of a foot.
$1''$ second is $\frac{1}{12}$ of $\frac{1}{12}$, ... $\frac{1}{144}$ of a foot.
$1'''$ third is $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$, ... $\frac{1}{1728}$ of a foot.
$1''''$ fourth is $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$, ... $\frac{1}{20736}$ of a foot.
$1'''''$ fifth is $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$, . . $\frac{1}{248832}$ of a foot.
$1''''''$ sixth is $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$ of $\frac{1}{12}$, $\frac{1}{2985984}$ of a foot, &c.

The marks $'$, $''$, $'''$, $''''$, &c., are the indices of the several denominations.

TABLE OF DENOMINATIONS.

$12''''''$ sixths make	$1'''''$ fifth.
$12'''''$ fifths make	$1''''$ fourth.
$12''''$ fourths make	$1'''$ third.
$12'''$ thirds make	$1''$ second.
$12''$ make	$1'$ prime or inch.
$12'$ inches or primes make	1 foot.

NOTE. — Duodecimals may be added or subtracted in the same manner as compound numbers; the denominations decreasing or increasing in the constant ratio of 12. These operations are so simple, that it is unnecessary to introduce any examples. The principle is the same as in Compound Addition and Subtraction.

MULTIPLICATION OF DUODECIMALS.

§ **133.** The greatest difficulty the scholar will here encounter, will be to determine *the denomination of the product of any two denominations.* Suppose it be required to multiply $6'$ inches or primes by $4'$ inches, their product is obviously 24; but of what denomination is it? By recurring to the preceding tables, it will be seen that $6'$ inches $= \frac{6}{12}$ of a foot, and $4'$ inches

$= \frac{4}{12}$ of a foot; and $\frac{6}{12} \times \frac{4}{12} = \frac{24}{144}$, that is, 24″. Hence, inches multiplied by inches, produce seconds. Again, let it be required to multiply 9″ seconds by 3′ inches. 9 seconds $= \frac{9}{144}$, and 3 inches $= \frac{3}{12}$; therefore, $\frac{9}{144} \times \frac{3}{12} = \frac{27}{1728}$, or 27‴. Lastly, what is the product of 7″ seconds multiplied by 9″ seconds? 7″ seconds $= \frac{7}{144}$, and 9″ seconds $= \frac{9}{144}$, and $\frac{7}{144} \times \frac{9}{144} = \frac{63}{20736}$, or 63⁗ fourths. From the above we draw the following conclusions, viz., *that feet multiplied by feet produce square feet; feet multiplied by inches produce inches; inches or primes multiplied by inches give seconds; seconds multiplied by seconds give fourths; seconds multiplied by thirds give fifths, &c.*; that is, *the product of any two denominations will always be of the denomination expressed by the sum of their indices.*

Rule.—*Begin with the lowest denomination of the multiplicand, and multiply it by the highest denomination of the multiplier, and place each term of the product according to its respective value. Multiply in the same manner by each remaining denomination of the multiplier, and place the product of each succeeding multiplication one or more places farther to the right, according to the denomination. The several products, thus obtained, when added together, will give the required product.*

NOTE 1.—It will be remembered to carry by 12 in all cases.

Ex. 1. Multiply 4 feet 4 inches by 4 feet 4 inches.

OPERATION.

ft.	in.	
4	4′	
4	4′	
17	4′	
1	5′	4″
18 ft.	9′	4″

Beginning with the 4 feet in the multiplier, we say, 4 times 4′ are 16′ = 1 ft. 4′. The 4′ is set down, and the 1 ft. carried to the next product, making it 17 feet. Then, multiplying by 4′, we say 4′ × 4′ = 16″ = 1′ and 4″; set down the 4″, and carry the 1′, and say, 4′ times 4 are 16′, and 1′ is 17′ = 1 ft. and 5′, which being set down, and the two products added, we obtain 18 ft. 9′ 4″ as the required product.

2.

	ft.	in.	
Mult.	8	6′	
by	4	3′	
	34	0′	
	2	1′	6″
Prod.	36	1′	6″

3.

	ft.	in.	
Mult.	9	10′	
	7	6′	
	68	10′	
	4	11′	0″
Prod.	73	9′	0″

4.

	ft.	in.
Mult.	3	8′
	7	6′
	27	6′

5.

	ft.	in	
Mult.	7	3′	
	4	7′	
	33	2′	9″

6.

ft.	in.	
3	11′	
9	5′	
36	10′	7″

7.

ft.	in.
4	6′
5	8′
25	6′

8.

ft.	in.	
9	7′	
3	6′	
33	6′	6″

9.				10.				
ft.	*in.*			*ft.*	*in.*			
6	4′	3″		3	9′	11″		
4	6′	4″		4	2′	3″		
28	9′	2″	11‴	16	0′	3″	3‴	9⁗

11. How many square feet does a board, 28 ft. 10′ 6″ long, and 3 ft. 2′ 4″ wide, contain? *Ans.* 92 ft. 2′ 10″ 6‴.

12. In a board, 16 ft. 9′ long, and 2 feet 3′ broad, how many square feet? *Ans.* 37 ft. 8′ 3″.

13. There is a wall 82 ft. 6 in. high, and 13 ft. 3 in. wide. How many square feet does it contain? *Ans.* 1093 ft. 1′ 6″.

14. There is a room, 20 feet square, and 7 ft. 6 in. high, to be plastered at 10 d., New York currency, per square yard. How many dollars will it cost? *Ans.* \$6.94 +.

15. There is a yard 58 ft. 6 in. in length, and 54 ft. 9 in. in breadth. How many dollars will it cost to pave it, at 5 d., New York currency, per square yard? *Ans.* \$18.53.

16. If a floor be 59 feet 9 inches long, and 24 feet 6 inches broad, how many square yards does it contain?
Ans. 162 yards, 5 ft. 10′ 6″.

NOTE 2. — If three dimensions, viz., length, breadth, and depth, be given, the solid content is found by multiplying them successively into each other.

17. There is a pile of wood 12 feet 6 inches long, 4 feet high, and 8 feet 6 inches wide. How many cords does it contain? *Ans.* 3 cords, 41 feet.

To reduce solid feet to cords, divide by 128, that being the number of solid feet in one cord. The required dimensions of the cord, are 8 feet long, 4 feet wide, and 4 feet high; since $8 \times 4 \times 4 = 128$.

18. How many solid feet are there in a block, 6 feet 8 inches in length, 4 feet 6 inches in height, and 3 feet 4 inches in width?
Ans 100 feet.

19. There is a certain pile of wood, measuring 24 feet in length, 16 feet 9 inches in depth, and 12 feet 6 inches in width. How many cords are there? and how many solid feet may be daily consumed, to have it last one year?
Ans. 39 cords, 33 feet; daily allowance, 13$\frac{7}{8}$ feet, nearly.

20. How many square feet are there in a board, which measures 16 feet 9 inches in length, and 2 feet 3 inches in breadth?
Ans. 37 feet, 8′ 3″.

QUESTIONS. — What are Duodecimals? How is the foot divided? What is each part called? How is the inch divided? What is each part called, &c.? When is the foot divided duodecimally? Repeat the table of denomi-

nations. How may duodecimals be added? What difficulty will be encountered in Multiplication of Duodecimals? Give an illustration of the difficulty. Of what denomination will the product of any two denominations be? What is the rule? What is Note 1? What is Note 2?

INVOLUTION.

§ **134.** A number is involved by being *multiplied into itself.*

The number thus multiplied by itself is called the *root.*

A power of any number is the product obtained by multiplying that number into itself. The particular power produced depends on *the number of successive multiplications*; the given number always being the first power, and also the root of the succeeding higher powers. The first multiplication then produces the second power; the second multiplication, the third power; the third multiplication, the fourth power, &c.; *the power obtained being always one in advance of the number of multiplications.*

Illustration: — $2 =$ the first power, and is also the root of the succeeding higher powers.

$2 \times 2 = 4$, the 2d power, or square of 2.
$2 \times 2 \times 2 = 8$, the third power, or cube of 2.
$2 \times 2 \times 2 \times 2 = 16$, the 4th power, or biquadrate of 2.
$2 \times 2 \times 2 \times 2 \times 2 = 32$, the 5th power of 2, &c.

The power to which a number is to be raised, is frequently expressed by a small figure, called *the index of the required power*, placed on the right of that number; thus 4^2 denotes the second power of $4 = 16$; and 4^3 denotes the third power of $4 = 64$; and 9^5 denotes the fifth power of $9 = 59049$, &c.

A fraction is involved by multiplying the numerator and the denominator each into itself the required number of times; thus the square of $\frac{2}{5}$ is $\frac{2}{5} \times \frac{2}{5} = \frac{4}{25}$. The square of $\frac{6}{7}$ is $\frac{36}{49}$; and the cube of the same is $\frac{216}{343}$, &c.

If the given quantity be a mixed number, it should first be reduced to an improper fraction, before being involved; thus the second power of $2\frac{1}{2}$ is $2\frac{1}{2} = \frac{5}{2}$, and $\frac{5}{2} \times \frac{5}{2} = \frac{25}{4} = 6\frac{1}{4}$. Or, proper and improper fractions may both be reduced to decimals, and then involved.

If any number be raised to two different powers, the power which is obtained by multiplying these two powers together, is expressed by adding their indices, thus: $2^2 \times 2^3 = 2^5 = 32$;

for $2^2 = 4$, and $2^3 = 8$, and $8 \times 4 = 32$, and $2 \times 2 \times 2 \times 2 \times 2 = 32$. Or, $3^3 \times 3^5 = 3^8 = 6561$; for $3^3 = 27$, and $3^5 = 243$, and $243 \times 27 = 6561$.

Again; any power of a given number is divided by another power of the same number, by subtracting the index of the divisor from the index of the dividend; thus: $2^5 \div 2^3 = 2^2$; for $2^5 = 32$, and $2^3 = 8$, and $32 \div 8 = 4$, the second power of 2. Or, $3^4 \div 3^2 = 3^2$; for $3^4 = 81$, and $3^2 = 9$, and $81 \div 9 = 9$, the second power of 3.

Ex. 1. What are the square, cube, and biquadrate of 3?
Ans. $3 \times 3 = 9$, the square; $3 \times 3 \times 3 = 27$, the cube; and $3 \times 3 \times 3 \times 3 = 81$, the biquadrate.

2. What are the square, cube, and biquadrate of 5?
Ans. 25, 125, and 625.

3. What are the cube and biquadrate of 12?
Ans. 1728 and 20736.

4. Multiply the second and third powers of 4 together. What is the product? and what power of 4 is it?
Ans. The product, 1024, or fifth power.

5. What power of 3 is obtained by multiplying its third power and its fourth power together? and what is the number?
Ans. 7th power, or 2187.

NOTE.—When the number to be raised to some given power consists of whole numbers and decimals, the number of decimals to be cut off in the required power is ascertained by multiplying the number of decimals in the given number by the index of the required power.

6. What is the square of 26.13? $26.13 \times 26.13 = 6827769$. Now, to determine how many decimals are to be cut off, we first notice that the number of decimals in the given number is 2, and also that the index of the required power is 2; therefore $2 \times 2 = 4$, the number of decimals to be cut off. Therefore, 682.7769 is the required power.

7. What is the cube of 25.4? *Ans.* 16387.064.

8. Divide 2^6 by 2^2, and what power of 2 will be obtained?
Ans. 8, the cube of 2.

§ 135. SHORT RULES FOR SQUARING ANY NUMBER NOT GREATER THAN 100.

Rule.—*Under the square of the* LEFT-*hand figure write* TWICE *the* PRODUCT *of the two figures, setting it one place to the right; also, under this, and one place still farther to the right, the* SQUARE *of the* RIGHT-*hand figure. The sum of the numbers thus written is the square of the given number.*

1. What is the square of 63?

6^2 36
Twice 6×3 . (one place to the right) . . . 36
3^2 (still one place farther to the right) 9

3969 = sqr. of 63.

2. What is the square of 78?

7^2 49
Twice 8×7 . (one place to the right) . 112
8^2 (one place farther to the right) 64

6084 = sqr. of 78.

In like manner square 49, 27, 36, 56, 72, 67, 89, 93, 55.

NOTE. — The above rule is equally applicable for squaring any number between 100 and 1000, if, to the square of the left-hand figure, twice the product of the left-hand figure into the *two* right-hand figures be taken, and the square of the *two* right-hand figures be written *two* places to the right.

TO SQUARE ANY NUMBER BETWEEN 50 AND 100.

Rule. — *Square the left-hand figure, and to its* SQUARE *add the right-hand figure, and on the right of this, write the* SQUARE *of the right-hand figure, preceded by a cipher, if less than* 10. *Then multiply the right-hand figure by the right-hand digit of* DOUBLE *the left-hand figure, and set the result one place to the* LEFT, *under the preceding number.* Their sum will be the square required.

What is the square of 86?

$8^2 + 6 = 70$ and $6^2 = 36$ $\Big\}$ which united, (see Rule,) $= 7036$
Twice the left-hand figure $= 16$; therefore, (see Rule,) $\Big\}$ $6 \times 6 = 36$

7396 = sqr. of 86.

In like manner square 58, 66, 72, 78, 83, 96, 59, 63, 72, 77, 88.

TO SQUARE ANY NUMBER ENDING IN 5.

Rule. — *Reject the* 5, *and to the remaining figures add their square; then, to the number thus produced, annex the square of* 5. The result will be the required square.

What is the square of 125?

$12 + 12^2 = 156$, and $5^2 = 25$. Therefore, 15625 is the square.

In like manner square 115, 85, 135, 165, 185, 25, 35, 45, 55, 105, 95, 155.

EVOLUTION.

§ **136.** Evolution is the reverse of Involution. In Involution we have the *root* given to find some required power; but in Evolution a *power* is given, and a *root* required.

The relation between roots and powers requires to be clearly understood.

A *root* of a number is obtained whenever that number is resolved into several equal factors; and a *power* of a number is obtained whenever that number, taken as a root, is multiplied into itself once or more. Thus, 2 is the cube root of 8, because 8 may be resolved into three 2's; or because 2 raised to its third power equals 8; for $2\times2\times2=8$. Also, 8 is the square root of 64, because the second power of 8 is 64, or $8\times8=64$. Again; 9 is the square of 3, and 3 is the square root of 9; 27 is the cube of 3, and 3 is the cube root of 27. Roots and powers are therefore correlative terms.

The exact root of some numbers cannot be obtained. Such numbers are called *irrational powers*, and their roots are called *surds*. Thus no root can be obtained, which, when multiplied into itself, will produce 2; 2 is therefore an irrational power, and its root is a surd. But a number whose root can be exactly extracted is a *perfect* or *complete power*, and its root is called a *rational number*. Thus 16 is a complete power, for 4 is its exact root; 4, therefore, is a rational number.

There are two methods of expressing roots. The first and more common method is, by using the character called the *radical sign*, written thus, $\sqrt{}$. This sign, without any accompanying index, always indicates the square root. If other roots are required, the same radical sign is used, with an index of the required root. Thus $\sqrt{9}$, is an expression for the square root; $\sqrt[3]{9}$, for the cube root; $\sqrt[4]{9}$, for the fourth root of 9, &c. $\sqrt{64}$, equals 8, because $8\times8=64$; and $\sqrt[3]{64}=4$, because $4\times4\times4=64$, or $\sqrt[6]{64}=2$, for $2\times2\times2\times2\times2\times2=64$, &c. Hence the root is to be taken as a factor in producing its corresponding power, as many times as there are units in the index of the required root.

The other mode of expressing roots is by means of fractional exponents. Thus, $6^{\frac{1}{2}}$ expresses the square root, $6^{\frac{1}{3}}$ the cube root, and $6^{\frac{1}{4}}$, the biquadrate or fourth root, of 6. The chief advantage of this mode arises from the fact, that not only *roots* of numbers may be expressed by it, but also *any required power of a given root.* The denominator of a fractional index always

denotes *a root* of the quantity to which it is applied, while the numerator expresses *some power of that root*; thus, $9^{\frac{3}{4}}$ implies that the fourth root of 9 is to be extracted, and that root raised to its third power. Again, $64^{\frac{5}{6}}$ implies that the sixth root of 64 is to be extracted, and the root then raised to its fifth power. But the sixth root of 64 is 2, and the fifth power of 2 is 32; therefore, $64^{\frac{5}{6}}=32$. Or a power higher than the given root may be expressed; thus, $16^{\frac{3}{2}}$ implies the third power of the square root of 16; but the square root of 16 is 4, and the third power of 4 is 64; therefore, $16^{\frac{3}{2}}=64$.

When several numbers are to be added, and the root of the sum obtained, they may be expressed thus: $\sqrt{65+16}$; which implies that the root of the sum of 65 and 16 is to be obtained. If the vinculum over the two numbers be rejected, the expression would imply, that 16 is to be added to the square root of 65. As the expression now stands, its value is $65+16=81$, and $\sqrt{81}=9$. Or the root of the difference of two quantities may be expressed in like manner, by placing the minus sign between them; thus, $\sqrt{90-26}$, the value of which is $90-26=64$, and $\sqrt{64}=8$. Without the vinculum over the two quantities, the expression would imply that 26 is to be taken from the square root of 90.

The root of the product of several numbers is equal to the product of their roots. As an illustration, take 9 and 16. Their product is 144, of which the square root is 12; that is, the root of their product is 12. The product of their roots is the same; for the square root of 9 is 3, and of 16, 4; and 4×3 is 12. The same is true of the cube roots, or of any roots whatever. Take the numbers 8 and 27. $\sqrt[3]{8}=2$, $\sqrt[3]{27}=3$, and $3\times2=6$, the product of their roots. Again; $27\times8=216$, and $\sqrt[3]{216}=6$, the root of their products.

EXTRACTION OF THE SQUARE ROOT.

§ **137.** The Square Root of any number is that number, which, being multiplied into itself once, will produce the number given.

The following table exhibits the square of all numbers from 1 to 12:—

Roots,	1	2	3	4	5	6	7	8	9	10	11	12
Squares,	1	4	9	16	25	36	49	64	81	100	121	144

Fig. 1.

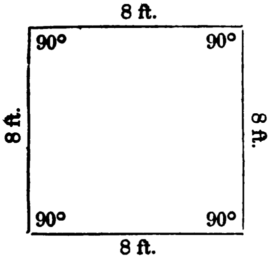


Fig. 2.

a	1	2	3	4	5	6	7	8	a
b	9	10	11	12	13	14	15	16	b
c	17	18	19	20	21	22	23	24	c
d	25	26	27	28	29	30	31	32	d
e	33	34	35	36	37	38	39	40	e
f	41	42	43	44	45	46	47	48	f
g	49	50	51	52	53	54	55	56	g
h	57	58	59	60	61	62	63	64	h

A square is a figure bounded by four equal sides, and has all its angles right angles, or angles of 90 degrees. This may be seen in Figure 1. Now, the area of a square, that is, the number of square feet, or rods, &c., it contains, is found by multiplying the length of any two sides together; or, since the sides are all equal, by multiplying the length of any one side into itself; or by squaring it. As each of the sides of the annexed figure is 8 feet in length, it is obvious they may each be divided into 8 equal parts, each of which will be 1 foot in length. Let each side be thus divided, and the points of division united, as seen in Figure 2. Now, since each side is 8 feet long, the divisions *a a*, *b b*, *c c*, *d d*, &c., must be just 1 foot each; the divisions made by the lines running at right angles to these, are also 1 foot each. The whole figure is, therefore, divided into 64 equal parts, each of which is just 1 foot square. But $8 \times 8 = 64$. Therefore, the area of a square is obtained by multiplying the length of the side by itself; or, in other words, by squaring it.

Now, to apply the above remarks to Evolution, suppose we have an area equal to what is given above, but placed in a different form. Suppose it to consist of a board 1 foot wide and 64 feet long; and that it is required to determine how large a square floor it will exactly cover. $\sqrt{64} = 8$ feet, is the length of the side of the required floor. Hence the area of a square is found by squaring one of its sides; and the length of its sides is found by extracting the square root of the given area.

The following is the rule for extracting the square root:—

Rule.—1. *Separate the given number into periods of two figures each, by placing a point or dot over the place of units, another over the place of hundreds, and another over the place of tens of thousands, &c.*

2. *Find by trial the greatest square root of the left-hand period, and place it for the first figure of the root, after the manner of the quotient in Division.*

3. *Subtract the square of this root from the left-hand period, and to the remainder bring down the next period of two figures for a dividend.*

4. *Double the root figure already obtained, for a divisor; then, omitting the right-hand figure of the dividend, divide as in Simple Division, and place the figure obtained, as the second figure of the root or quotient, and also on the right hand of the divisor.*

5. *Multiply the divisor, thus increased, by the figure last placed in the root, and place the product under the dividend, as in Division; then subtract, and to the remainder bring down the next period of two figures.*

6. *Double the root already found for a new divisor, with which divide as before. Continue the operation till the periods are all brought down: the number obtained will be the root required.*

NOTE 1.—If the number, the root of which is required, consist in part of a decimal, place the first point over the unit figure, as already directed, and point off both ways from that figure, allowing two figures to each point. If the decimal consist of an odd number of figures, annex one cipher to complete the last period.

2. If, after all the periods have been brought down, there be a remainder, periods of two ciphers each may be annexed, and the operation continued. The root figures obtained by thus annexing ciphers will be decimals, and must be so marked.

3. The number of dots employed in pointing off the given number, always determines the number of figures in the required root.

Ex. 1. What is the square root of 1296?

1296, pointed according to the rule, is $1\dot{2}9\dot{6}$. Hence we know that the root will consist of two figures. The next step is to determine the root of 12, the left-hand period. This is done by trial. If 4 be taken, it will be found too large, since $4 \times 4 = 16$. We will, therefore, take 3. $3 \times 3 = 9$; and since 9 is less than 12, it is not too large, and yet it is the greatest integral quantity, whose square is less than 12, and is, therefore, the root we want.

OPERATION CONTINUED.

$$\begin{array}{rl} & 1\dot{2}9\dot{6}(3 \\ \text{Root squared} = & \underline{9} \\ & 396 \text{ remainder, increased by the next period.} \end{array}$$

Now, since we are to have another figure in the root, the 3 already obtained is 3 tens, or 30, the square of which is $30 \times 30 = 900$; the 12 is also 1200. After subtracting the 9, therefore, there remains 3, or 300, and, the next period being brought down, we obtain the number 396.

The next step is to find a divisor, which, by the rule, is $3+3=6$; therefore,

$$\begin{array}{rl} & 1\dot{2}9\dot{6}(36 \\ & \underline{9} \\ 6) & 396 \end{array}$$

In dividing, the right-hand figure, viz., 6, is omitted.

```
Again, 12̇96̇(36
        9
      ----
  66) 396
      396
      ----
      000
```

The last root figure is placed on the right of the divisor, making it 66, and the whole is multiplied by the root figure, that is, $66 \times 6 = 396$. I then subtract, and nothing remains. Therefore, 36 is the root required.

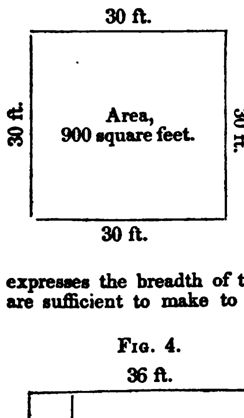
The operation is proved by squaring the root; thus, $36 \times 36 = 1296$.

If the given number had consisted of three or four periods, instead of two, the operation would have been continued by bringing down another period, and then doubling the root, 36, for a new divisor.

§ **138.** EXPLANATION. — We will suppose the 1296, in the preceding operation, to be so many feet of boards 1 foot in breadth; and that it is required to know how large a floor, exactly square, they will cover.

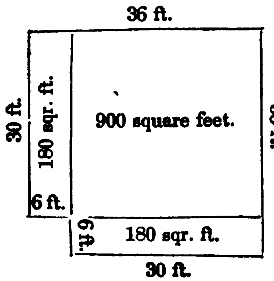
As has already been said, the 12 in the number 1296 is 1200, and the root, 3, is so many tens, or 30. This, therefore, is the length, in feet, of one side of a square floor, which 1200 feet of the boards will cover, and leave a remainder of 3, or 300.

FIG. 3.



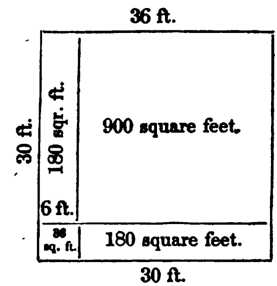
Now, to find the area of a square, (see Fig. 3,) we multiply the length of any two sides together, or square the length of one side. Therefore, $30 \times 30 = 900$. We have, then, disposed of 900 of 1296 feet given, and there remain 396 feet to be so added to this figure as to preserve its square form. This is done by making equal additions upon any two adjacent sides; and hence we see the obvious reason for doubling the root, (which is the length of one side of the square,) for a divisor. But the root, (Fig. 3,) being doubled, gives 6 for a divisor, and this is contained in the remaining figures, 396, 6 times, (the right-hand figure, viz., 6, being omitted, agreeably to the rule.) Now, this figure, 6, expresses the breadth of the addition which the remaining feet of boards are sufficient to make to the original square, as seen in Fig. 3. (For this addition, see Fig. 4.) This diagram is not a perfect square; a corner remains to be filled up. We will, however, before completing it, ascertain how many of 396 feet, that remained after the first square of 900 feet was completed, are here disposed of. The length of each addition being 30 feet, and the breadth 6 feet, the area is $30 \times 6 = 180$ sq. feet; and the area of both, consequently, is $180 + 180 = 360$. But $396 - 360 = 36$. Therefore, 36 feet still remain to be added. The scholar, by reference to the preceding diagram, will perceive that the corner, which yet remains to be filled, to complete the square, is just 6 feet from corner to cor-

FIG. 4.



ner; therefore, $6 \times 6 = 36$. This addition just disposes of the remaining feet of boards, and completes the square. (See Fig. 5.) The area of the original square, (see Fig. 3,) and also, of the several additions made (see Fig. 4 and 5,) disposes of the whole of the given quantity of board; for $900 + 180 + 180 + 36 = 1296$.

FIG. 5.



But why, in dividing, is the right-hand figure of the dividend omitted?

The scholar will remember that the points placed over any number, determine the number of figures in its root. (See Note 3.) Before performing the operation, we therefore know that the required root of the preceding number will consist of two figures. The 3, or left-hand figure of that root, is therefore 3 tens, or 30. This number, viz., 30, is consequently the true value of the root already found, which, if doubled, will give 60 as the divisor, and not 6, as in the sum. The true value of the divisor is therefore ten times as great as represented in the operation; and hence the dividend is divided by 10; that is, its right-hand figure is omitted, to make its value correspond with the apparent value of the divisor.

Another peculiar feature of the operation consists in placing the quotient, or root figure, on the right of the divisor, thereby multiplying it into itself. For this there must also be a reason. If the scholar will examine Fig. 4, he will notice a vacant corner, which, as the addition made to each of the two adjacent sides is 6 feet, must be just 6 feet square. By placing the root figure on the right of the divisor, and multiplying it into itself, this corner is filled up, and the square completed.

2. What is the square root of 9801?

OPERATION.

```
      9 8̇ 0 1̇ ( 9 9 root.
      8 1
      -------
1 8 9)1 7 0 1
      1 7 0 1
      -------
```

The 9 in the divisor is the last root figure, placed there by the rule.

3. What is the square root of 30138.696025?

OPERATION.

```
         3̇ 0 1̇ 3 8̇ . 6 9̇ 6 0̇ 2 5̇ ( 1 7 3 . 6 0 5
         1
         -----
   2 7 ) 2 0 1
         1 8 9
         -------
  3 4 3 ) 1 2 3 8
          1 0 2 9
          ---------
 3 4 6 6 ) 2 0 9 6 9
           2 0 7 9 6
           -------------
3 4 7 2 0 5 ) 1 7 3 6 0 2 5
              1 7 3 6 0 2 5
              -------------
```

NOTE 4.—It will be observed, from this example, that when a period is brought down, and the number obtained is not sufficient to contain the divisor, a cipher is to be placed in the root, and also on the right hand of the divisor, and the next period brought down. For arranging the points in this sum, see Note 1.

4. What is the square root of 5499025? *Ans.* 2345.
5. What is the square root of 1522756? *Ans.* 1234.
6. What is the square root of 207936? *Ans.* 456.
7. What is the square root of 5700.25? *Ans.* 75.5.
8. What is the square root of 74770609? *Ans.* 8647.
9. What is the square root of 4191125517321? *Ans.* 647389.
10. What is the square root of 10? *Ans.* 3.16227+.
11. What must be the length of each side of a square field, in rods, in order that the field may contain 40 acres?
Ans. 80 rods.

12. A certain regiment consists of 6561 men. How many must be placed in rank and file, to form them into a square?
Ans. 81 men.

13. A company of men spent 6£. 13s. 4d., which was just as many pence for each man as there were men in the company. How many men were there, and how many pence did each man spend?
Ans. There were 40 men, and each man spent 40d.

14. If a man were to plant 3969 hills of corn in a square, how many rows must he have, and how many hills in a row?
Ans. 63 of each.

§ **139.** NOTE 5.—To extract the square root of a vulgar fraction, *first reduce the fraction to its lowest term*, and then extract the root both of the numerator and denominator.

15. What is the square root of $\frac{72}{128}$?

$$\frac{72}{128}=\frac{9}{16}, \text{ and the square root of } 9 \text{ is } 3, \text{ and } \ldots\ldots 16 \text{ is } 4 = \frac{3}{4}, \textit{ Ans.}$$

16. What is the square root of $\frac{98}{128}$? *Ans.* $\frac{7}{8}$.
17. What is the square root of $\frac{112}{175}$? *Ans.* $\frac{4}{5}$.
18. There is an army containing 9216 men. How many men must be placed rank and file, to form a square which shall contain every man? *Ans.* 96.
19. What is the square root of $\frac{7056}{9216}$? *Ans.* $\frac{7}{8}$.

APPLICATION OF SQUARE ROOT TO PARALLELOGRAMS AND TRIANGLES.

§ **140.** A parallelogram is an oblong figure having its opposite sides equal and parallel.

FIG. 6.

10 ft.

2 ft.	1	2	3	4	5	6	7	8	9	10	2 ft.
	11	12	13	14	15	16	17	18	19	20	

10 ft

The adjoining figure represents the parallelogram. If we suppose this figure to be 10 feet long, and 2 feet broad, the area of the whole figure is evidently $10 \times 2 = 20$ square feet. *The area of a parallelogram is therefore found by multiplying its length into its breadth.* Had the length of the above figure been 18 feet, and its breadth as given above, the area would have been $18 \times 2 = 36$ square feet; and this equals a square figure, the sides of which are 6 feet long; for $\sqrt{36} = 6$.

A square, equal in area to a given parallelogram, is found by extracting the square root of the area of that parallelogram.

20. What is the length of the sides of a square, whose area shall be equal to the area of a parallelogram 32 feet in length, and 2 in breadth?

Operation: $32 \times 2 = 64$, and $\sqrt{64} = 8$ feet, the side of the required square.

21. There is a parallelogram, 27 feet in length and 3 in breadth. Required the sides of a square of equal dimensions.

Ans. 9 feet.

22. Required the dimensions of a square, the area of which shall be equal to the area of a parallelogram 18 feet in length and 8 in breadth. *Ans.* 12 feet square.

§ 141. NOTE 6. — When the area of a parallelogram is given, and also the ratio of its length and breadth, the sides may be found by *dividing the area by that ratio, and extracting the square root of the quotient.* The root obtained will be the required *breadth*, by which *divide the area*, and the quotient will be the length.

23. If the area of a parallelogram be 288 rods, and its length be twice as much as its breadth, what is its length, and what its breadth?

Operation: $288 \div 2 = 144$, and $\sqrt{144} = 12$, the required breadth; and $238 \div 12 = 24$, the required length. The reason of the above operation is obvious. The length being twice its breadth, if the parallelogram be divided in the middle, the two parts will be equal and square, and the square root of one of these parts will evidently be the breadth of the parallelogram. The same reasoning may be applied to parallelograms of any dimensions.

24. There is a field, containing 30 acres, lying in the form of a parallelogram, of which the length is three times the width. What is its length, and what is its breadth?

Ans. The length is 120 rods, and the breadth 40 rods.

§ 142. NOTE 7.—The side of a square of equal area with a geometrical figure of any form, may be found by extracting the square root of the area of that figure.

25. I have an irregular piece of land, containing $50\frac{5}{8}$ acres, which I am desirous to exchange for an equal number of acres lying in a square form. What must be the length of the sides of that square? *Ans.* 90 rods.

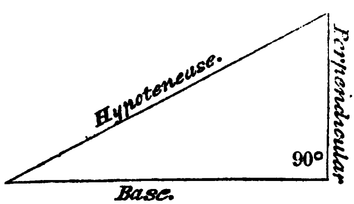
26. A certain triangular field contains 10 acres of land. What is the length of the sides of a square containing the same number of acres? *Ans.* 40 rods.

NOTE 8.—If the measure of the sides of an *oblique*-angled parallelogram, and also the measure of one of its diagonals, be given, the other diagonal may be found, by subtracting from the *sum* of the *squares* of all the sides of the parallelogram, the *square* of the given diagonal, and extracting the square root of the remainder.

27. If the length of an oblique-angled parallelogram be 30 feet, its breadth 20 feet, and its shorter diagonal 30 feet, what is the length of the longer diagonal? *Ans.* 41.23 ft.

The principle of the square root may be applied to find the length of the sides of a right-angled triangle, the measure of either two being given.

The square of the hypotenuse, or side opposite the right-angle, is always equal to the sum of the squares of the base and perpendicular. Therefore,



If the base and perpendicular be given, and the hypotenuse required, *square the given sides, and extract the square root of their sum.* If the hypotenuse and one of the legs of the triangle be given, to find the other leg, *square the hypotenuse, and from its square subtract the square of the given side. The square root of the remainder will be the length of the required side.*

The following are among the curious facts, which relate to the properties of the right-angled triangle: —

1. If the measure of the two legs be each expressed by an odd number, the hypotenuse cannot be expressed by a whole number.

2. If the measure of the hypotenuse be expressed by a whole number, the number expressing the measure of *one* of the legs, must be divisible by 4 without remainder.

3. If the legs of a triangle be equal in length, or if one of them be any multiple of the other by a whole number, the hypotenuse cannot be a whole number.

28. There is a wall 15 feet high, and in front of it is a pavement 24 feet wide. How long a ladder is required to reach from the outside of the pavement to the top of the wall? The hypotenuse is required; therefore, $15 \times 15 = 225$; and $24 \times 24 = 576$; then, $225 + 576 = 801$; and $\sqrt{801} = 28.3+$. *Ans.*

29. A certain tree is broken off 8 feet from the ground, and, resting on the stump, touches the ground at the distance of 12 feet. What is the length of the part broken off?

Ans. 14.42 + feet.

30. There is a fort standing by the side of a river, 24 yards high; and a line 36 yards long will just reach from the top of the fort to the opposite side of the river. What is the width of the river? *Ans.* 26.832 yards.

31. Two ships sail from the same port, one due east, and the other due north. What is the distance between them, when one has sailed 100 miles, and the other 168 miles?

Ans. 195.5 + miles.

32. A man shot a bird sitting on the top of a steeple 80 feet high, while standing at the distance of 60 feet from its base. How far did he shoot? *Ans.* 100 feet.

33. A rope 100 feet long, attached to the top of a steeple, touches the ground when drawn perfectly straight, 20 feet from its base. How high is the steeple? *Ans.* 98 feet, nearly.

34. Two boys were playing with a kite, the line of which was 520 feet in length. When the string was all out, one of them standing directly under the kite, and the other holding the string, the distance between them was 312 feet. What was the perpendicular height of the kite? *Ans.* 416 feet.

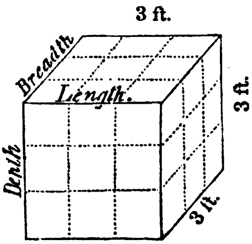
QUESTIONS.— When is a number involved? What is a root? What is a power of any number? On what does the particular power produced depend? How does the power obtained compare with the number of multiplications in producing it? How is a required power expressed? What is the figure denoting the power called? How is a fraction involved? If the given quantity be a mixed number, what must be done? If a number be raised to two different powers, how is the power obtained by multiplying these two powers together, expressed? How is any power of a given number divided by another power of the same number? When the number to be raised to a power is in part a decimal, how is the number of decimals to be cut off from the required power ascertained? What is Evolution? What is a root of a number? What is a power of any number? What are irrational powers? What are surds? How many methods are there of expressing roots? What is the first method? What root is expressed by the radical sign without any index or exponent? If any other root is to be expressed, how is it done? How many times is the root to be taken as a factor in producing its corresponding power? What is the other mode of expressing roots? What is an advantage of this mode? When a fractional index is used, what does the denominator denote? What does the numerator? When several numbers are to be added, and the root of the sum extracted, how is the operation expressed? What does the root of the product of several numbers equal? Give an illustration. What is the square root of any number? What is a square? How is the area of a square found? Give the illustration. How is the length of the sides of a square found? What is the rule for extracting the square root? What is Note 1? Note 2? Note 3? Why do we take twice the root for a divisor? Why, in dividing, do we omit the right-hand figure of the dividend? Why do we place the quotient figure on the right of the divisor? How is the square root of a vulgar fraction extracted? The vulgar fraction may first be reduced to a decimal, and the root of the decimal extracted, if preferred. What is a parallelogram? How is its

area found? How may a square equal in area to a given parallelogram be found? What is Note 5? Note 6? To what is the square of the hypotenuse of a right-angled triangle equal? If the base and perpendicular be given, how may the hypotenuse be found? If the hypotenuse and one of the legs of a triangle be given, how may the other leg be found? The base and perpendicular are called the *legs* of a triangle. How are the operations in Square Root proved? *Ans.* By multiplying the root into itself.

EXTRACTION OF THE CUBE ROOT.

§ **143.** A CUBE *is bounded by six equal, plane surfaces, each of which is a square; that is, the length, breadth, and depth of a cube are equal.* (See Fig. 1.) The area of each of the six equal surfaces is found by squaring the measure of its side, as has already been explained in Square Root.

FIG. 1.



If the adjoining figure represent a cubic block measuring three feet in length, breadth, and thickness, the superficial area of each face is $3 \times 3 = 9$ square feet. Now, if the divisions marked by the dotted lines on each face of the block, were extended through it, in either direction, the whole would be divided into 9 parts, each 1 foot square and 3 feet long, and susceptible of being divided each into 3, and consequently the whole into 27 blocks, each 1 cubic foot.

We have, then, the following general principle:— *The content of a solid or cubic body is found by multiplying its length, breadth, and thickness into each other;* or, what is the same in effect, *by cubing one of these dimensions.*

Extracting the cube root, is a process the reverse of the preceding; that is, *it is finding the length of one of the sides of a cubic body, the solid content of that body being given; or it is finding, from a given number, another number, whose cube or third power shall equal that number.*

Rule.—1. *Separate the given number into periods of three figures each, by placing a point first over the unit figure, and advancing toward the left when the number consists of integers only; but to the right and left both, when it consists of integers and decimals, and make the last period of the decimal complete, by annexing ciphers, whenever necessary.*

2. *Find by trial the greatest cube root of the left-hand pe-*

riod, and place it as in square root; then subtract its cube from the same period, and bring down the next period of three figures to the remainder, for a dividend.

3. *Square the root figure and multiply its square by 3 for a divisor, and see how many times it is contained in the dividend, omitting the first two right-hand figures, and place the result as the second figure in the root.*

4. *Multiply the divisor by the last quotient figure, and, placing two ciphers on the right of the product, place the result under the dividend. Also, multiply the square of this same last quotient figure by the former figure or figures of the root, and also by 3; and, placing one cipher on the right of the product, write it under the preceding product. Lastly, under this, write the cube of the last quotient or root figure, and make the sum of these three numbers a subtrahend.*

5. *Subtract the subtrahend from the dividend, and to the remainder bring down the next period for a new dividend.*

6. *To obtain a new divisor, proceed as before; and thus continue the operation, till all the periods of the given number have been brought down.*

NOTE 1.—In obtaining each divisor, *square the whole root obtained*, and multiply that square by 3.

Ex. 1. What is the cube root of 10648?

OPERATION.

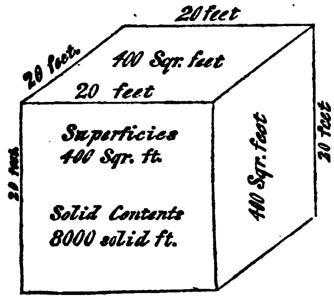
$$\begin{array}{lrr}
 & & 1\dot{0}64\dot{8}\,(22 \\
 & 2^3 = & 8 \\
\text{Article 3, rule, } 2^2\times 3=12 \text{ (Div.)} & 12) & \overline{2648} \quad \text{(See art. 2.)} \\
\text{Rule, article 4,}\left\{\begin{array}{r} 12\times 2+00= \\ 2^2\times 2\times 3+0= \\ 2^3= \end{array}\right. & & \begin{array}{r} 2400 \\ 240 \\ 8 \end{array} \\
 & \text{Subtrahend,} & \overline{2648} \\
 & & \overline{0000}
\end{array}$$

In dividing, the two right-hand figures of the dividend are omitted. (See rule, art. 3.) Proof, $22^3 = 10648$.

§ **144.** EXPLANATION.—We will suppose the number 10648, in the preceding sum, to be so many feet of timber, one foot square, and that it is required to find how large a cubic pile they will form; that is, what will be the length, breadth, and depth of a cubic pile containing that number of solid feet. To make each step as clear as possible, we will repeat, in part, the preceding operation.

The number given, when pointed, (see rule,) is divided into two periods, viz., 10 and 648. We therefore know that the required root will consist of two figures, and by trial we find the root of the first or left-hand period to be 2, that is, 2 tens, or 20. Hence, $1\dot{0}64\dot{8}$(20; the same as is

FIG. 2.



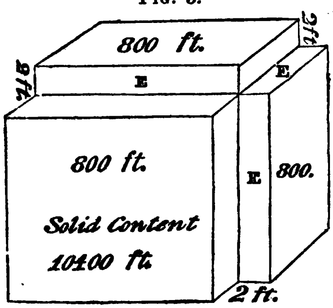
seen in the above operation, excepting that a cipher is placed on the right of the root figure, 2, to give it its true value. This gives the linear measure of a cubic block, which the 10 (10.000) of the given number will make. Now, since 20 is the linear measure of the *cube*, (that is, the direct and not diagonal measure from corner to corner,) it is also the linear measure of *each of the six equal square faces of that cube.* Therefore, $20 \times 20 = 400$, the area of each face; and $400 \times 20 = 8000$, the number of cubic feet required to make a cubic body, whose linear measure is 20 ft. (See Fig. 2.) By this step we have, then, disposed of 8000 of the 10648 solid feet. Hence,

$$
\begin{array}{l}
1\dot{0}64\dot{8}\,(20 \\
\underline{8000} \\
2648
\end{array}
$$

(This same effect is obviously produced in the first solution of this sum, by cubing the 2, and subtracting it from the left-hand period, 10, and then bringing down the next period, 648, to the remainder.) There now remains 2648 feet to be so added to the block already formed, that the whole shall be a perfect cube. This is done by making equal additions on any three of the equal faces *which lie contiguous to each other.* The reason of this is obvious. A solid body has length, breadth, and thickness; and in a cubic body, these dimensions are all equal, and by making the additions as here directed, *they are equally increased.*

The scholar will now understand why three times the square of the root obtained is taken as a divisor. The *square of the root* is the superficial area of one of the sides or faces of the cubic block, and this multiplied by 3 gives the area of three faces or sides, which is the number of sides to which equal additions are to be made. Hence, dividing the quantity to be added to the cube now obtained, by this area, determines the thickness of the addition. This, in the sum now under consideration, is 2 feet, as seen at Fig. 3. But these additions are evidently limited in size to the original block; consequently, the corners E, E, E, (Fig. 3,) remain to be filled before a perfect cube is produced.

FIG. 3.



Now, to determine the quantity here added:—Each face of the original cube contains 400 square feet, (see Fig. 2,) and this multiplied by 2, the depth of the addition, gives 800 solid feet as the content of the addition made to each face; the whole addition, therefore, is $800 \times 3 = 2400$ solid feet, and $2648 - 2400 = 248$ solid feet, the quantity yet remaining to be disposed of.

OPERATION CONTINUED.

$$\begin{array}{lr} & 1\dot{0}64\dot{8}(22 \\ & 8 \\ 2^2 \times 3 = 12 \text{ div.} \ldots\ldots 12) & \underline{2648} \\ \text{Solid con. of 3 additions, } 12 \times 2 + 00, \text{ (rule,)} & 2400 \end{array}$$

The reason for placing two ciphers on the right of the divisor multiplied by the root figure, is obvious. By referring to a previous statement of this sum, it will be seen that the root figure, 2, which, when squared and multiplied by 3, forms the divisor, is 2 tens, or 20. Hence, $20 \times 20 = 400$, and $400 \times 3 = 1200$, which would be the divisor, were the full value of the root figure expressed. This deficiency in the divisor is made up *by omitting the two right-hand figures of the dividend in dividing, and by placing two ciphers on the right of the product of the divisor multiplied by the root figure.*

Our next step will be to fill the vacant corners as seen at E, E, E, (Fig. 3.)

The rule, after directing the preceding step, says—"*Also multiply the square of this same last quotient figure by the former quotient figure or figures, and also by 3, and, placing one cipher on the right of the product, write it under the preceding product.*" This operation fills the corners here alluded to. For, since the last quotient figure, 2, is the thickness of the addition made, its square, viz., 4, is the measure of the corner to be filled, and the preceding quotient figure is the length of each side; hence, $4 \times 2 + 0 = 80$, the solid content of one corner; (see Fig. 4.) The cipher here is added, because 2, the preceding quotient figure, is 2 tens, or 20. The three corners, therefore, require $80 \times 3 = 240$ feet, to fill them. The corner C still remains to be filled, and 8 feet of timber also remain, for $10648 - 10640 = 8$. This vacant corner, C, measures exactly 2 feet in each of its three dimensions; hence $2^3 = 8$, is the solidity of that corner; and this exactly disposes of the remaining timber. The cube completed is seen at Fig. 5. The contents of the different parts of the completed block are, Fig. 2, 8000+; Fig. 3, 2400+; Fig. 4, 240+; Fig. 5, $8 = 10648$ feet.

FIG. 4.

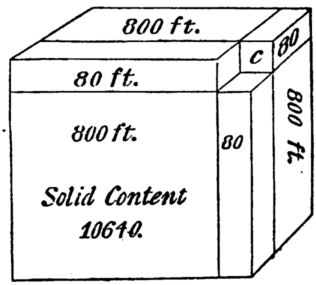
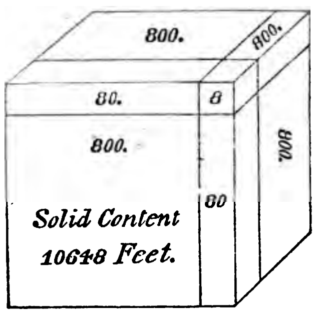


FIG. 5.



§ **145.** NOTE 2.—In Square Root, we were directed to point off the given number into periods of two figures each; and in Cube Root, the direction is, to allow three figures to each period. The following is the reason:—The square of any number always consists of *twice as many fig-*

ures as the number itself, or one less than twice as many. The cube of any number always consists of *three times as many figures as the number itself, or one or two less than three times as many.* That is, in Square Root, the left-hand period may consist of *one or two figures*; and in Cube Root, the same period may consist of *one, two, or three figures.*

ILLUSTRATION.—$13^2=169$, one less than twice the number of figures squared; and to extract its root it would be thus pointed, $\dot{1}6\dot{9}$. $46^2=2\dot{1}1\dot{6}$, twice the number of figures squared. $13^3=\dot{2}19\dot{7}$, two less than three times the figures cubed, and the left-hand period consists of one figure only. $25^3=1\dot{5}62\dot{5}$, one less than three times the figures cubed, and the left-hand period consists of two figures. $99^3=97\dot{0}29\dot{9}$, three times the number of figures cubed, and the left-hand period consists of three figures.

In the following solution, the scholar will carefully compare each step of the operation with the rule. The first thing to be done, is to form the periods. (Art. 1, Rule.) The first figure of the root is then to be determined, its cube subtracted, and to the remainder, the next period of three figures to be brought down. (Art. 2, Rule.) He must then proceed to obtain a divisor, as directed by Art. 3; and, lastly, to determine the solidity of the several additions made, and to make the result a subtrahend (Art. 4.)

Ex. 2. What is the cube root of 12812904?

OPERATION.

$$1\dot{2}81\dot{2}90\dot{4}\,(234$$

2^3	.....	8
$2^2\times3=12$ (Divisor)	..12)	4812
$12\times3+00$	....	3600
$3^2\times2\times3+0=540$	....	540
3^3	....	27
		4167 = Subtrahend.
$23^2\times3=1587$ (Divisor)	1587)	645904 = New dividend.
$1587\times4+00$	....	634800
$4^2\times23\times3+0$	....	11040
4^3	....	64
		645904 = Subtrahend.
		000000

Proof, $234^3=12812904$.

It is obvious, from the first division by 12, that the divisor is not contained in the dividend, in all instances, as many times as it would be in simple division.

In dividing, it must be remembered to omit the first two figures on the right hand of the dividend.

3. What is the cube root of 250047? *Ans.* 63. Proof, $63^3=250047$.

4. What is the cube root of 970299? *Ans.* 99. Proof as before.

5. What is the cube root of 1.953125? *Ans.* 1.25.
6. What is the cube root of 22069810125? *Ans.* 2805.
7. What is the cube root of 183250432? *Ans.* 568.
8. What is the cube root of 84.027672? *Ans.* 4.38.
9. What is the cube root of 6859? *Ans.* 19.
10. What is the cube root of 205379? *Ans.* 59.
11. What is the cube root of 432081216? *Ans.* 756.

12. There is a cubic rock containing 8000 solid feet. What is the distance from corner to corner? *Ans.* 20 feet.

13. What is the difference between half of a solid foot, and a solid half foot? *Ans.* 3 solid half feet.

14. What is the superficial area of one of the faces of a cubic block containing 4096 solid feet? *Ans.* 256 square feet.

15. What is the side of a cubical mound, equal to one, 144 feet long, 108 feet broad, and 24 feet deep? *Ans.* 72 feet.

Multiply together the several dimensions of the given mound, and extract the cube root of their product.

NOTE 3.—*All solid bodies are to each other as the cubes of their similar sides or diameters.*

16. If a ball, weighing 8 lb., be 6 inches in diameter, what will be the diameter of another ball, of the same material, weighing 64 lb.?

$$8 : 64 :: 6^3 : 1728 \quad \sqrt[3]{1728} = \textit{Ans. } 12 \text{ inches.}$$

NOTE 4.—Instead of cubing the linear measure of each of the similar bodies, the operation may be performed by *canceling*, as illustrated in Proportion, if each term of linear measure be repeated three times; that is, if it be written down as many times as it is taken as a factor, in producing the required power.

The preceding sum would be stated as follows: $\frac{6.\ 6.\ 6.\ 64}{8} = 1728.$

17. If a ball, 6 inches in diameter, weigh 8 lb., what is the weight of another ball, of the same kind, measuring 12 inches in diameter? *Ans.* 64 lb. $6^3 : 12^3 :: 8 : 64$.

18. What would be the value of a globe of silver, one foot in diameter, if a globe of the same, one inch in diameter, be worth \$6? *Ans.* \$10368.

19. If a globe of silver, one inch in diameter, be worth \$6, what is the diameter of another globe of the same metal, worth \$10368? *Ans* 12 inches.

20. How many globes, one foot in diameter, would be required to make one globe, 27 feet in diameter? *Ans.* 19683.

21. Suppose the diameter of the sun to be 110 times as large as that of the earth; how many bodies like the earth would be required to make one as large as the sun? *Ans.* 1331000.

22. If a man dig a square cellar, that will measure 5 feet each way, in one day, how long would it take him to dig one, measuring 15 feet each way? *Ans.* 27 days.

QUESTIONS.—What is a cube? How is the area of each face of a cubic body found? How is the content of a cubic body found? What is the extraction of the cube root? How is the number whose root is to be extracted, to be pointed? Of which period is the root first found? What is then done with this root? How many figures are to be brought down to what remains? How is a divisor found? By what do you multiply the divisor? How many ciphers do you place on the right of the product? Where do you place the product? What further is done with the quotient or root figure? What does the sum of all these products form? What is the fifth step of the rule? How is a second divisor obtained? What is Note 1? Can we know of how many figures the root will consist? Of what is the root figure the linear measure? How much of the whole timber is disposed of by subtracting the cube of the quotient figure from the left-hand period, in the operation taken for explanation? How many feet remain to be added? To how many sides of a cube must equal additions be made, to preserve its cubic form? Why is three times the square of the root taken for a divisor? What is determined by dividing? How many solid feet are disposed of by the first addition made to the three faces of the cube? Explain how the solid content of the addition to each face is obtained. Why, in multiplying the divisor by the root figure, are two ciphers placed on the right of the product? How is the deficiency of the divisor made up in dividing? What operation, as directed by the rule, fills up the corners left vacant? Why is the square of the quotient figure multiplied by the previous root figures, and a cipher placed on the right hand of the product? How much of the remaining timber is required to fill the three vacant corners? What is the measure of the corner left vacant, after the preceding additions were made? How is it filled? Why, in Square Root, do we divide the given number into periods of two figures each? Why is the given number divided into periods of three figures each, in Cube Root? Of how many figures does the square of any number consist? Of how many does the cube? How are the roots proved? *Ans.* By raising the root to a power of the same name as the root itself.

ARITHMETICAL PROGRESSION.

§ **146.** Any series of numbers, more than two, increasing or decreasing by a constant and uniform difference, is called *Arithmetical Progression*, or *Arithmetical Series.*

The series formed by a continual addition of any number, (called the common difference,) is called the *ascending series.* Thus, 3, 5, 7, 9, 11, 13, &c., is an ascending series, of which the common difference is 2. The reverse of this forms the *descending series*; that is, a series decreasing by a continual subtraction of the common difference. Thus, 13, 11, 9, 7, 5, 3, &c., is a descending series.

The numbers constituting the series are called *terms.* The *first* and *last* term of the series are called *extremes*; all the intervening terms are called the *means*, and the number constantly added or subtracted, is called the *common difference.* When the first term and common difference are given, the series may be indefinitely extended.

In every arithmetical progression, *five particulars are to be noticed;* of which, if any *three* be given, the remaining two may be found. The five particulars are, *the first term, the last term, the common difference, the number of terms, and the sum of all the terms.*

CASE I.

§ **147.** The First Term, the Number of Terms, and the Last Term given, to find the Common Difference.

Ex. 1. The first term of an arithmetical series is 2; the number of terms, 13; and the last term, 38. What is the common difference?

The series commences with 2, as the first term; therefore, $38-2=36$, is the amount of all the additions made to this number. But, the whole number of terms being 13, and one of these being given, 12 terms must be formed by adding the common difference. Therefore, $36 \div 12 = 3$, the common difference required. The whole series, therefore, is 2, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35, 38; thirteen in number.

We have, then, the following rule for solving sums like the preceding: —

Rule. — *Divide the difference of the extremes by the number of terms, less one. The quotient will be the common difference.*

2. A man, in feeble health, commenced a journey, and traveled 9 days. On the first day, he traveled only 3 miles, but afterwards continued to gain each day an equal number of miles on the journey of the preceding day, till the last day, on which he traveled 43 miles. The daily increase is required.

Ans. 5 miles per day; that is, $43-3 \div 8 = 5$.

3. I owe a debt, which, by agreement, I am to pay at 17 different periods. The first payment is to be \$20, and the last, \$100. Required the common difference of the several payments. *Ans.* \$5.

4. A man had 10 sons, whose ages differed alike; the youngest of whom was 2 years old, and the oldest, 29. What was the difference of their ages? *Ans.* 3 years.

CASE II.

§ **148.** The First Term, the Common Difference, and the Last Term given, to find the Number of Terms.

Ex. 1. If the extremes be 3 and 51, and the common difference 6, what is the number of terms?

T*

$51-3=48$, the amount of all the additions made to the first term, 3; and, since each addition is 6, $48 \div 6=8$, the number of additions; that is, the number of terms formed by adding the common difference to the first term; therefore $8+1=9$, the whole number of terms, or answer required.

From the above we derive the following rule: —

Rule. — *Divide the difference of the extremes by the common difference, and add one to the quotient.*

2. A man commenced a journey, and traveled the first day only 4 miles; after which he gained each day 6 miles on the journey of the preceding day, and on the last day he traveled 94 miles. How many days did he travel? *Ans.* 16.

Operation: $94-4=90$, and $90 \div 6=15$, and $15+1=16$.

3. A man commenced a journey in great haste, and traveled 63 miles the first day; but, being unable to continue at the same rate, the second day he traveled only 59 miles; and thence continued to lose 4 miles per day, till the last day of his journey, on which he traveled 11 miles. How many days did he travel? *Ans.* 14 days.

4. A man set out on a journey for the improvement of his health. The first day, he traveled 10 miles, and the last, 65 miles, making each day an advance of 5 miles on the journey of the preceding day. How many days did he travel? *Ans.* 12 days.

CASE III.

§ 149. The First Term, the Last Term, and the Number of Terms given, to find the Sum of all the Terms.

Ex. 1. Bought 30 yards of cloth, paying 20 cents for the first yard and 50 cents for the last. What was the whole cost, allowing each succeeding yard to increase in price by a constant access?

The price of each succeeding yard increases by a constant access; therefore the price of the last is as much above the average price, as the price of the first is below it; hence, one half of the price of the first and last yard is the average price. Therefore, $20+50=70$, and $70 \div 2=35$ cents, average price; hence, $30 \times 35=\$10.50$, whole cost.

We have the following rule: —

Rule. — *Multiply half the sum of the extremes by the number of terms; the product will be the answer.*

2. Paid 4 cents for the first, and \$1.21 for the last, yard of a piece of cloth containing 86 yards. What was the whole cost? *Ans.* \$53.75.

3. How many strokes does a regular clock strike in 24 hours? *Ans.* 156.

4. If a person walk 3 miles the first, and 91 miles the last, day of his journey, how far will he have walked, allowing him to have been on his journey 24 days? *Ans.* 1128 miles.

QUESTIONS.—What is Arithmetical Progression? What is the ascending series? What is the descending series? What is denoted by the word *terms*? What terms are the extremes? What are the means? What is the number constantly added or subtracted called? How many particulars require to be noticed? What are they? What is Case I.? What is the rule? What is Case II.? What is the rule? What is Case III.? What is the rule?

GEOMETRICAL PROGRESSION.

§ **150.** Any series of numbers, increasing by a common multiplier, or decreasing by a common divisor, is called *Geometrical Progression*, or *Geometrical Series.* The common multiplier of the *ascending series*, and the common divisor of the *descending series*, is called the *ratio* of the series, or the *common ratio.* Thus, if we take 3 as a first term, and multiply it continually by 2, as a common ratio, we obtain the series 3, 6, 12, 24, 48, 96, 192, 384, 768, &c., in which series each term is obtained by multiplying the preceding term by 2. We have here an ascending series. Or the series may be 768, 384, 192, 96, 48, 24, 12, 6, 3, &c., in which each term is obtained by dividing the preceding term by 2. This forms a descending series. The several numbers thus produced constitute the *terms* of the series; the *first* and *last* of which are called the *extremes;* and the intervening ones are called the *means.*

In geometrical, as in arithmetical progression, there are *five things* to be considered; of which, if any three be given, the other two may be found. The five things are, *the first term, the last term, the number of terms, the sum of all the terms, and the ratio.*

CASE I.

§ 151. The First Term, the Ratio, and the Number of Terms given, to find the Last Term.

Rule.—*Raise the ratio to a power whose index is one less than the number of terms, and multiply this power by the first term. The product will be the number required.*

Ex. 1. The first term of a geometrical series is 2, and the ratio 3. What is the 12th term?

Since the given term, 2, is the first of the required series of 12 terms, and since each succeeding term is found by multiplying the preceding one by 3, 3 is evidently to be taken, as multiplier or factor, eleven times; that is, any term of a series is equal to the first term multiplied by the ratio raised to a power *one less than the number of terms.* Thus, $2 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 3^{11} \times 2 = 354294$, *Ans.*, or twelfth term.

2. A person purchased a house having 8 doors, and agreed to pay for the whole, whatever value might be attached to the eighth door, by allowing $4 for the first, $16 for the second, and $64 for the third door, &c. What did his house cost him?

Ans. $4^7 \times 4 = \$65536$.

NOTE 1.—It is obvious, from what was said of Involution, that, if the *ratio be raised to two or three different powers, whose indices, when added together, equal the index of the required power, the product of these several powers will be the power or number required.* In the above example, the answer is obtained thus: $4^3 \times 4^4 \times 4 = \$65536$, the answer as before.

3. A boy purchased 12 oranges, and agreed to pay 1 cent for the first, 4 cents for the second, 16 cents for the third, &c. What was the value of the twelfth orange? *Ans.* $41943.04.

4. A man, wishing to get his horse shod, agreed to allow 3 cents for the first nail, 9 cents for the second, and 27 cents for the third, &c.; and to pay for the whole the value of the last nail, the number of nails being 32. What was the cost of shoeing his horse? *Ans.* $18530201888518.41.

5. A person sold 20 yards of cloth as follows:—For the first yard he received 3 d.; for the second, 9 d.; for the third, 27 d., &c. What was the cost of the twentieth yard?

Ans. 14528268 £. 6 s. 9 d.

6. A man purchased 12 horses. For the first horse he gave only 4 cents; for the second, he gave 16 cents; for the third, 64 cents, &c.; and thus in a quadruple ratio to the last. What did the twelfth horse cost him? *Ans.* $167772.16.

CASE II.

§ 152. THE EXTREMES AND RATIO BEING GIVEN, TO FIND THE SUM OF ALL THE TERMS.

Rule.—*Divide the difference of the extremes by the ratio, less 1, and the quotient, increased by the greater extreme, will be the sum of the series.*

Ex. 1. The extremes of a geometrical series are 2 and 1458. What is the sum of all the terms, the ratio being 3?

$1458-2=1456$, the difference of the extremes; and $3-1=2$, the ratio, less 1. Therefore, $1456\div2=728$; and $728+1458=2186$, the sum of all the terms, *Ans.*

2. A farmer has sheep in 6 different pastures. In the first pasture there are 3, and in the sixth, 729. If the ratio of increase be 3, how many sheep has he in all his pastures?

Ans. 1092.

3. There is a cherry-tree with 10 branches. On the first branch there are only 2 cherries; on the tenth branch, there are 524288. Now, the ratio of increase from one branch to another being 4, how many cherries are there on the tree?

Ans. 699050.

CASE III.

§ **153.** The First Term, the Ratio, and the Number of Terms given, to find the Sum of the Series.

Rule. — *Find the last term by Case I., and the sum of the series, or of all the terms, by Case II.*

Ex. 1. A gentleman sold 20 yards of cloth, receiving for the first yard, 3 d.; for the second yard, 9 d.; and for the third yard, 27 d. How much did he receive for the whole, at that rate?

Case I., $3^{19}\times3=3486784401$, the last term. Case II., $\overline{3486784401-3}\div2=1743392199+3486784401=5230176600$, the sum of all the terms in pence $=21792402$ £. 10 s.

2. What would 12 horses cost, if 4 cents were allowed for the first, 16 cents for the second, and 64 cents for the third horse, &c., the value thus increasing in a quadruple ratio to the last or twelfth horse? *Ans.* $223696.20.

3. A gentleman gave his daughter, on the day of her marriage, one dollar, promising to triple it on the first day of each month in the year. What was the amount of her portion?

Ans. $265720.

CASE IV.

§ **154.** The Extremes and Number of Terms given, to find the Ratio.

Rule. — *Divide the greater extreme by the less, and the quotient will be that power of the ratio which is equal to the number of terms, less 1. The corresponding root will, therefore, be the ratio.*

Ex. 1. The first term of a geometrical series is 2, and the

last term, 354294, and the number of terms 12. What is the ratio?

$354294 \div 2 = 177147$, and the eleventh root of this number is the ratio required; therefore, $\sqrt[11]{177147} = 3$, the ratio.

2. The first term of a certain series is 4, and the last, 65536, and the number of terms, 8. What was the ratio? *Ans.* 4.

QUESTIONS.—What is Geometrical Progression? What is the ratio of the series? What are the terms of the series? What terms of a series are called *extremes*? And what are called *means*? In geometrical progression, how many things are to be considered? How many of these must be given, to find the others? What are the five things given? What is Case I.? What is the rule for Case I.? What is Note 1? What is Case II.? What is the rule? What is Case III.? What is the rule? What is Case IV.? What is the rule?

ALLIGATION.

§ **155.** Alligation is the method of mixing several simples, of different qualities, so as to obtain a compound of a mean or middle quality.

CASE I.

When the Quantities and Prices of several Simples are given, to find the Mean Price of the Mixture.

Rule.—*Find the total value of the several kinds to be mixed, and divide the amount of this value by the whole number of articles.*

Ex. 1. A farmer mixed together 8 bushels of rye, worth $0.50 per bushel; 12 bushels of corn, worth $0.65 per bushel; and 6 bushels of oats, worth $0.30. What was the value of one bushel of the mixture?

8 bushels of rye, at 50 cts. = $4.00; 12 bushels of corn, at 65 cts. = $7.80; and 6 bushels of oats, at 30 cts. = $1.80. And $8 + 12 + 6 = 26$ bushels; and $4.00 + $7.80 + $1.80 = $13.60; and $13.30 ÷ 26 bushels = $0.523 +, price of one bushel of the mixture.

2. A grocer mixed 6 lb. of tea, at $1.20 per lb.; 12 lb., at $1.60; and 8 lb., at $1.80. What was the value of 1 lb. of the mixture? *Ans.* $1.569.

3. If 15 bushels of wheat, worth $1.40 per bushel, be mixed with 12 bushels of rye, at $0.60 per bushel, and 10 bushels of

oats, at $0.35, what is the value of one bushel of the mixture?

Ans $0.856+.

4. If 6 lb. of gold, 20 carats fine, be mixed with 12 lb. at 18 carats fine, what is the fineness of the mixture?

Ans. $18\frac{2}{3}$ carats fine.

5. If 6 gallons of wine, at $0.67 per gallon; 7 gallons, at $0.80 per gallon; and 5 gallons, at $1.20 per gallon, be mixed together, what will be the value of one gallon of the mixture?

Ans. $0.867+.

CASE II.

§ 156. The Prices of several Commodities being given, to determine how much of each Commodity must be taken, to form a Compound of a certain proposed Medium Value.

Rule.—*Write down the prices of the several simples under each other, placing that price which is least in value uppermost, and the remaining prices in the order of their values.*

Connect, by a line, any price less than the given mean price, with one that is greater, and continue thus to do till they are all connected; then place the mean price on the left, and separate it from the other numbers by a perpendicular line. Write the difference between the proposed price of the mixture and the price of each simple, opposite the number or numbers with which that simple is connected. And, finally,

Notice whether more than one difference stands opposite any one price; if so, their sum will express the quantity of that price to be taken; but if only one difference stands there, that will be the quantity required.

Note.—One difference, at least, must stand against each price.

Ex. 1. How much corn, at 48 cents, barley, at 36 cents, and oats, at 24 cents, per bushel, must be taken to make a compound worth 30 cents per bushel?

Mean price, 30	24	6 + 18 = 24 bushels, at	24 cents.
	36	6 . . . . . 6	36 cents.
	48	6 . . . . . 6	48 cents.

The difference between 30, the mean, and 24, is placed opposite both 36 and 48, as it is connected with them both; and the difference between 30, the mean, and 36, and also between 30 and 48, are both placed opposite 24, because these numbers are both linked with 24, and the sum of their differences determines the number of bushels required of that price. Of the oats, therefore, 24 bushels are required, and of the corn and barley, only 6 bushels of each.

2. I have four kinds of sugar, valued at 8, 12, 15, and 18

cents per pound. How much of each kind must be taken to make a mixture, worth 14 cents per pound?

14	8	4, number of pounds at 8 cents.
	12	1, ... 12 cents.
	15	2, ... 15 cents.
	18	6, ... 18 cents.

3. A grocer mixed together three kinds of tea, valued at 6, 9, and 10 shillings per pound, so that the compound was worth 8 shillings per pound. How much of each sort did he take?

Ans. 3 lb. at 6 s., 2 lb. at 9 s., and 2 lb. at 10 s.

4. A merchant has three kinds of wine. For the first kind, he charges 3 s. 4 d., for the second, 5 s., and for the third, 7 s. per gallon. How much of each is required to form a mixture worth 6 s. per gallon?

Ans. 12 gal. at 3 s. 4 d., 12 gal. at 5 s., and 44 gal. at 7 s.

5. How much gold, at 16, 19, 21, and 24 carats fine, will be required to form a compound of 20 carats fine?

Ans. 4 parts of 16, 1 of 19, 1 of 21, and 4 of 24, carats fine.

CASE III.

§ **157.** THE PRICE OF EACH OF SEVERAL SIMPLES, THE QUANTITY OF ONE, AND THE PRICE OF THE COMPOUND BEING GIVEN, TO FIND HOW MUCH OF EACH OF THE OTHER SIMPLES IS REQUIRED.

Rule. — *Link the several prices together, as in the last Case, and find their differences; then multiply the given quantity by the differences standing severally against the other quantities, and divide the product by the difference standing against itself. Or say, As the difference opposite the given quantity is to the given quantity, so are the other differences severally to their required quantities.*

Ex. 1. How much barley, at 30 cents, rye, at 36 cents, and corn, at 48 cents, per bushel, must be mixed with 12 bushels of oats, at 18 cents per bushel, so that the compound may be worth 22 cents per bushel?

Mean price, 22	30	. . . . . . 4 = 4.
	36	. . . . . . 4 = 4.
	48	. . . . . . 4 = 4.
	18	8 + 14 + 26 = 48.

The price of the given quantity is 48; therefore, 48 : 12 : : 4 : 1, the quantity required at 30 cents per bushel. The remaining statements and *answers* are the same, since the differences are all the same. Therefore, 1 bushel at 30 cents, 1 at 36 cents, and 1 at 48 cents, would be required to be mixed with 12 bushels at 18 cents, to form a mixture worth 22 cents.

2. A grocer has three kinds of beer for sale, valued at 7 s., 5 s., and 3 s. per gallon, which he proposes to mix with 20 gallons of a superior quality, worth 6 s. per gallon, so that the mixture may be sold at 4 s. per gallon. How much of the first three kinds must he take?

Ans. 120 gal. at 3 s., and 20 gal. at 5 s. and 7 s.

3. How much tea, at 80, 60, and 40 cents per lb., must be mixed with 30 lb. at $1.00 per lb., so that the mixture may be sold at 70 cents per lb.?

Ans. 10 lb. at 80 cts. and 60 cts. and 30 lb. at 40 cts.

4. How much water, of no value, must be mixed with 100 gal. of wine, at 7 s. 6 d. per gal., to reduce the price to 6 s. 3 d. per gallon? *Ans.* 20 gal.

CASE IV.

§ 158. The Price of the Simples being given, and also the Compound to be formed, to find how much of each Simple must be taken.

Rule.—*Connect the prices of the simples as in the preceding Cases, and find the amount of the differences; then say, As the amount of the differences is to each of the differences taken separately, so is the whole compound to the part required.*

Ex. 1. A compound of 15 gallons, which shall be worth 8 shillings per gallon, is to be made of three sorts of wine, valued at 5, 7, and 12 shillings per gallon. How much of each kind will be required?

8	5	4	4
	7	4	4
	12	3+1	4
			12

Then, 12 : 4 : : 15 : 5, *Ans.* 5 gal. of each kind are required.

Proof, 5 s. $\times$ 5 $=$ 25 s.; 7 s. $\times$ 5 $=$ 35 s.; and 12 s. $\times$ 5 $=$ 60 s.; and 25 $+$ 35 $+$ 60 $=$ 120 s.; and 120 $\div$ 8 $=$ 15 gallons.

2. I have four sorts of tea, of which the first kind is worth 1 s. per lb.; the second kind, 3 s.; the third, 6 s.; and the fourth, 10 s. How much of each kind will be required to make a compound of 120 lb., worth 4 s. per lb.?

Ans. 60 lb., at 1 s.; 20 lb., at 3 s.; 10 lb., at 6 s.; and 30 lb., at 10 s.

3. How much of each of four kinds of coffee, worth 8, 12,

18, and 22 cents per lb., will be required to make a compound of 120 lb., worth 16 cents per lb.?

Ans. 36 lb., at 8 cents; 12 lb., at 12 cents; 24 lb., at 18 cents; 48 lb., at 22 cents.

4. A gold-beater has gold, 15, 17, 18, and 22 carats fine, of which he wishes to make a compound of 40 oz., 20 carats fine. How much of each kind must he take?

Ans. 25 oz., 22 carats fine; and 5 oz. of 15, 17, and 18 carats fine.

5. How much water, of no value, and how much wine, at 90 cents per gallon, must be taken to make 100 gallons, worth 60 cents per gallon?

Ans. $33\frac{1}{3}$ gallons of water, and $66\frac{2}{3}$ gallons of wine.

QUESTIONS. — What is Alligation? What is Case I.? What is the rule? What is Case II.? What is the rule? What is the note? What is Case III.? What is the rule? What is Case IV.? What is the rule?

POSITION.

§ **159.** Position is a rule by which answers are obtained to such questions as cannot be solved by the common direct rules, by assuming any convenient number or numbers, and then working according to the nature of the question.

SINGLE POSITION.

§ **160.** When the question can be solved by the assumption of a *single number*, the operation is called *Single Position.*

The following sum will serve for an illustration: —

Ex. 1. A teacher, being asked how many scholars he had, replied, "If I had once, one half, one third, and one fourth as many more as I now have, I should have 185." How many had he?

We will suppose the number to be 24. As he first doubles his number, 24 must be doubled. To this amount one half his original number, 12, must also be added. He then increases his number by one third of his original number, viz., 8, and also by one fourth, viz., 6. Whole amount, 74.

Now, it is evident that we have not supposed the right number; otherwise the amount would have been 185, as given in the sum. We have, however, *increased the number* we supposed, viz., 24, by the *same* or *similar additions* as the *teacher did the true number of his scholars;* consequently, 74, the number we obtained, must have the same ratio to 24, the number assumed, as 185 has to the real number of scholars in the school. Therefore, 74 : 24 : : 185 : the number required, viz., 60. Proof, 60 + 60 + 30 + 20 + 15 = 185.

We have, then, the following rule: —

Rule. — *Take any convenient number, and proceed with it according to the conditions of the question, and observe the result; then say, As the number thus obtained is to the given number, so is the assumed number to the true one.* Or, the mumbers may be canceled, by *arranging the terms as directed in Simple Proportion.*

Ex. 2. A man, being asked how much money he had, replied, that $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$, $\frac{1}{6}$ of his money added, made $57. How much money had he? 12 *Ans.* $60.

3. What number is that, which being multiplied by 9 and divided by 4, the quotient will be 27? 96 *Ans.* 12.

4. A man borrowed a sum of money on interest, which, in 10 years, amounted to $1800, at 6 per cent. What was the sum? 6 *Ans.* $1125.

5. Two boys were playing at marbles. Says one to the other, "$\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$ of my marbles, added together, make 45; and if you can now tell how many I have, you may have them." How many had he? 72 *Ans.* 60.

6. A boy, wishing to try the skill of his companions in figures, said he had a pile of apples, of which, if he gave $\frac{1}{3}$ to A, $\frac{1}{4}$ to B, and $\frac{1}{6}$ to C, there would remain 28 for D; and requested them to tell him how many there were in all. What was the number? 60 *Ans.* 112.

7. A person, being asked his age, said that if $\frac{3}{5}$ of the years he had lived were multiplied by 7, and $\frac{2}{3}$ of them added to the product, the sum would be 292. How old was he? 45

Ans. 60 years.

8. A saves $\frac{1}{3}$ of his income; but B, who has the same income, spends twice as fast as A, and thereby contracts a debt of $120, annually. What is their income? 60 *Ans.* $360.

9. The sum of A, B, and C's ages is 132 years. B's age is $1\frac{1}{2}$ the age of A; and C's age is twice as great as B's. What are their respective ages? 60

Ans. A's age is 24; B's, 36; and C's, 72 years.

QUESTIONS. — What is Position? What is Single Position? What is the rule? How may the operations be canceled?

DOUBLE POSITION.

§ 161. By Double Position we solve such sums as require two suppositions.

In this rule, the numbers supposed to be the true ones bear no certain or definite proportion to the required answers.

Rule. — *Assume any two convenient numbers, and proceed with each according to the conditions of the question, and compare the result of each with the sum or result given in the question, and find their differences. Call each difference an* error.

Multiply the first assumed number by the last error, and the last assumed number by the first error.

If both errors are too great or too small, divide the difference of these products by the difference of the errors, and the quotient will be the number sought. But if one of the errors be too large, and the other too small, divide the sum of the products by the sum of the errors.

NOTE 1. — The errors are said to be too large or too small, when, by operating on each supposed number according to the nature of the question, the number obtained is greater or less than the corresponding number in the sum.

NOTE 2. — When the errors are too large, to avoid mistakes, they may be marked thus, +; and when they are too small, they may be designated thus, —.

NOTE 3. — When the errors are the same *number*, but differently marked, that is, if one is marked +, and the other —, the operation need not be continued; for *half the sum* of the suppositions is the number sought.

NOTE 4. — If the errors can both be divided by the same number without remainders, the quotients may be used instead of the numbers themselves; or, when desirable, they may each be multiplied by the same number, and their products used instead of the errors themselves.

Ex. 1. Three men found a purse of money, containing $80, which they agreed to divide in such a manner that A should have $5 more than B, and that B should have $10 more than C. What was each man's share of the money?

Suppose, first, that C had . $15
then B had, by the conditions, 25
and A, ... 30

$70, a sum of money less than that found; therefore, $80 — $70 = $10, first error.

Again, suppose that C had $20
B, of course, must have had . 30
and A, ... 35

$85, a sum of money greater than that found; therefore, $85 — $80 = $5, second error.

If, now, the above operations be compared with the rule and the note following, it will be seen that the first error is too small, and the last one too large; therefore, 15, number first supposed, $\times$ 5, the last error, $=$ 75; and 20, the number last supposed, $\times$ 10, the first error, $=$ 200; and 200 $+$ 75 $=$ 275, the sum of the products; and 10 $+$ 5 $=$ 15, the sum of errors. Therefore, 275 $\div$ 15 $=$ \$18.333 $+$, C's share; and \$18.333 $+$ \$10 $=$ \$28.333 $+$, B's share; and \$28.333 $+$ \$5 $=$ \$33.333, A's share.

2. Four individuals, having \$100 to divide among themselves, agree that B shall have \$4 more than A; C, \$8 more than B; and D, twice as much as C. What is each man's share?

First, suppose A had	\$6	Second, suppose A had	\$8
then B had	10	then B had	12
C, ...	18	C, ...	20
and D, ...	36	and D, ...	40
	\$70		\$80

and 100 — 70 $=$ 30, first error. Hence, 100 — 80 $=$ 20, second error.

Here both errors are too small: therefore, 6 $\times$ 20 $=$ 120; and 8 $\times$ 30 $=$ 240; then, 240 — 120 $=$ 120, the difference of the products; and 30 — 20 $=$ 10, the difference of errors. Therefore, 120 $\div$ 10 $=$ 12, A's share; 12 $+$ 4 $=$ 16, B's share; and 16 $+$ 8 $=$ 24, C's share; and 24 $+$ 24 $=$ 48, D's share. Proof, 12 $+$ 16 $+$ 24 $+$ 48 $=$ 100.

3. Three men hired a piece of wall built, for which they paid \$500. Of this, A paid a certain part; B paid \$10 more than A; and C paid as much as A and B both. What did each man pay? *Ans.* A paid \$120; B, \$130; and C, \$250.

Sums like the preceding are solved with ease by analysis. Since we have the sum they all paid, we know that C paid \$250, because he has paid as much as the other two, that is, one half of the whole. Therefore, A and B together paid \$250. But B paid \$10 more than A; hence, 250 — 10 $=$ 240, twice the number of dollars A paid, and 240 $\div$ 2 $=$ 120, A's share: then, 120 $+$ 10 $=$ 130, B's share; and 120 $+$ 130 $=$ 250, C's share.

4. Two persons lay out equal sums of money in trade. A gains 120£., and B loses 80£. A's money was then treble B's. With what sum did they commence? *Ans.* £180.

5. A farmer hired a laborer 40 days, on condition that he should receive 20 cents for every day he wrought, and forfeit 10 cents every day he was idle. At the expiration of the 40 days, he received \$5. How many days did he work, and how many was he idle? *Ans.* He wrought 30 days, and was idle 10 days.

6. What is the length of a fish, whose head is 10 inches long, his tail as long as his head and half the length of his body, and his body as long as his head and tail both? *Ans.* 80 inches.

7. Two persons, A and B, have the same income. A saves

$\frac{1}{4}$ of his; but B, by spending \$150 per annum more than A, at the end of 8 years, finds himself \$400 in debt. What was their income, and how much did each spend annually?

Ans. Income, \$400. A spends \$300, and B, \$450.

8. A man bequeathed his property to his three sons, on the following conditions, viz., to A, one half, wanting \$50; to B, one third; and to C, the remainder, which was \$10 less than B's share. How much did each son receive, and what was the whole estate?

Ans. A received \$130; B, \$120; and C, \$110. The whole estate was \$360.

9. A farmer bought a certain number of oxen, cows, and calves; for which he paid 130 £.. For every ox he paid 7 £.; for every cow, 5 £.; and for every calf, 1 £. 10 s. There were two cows for every ox, and three calves for every cow. How many were there of each kind?

Ans. 5 oxen, 10 cows, and 30 calves.

10. A person, after spending \$10 more than $\frac{1}{5}$ of his annual income, had \$35 more than $\frac{1}{2}$ of it remaining. What was his income? *Ans.* \$150.

11. A person has two horses; he also has a saddle worth 10 £. If the saddle be placed on the first horse, the horse and saddle are worth twice as much as the second horse; but the value of the second horse, with the saddle, is 13 £. less than the value of the first horse. How much is each horse worth?

Ans. The first is worth 56 £., and the second, 33 £.

QUESTIONS. — What is Double Position? What relation do the supposed numbers bear to the true ones? What is the rule? When are the errors said to be too large or too small?

PROMISCUOUS EXAMPLES.

Ex. 1. If 460 be multiplied by 36, and the product divided by 9, what will the quotient be? *Ans.* 1840.

2. What number is that, which, when increased by $\frac{3}{4}$ of itself, will be 126? *Ans.* 72.

3. What number, multiplied by $\frac{3}{4}$, will produce 16?

Ans. $21\frac{1}{3}$.

4. What fraction, multiplied by 15, will produce $\frac{1}{2}$?

Ans. $\frac{1}{30}$.

5. What number, multiplied by 32, will produce 2912?

Ans. 91.

6. What number, divided by 21, will give 65 as a quotient? *Ans.* 1365.

7. How many nails are required to shoe 27 horses, each shoe requiring 8 nails? *Ans.* 864.

8. In the counter of a merchant, there are four drawers, in each drawer, 4 divisions, and in each division, $23.75. How many dollars do the four drawers contain? *Ans.* $380.00.

9. Two men depart from the same place, and travel the same way; one travels 36 miles per day, and the other 42. What will be the distance between them at the end of the 8th day, and how far will each have traveled?

Ans. 48 miles apart, the one having traveled 288, and the other 336 miles.

10. A person, owning $\frac{3}{5}$ of a ship, sold $\frac{5}{8}$ of his share for $474. What was the value of the whole ship, at the same rate? *Ans.* $1264.

11. How many men must be employed to finish a piece of work in 15 days, which would require 5 men 24 days? *Ans.* 8 men.

12. A person, being asked the time of day, answered, "The time past noon is equal to $\frac{7}{8}$ the time till midnight." What was the time? *Ans.* 36 minutes past 5.

13. In a certain school, $\frac{1}{2}$ the scholars learn to read and write, $\frac{1}{4}$ learn geography, $\frac{1}{8}$ learn grammar, and 16 study astronomy. What is the number in the school? *Ans.* 128.

14. What is the whole length of a pole, $\frac{1}{8}$ of which stands in the ground, 16 feet in the water, and $\frac{4}{5}$ in the air? *Ans.* 213 feet, 4 inches.

15. There is a room, 12 feet long, 8 feet wide, and 7 feet high. How much paper, 2 feet wide, will be required to paper the same? *Ans.* 46 yards, 2 feet.

16. My horse and saddle are both worth 36 £. 12 s.; and my horse is worth 7 times as much as my saddle. What is the value of each?

Ans. My horse is worth 32 £. 0 s. 6 d., and my saddle, 4 £. 11 s. 6 d.

17. There is a cistern having 3 faucets, the largest of which will empty it in 1 hour, the second in 2 hours, and the third in 3 hours. In what time will they all empty it, if opened at the same time? *Ans.* $32\frac{8}{11}$ minutes.

18. Divide 1500 acres of land between A, B, and C, so that A shall have 150 acres more than B, and B, 100 acres more than C. *Ans.* A has $633\frac{1}{3}$, and B, $483\frac{1}{3}$, and C, $383\frac{1}{3}$.

19. A certain pasture will feed 324 sheep 7 weeks. How many must be turned away, in order that it may be sufficient for the remainder 9 weeks? *Ans.* 72

20. A merchant bought 120 gallons of melasses, for \$45. How must he sell the same, per gallon, to gain 15 per cent.? *Ans.* 0.43 +

21. If a family of 8 persons consume \$200 worth of provision in 9 months, how much will 18 persons consume in a year? *Ans.* \$600.

22. A man left his son a fortune, $\frac{1}{4}$ of which he spent in 3 months; and in 6 months more he spent $\frac{5}{6}$ of the remainder, when he had only \$1500 remaining. What was his fortune? *Ans.* \$13500.

23. A young man received 350 £., as his share of his father's estate, which was $\frac{3}{5}$ of his elder brother's portion; and the elder brother's portion was $\frac{1}{4}$ of the whole estate. What was the whole estate? *Ans.* 2333 £. 6 s. 8 d.

24. If 365 persons consume 75 barrels of provision in 9 months, how many barrels will 500 men consume in the same time? *Ans.* 102$\frac{54}{73}$ barrels.

25. A can mow an acre of grass in 5$\frac{1}{2}$ hours; B can mow 2 acres in 9 hours. In what time will they both mow 12$\frac{1}{2}$ acres? *Ans.* 30$\frac{5}{16}$ hours.

26. If 6 men build a wall 20 feet long, 6 feet high, and 4 feet thick, in 16 days, in what time will 24 men build one 200 feet long, 8 feet high, and 6 feet thick? *Ans.* 80 days.

27. A farmer, being asked how many sheep he had, replied, that in one pasture he had $\frac{1}{4}$ of his whole flock; in another, $\frac{1}{6}$; in another, $\frac{1}{8}$; and $\frac{1}{12}$ in another; and that a fifth pasture contained 450 sheep. How many did the five pastures contain? *Ans.* 1200.

28. If $\frac{5}{8}$ of a gallon of wine cost $\frac{3}{8}$ of a pound, New York currency, what will $\frac{5}{8}$ of a tun cost, in dollars and cents? *Ans.* \$262.50.

29. If $\frac{2}{3}$ of an ounce cost $\frac{7}{8}$ of a shilling, how many dollars, each 8s., will a pound cost? *Ans.* \$2.62$\frac{1}{2}$.

30. Bought 36 bags of rice, each weighing 84 lb., tret 4 lb. per 104. What will the whole net weight amount to, in Federal Money, at 8 d., New York currency, per pound? *Ans.* \$242.307 +.

31. In a certain orchard, $\frac{1}{2}$ of the trees bear apples, $\frac{1}{4}$ pears, $\frac{1}{6}$ plums; and 60 bear peaches; and 40, cherries. How many trees are there? *Ans.* 1200.

32. Sold goods to the amount of \$560, by which I lost 18 per cent., whereas I ought to have gained 12 per cent. What was my real loss? *Ans.* \$180.096 +.

33. What is that number of beggars, to whom if I give 3 pence apiece, I shall want 8 pence more than I now have, but if I give them 2 pence apiece, I shall have 3 pence left? *Ans.* 11.

34. How much sugar, at 9 d. per lb., must be given in exchange for 492 lb. of rice, at 3 d. per lb.? *Ans.* 164 lb.

35. Reduce $\frac{1}{2}$ of $\frac{3}{4}$ of $\frac{8}{9}$ of $\frac{7}{12}$ of $\frac{3}{14}$ of $\frac{21}{24}$, to a simple fraction. *Ans.* $\frac{7}{192}$.

36. What is the premium on $1800, at 15 per cent.? *Ans.* $270.

37. A father of 12 children said he was 24 years old when his oldest child was born, and that just a year and a half intervened between the birth of each two of his children. What was the age of the father, at the birth of his youngest child? *Ans.* $40\frac{1}{2}$.

38. Bought 16 bales of goods in London, for 96 £.; paid 4 £. for shipment to New York. How ought I to sell the same, in Federal Money, to gain 20 per cent.? *Ans.* $533.333+.

39. Bought a quantity of Irish linen in Dublin, for 50 £., and paid 10 £. for the shipment of the same. How must I sell the whole, in Federal Money, to gain 30 per cent.? *Ans.* $319.80.

40. What is the interest on $462.50, for 3 years, 6 months, and 12 days? *Ans.* $98.05.

41. Suppose there to be two silver cups, having one cover, which weighs 5 oz.; and suppose them to be such, that if the cover be placed on the smaller cup, the whole weighs twice as much as the greater cup; but if it be placed on the greater cup, the whole weighs three times as much as the smaller cup. What is the weight of each cup?

Ans. The smaller cup weighs 3 oz., and the larger, 4 oz.

42. A can do a piece of work in 6 days; B can do the same in 11 days. In what time will they both accomplish the work, if they labor together? *Ans.* $3\frac{15}{17}$ days.

43. A can do a piece of work in 7 days; B, in 12 days; C, in 6 days; and D, in 4 days. In what time will they accomplish the same work, if they all labor upon it at the same time? *Ans.* $1\frac{5}{9}$ days.

44. A person, being asked the time of day, replied that it was between 5 and 6 o'clock, and that the hour and minute hand were precisely together. What was the time? *Ans.* $27\frac{3}{11}$ minutes past 5 o'clock.

45. How many square feet are there in a board 16 feet 8 inches long, and 9 inches broad? *Ans.* $12\frac{1}{2}$ square feet.

46. The linear measure of a cubic block being 8 inches, how many cubic blocks, each one solid inch, does it contain? *Ans.* 512.

47. If a person own $\frac{2}{5}$ of a ship, and sell $\frac{1}{8}$ of his share for $1500, what is the value of the whole ship, at the same rate? *Ans.* $6000.

48. Divide $\frac{3}{4}$ of $\frac{8}{9}$ of $\frac{7}{12}$ of $\frac{15}{18}$ by $\frac{4}{5}$ of $\frac{6}{7}$ of $\frac{1}{4}$. *Ans.* $1\frac{577}{648}$.

49. In a thunder storm, I observed the time between the flash of the lightning and the report to be one minute and a half. How far distant was the lightning, allowing sound to travel 1142 feet in a second? *Ans.* $19\frac{41}{88}$ miles.

50. Bought 84 apples, at the rate of 2 for a penny, and 114, at the rate of 3 for a penny; the whole of which I afterwards sold at the rate of 9 for 4 pence. Did I gain, or lose? and how much? *Ans.* I gained 8 d.

51. A line, 44 yards in length, will just reach from the top of a steeple to the opposite side of the street, which is 24 yards wide. How high is the steeple? *Ans.* 36.87 + yards.

52. A tree, 36 feet high, stands by the side of a stream, 27 feet wide. How many feet from the top of the tree to the opposite side of the stream? *Ans.* 45.

53. There is a cistern having two pipes leading into it, one of which will fill it in 30 minutes, and the other in 45 minutes. In what time will both, running together, fill the same? *Ans.* 18 minutes.

54. How many bricks, 9 inches long and 4 inches broad, will pave a yard 40 feet square? *Ans.* 6400.

55. A certain cistern has two pipes leading into it, and one leading out of it. Of the two leading into it, one will fill it in 50 minutes, and the other in 75 minutes; while the one leading from it, will empty it in 60 minutes. Now, if all three be opened at the same time, in what time will the cistern be filled? *Ans.* 1 hour.

56. What is the difference between 6 dozen dozen, and half a dozen dozen? *Ans.* 792.

57. If $\frac{5}{7}$ of $\frac{1}{4}$ of $\frac{4}{5}$ of a ship be worth $\frac{2}{3}$ of $\frac{7}{8}$ of $\frac{12}{13}$ of her cargo, valued at \$2400, what is the value of both ship and cargo? *Ans.* \$5415.384 +.

58. If 11 men can build a house in 5 months, by working 12 hours per day, in what time will they complete it, if they work only 8 hours per day? *Ans.* $7\frac{1}{2}$ months.

59. Bought a pipe of wine, for \$84, from which 12 gallons leaked out. Now, what shall I gain, if I sell the remainder at $12\frac{1}{2}$ cents a pint? *Ans.* \$30.

60. A alone can do a piece of work in 12 days, and, in connection with B, in 8 days. In what time can B perform the same alone? *Ans.* 24 days.

61. If A can do a piece of work in 12 days, and B in 24 days, in what time will they both do it? *Ans.* 8 days.

62. Three men, A, B, and C, can do a piece of work in 18 days. A alone can do it in 36 days; B, in 54 days. In what time can C do it? *Ans.* 108 days.

63. Four men can do a piece of work in 15 days. A alone can do it in 40 days; B, in 60 days; and C, in 80 days. In what time will D do the work alone? *Ans.* 80 days.

64. A gentleman left his son a fortune, $\frac{1}{2}$ of which he spent in 3 months; $\frac{3}{4}$ of $\frac{4}{6}$ of the remainder lasted him 6 months longer, when he had only $1200 left. What was his whole fortune? *Ans.* $6400.

65. A farmer bought a yoke of oxen, a horse, and a cow, for $250. For the oxen he paid twice as much as for the horse, and for the horse three times as much as for the cow. What did he pay for each?

Ans. For the oxen, $150; for the horse, $75; and $25 for the cow.

66. Three men start, at the same time, to travel round an island 80 miles in circumference, and agree that each shall continue to travel at the same rate till they all come together again, and that the first shall travel 5 miles, the second, 6 miles, and the third, 7 miles per day. In what time will the three come together, and how far will each have traveled?

Ans. 80 days. The first will have traveled 400 miles; the second, 480 miles; and the third, 560 miles.

67. How will it affect the distance traveled by each, provided they travel 5, 7, and 9 miles, respectively?

Ans. The distance of the first will be 400 miles; of the second, 560; and of the third, 720.

68. Divide 1200 acres of land between A, B, and C, so that B may have 75 acres more than A, and C 95 acres more than B. What will be the share of each one?

Ans. A's share will be $318\frac{1}{3}$; B's, $393\frac{1}{3}$; and C's, $488\frac{1}{3}$ acres.

69. Three men met at an inn, two of whom brought provision with them. The third, not having brought any, proposed that they should eat together, and then he would pay his proportion. The proposal being accepted, A produced 5 loaves, and B, 4 loaves, all of which they ate up; and C, as his share, paid 9 pieces of money. With this, A and B were satisfied, but could not agree as to the division. What should each have received? *Ans.* A, 6, and B, 3 pieces of the money.

70. If A can reap a field of grain in 12 days, and B in 16 days, in what time will both do it, working together?

Ans. $6\frac{6}{7}$ days.

71. What number must be added to $\frac{1}{35}$ of 4560, to make the same 500? *Ans.* $369\frac{5}{7}$.

72. If the head of a fish be 9 inches long, its tail as long as its head and half the length of its body, and its body as long as its head and tail both, how long is the fish? *Ans.* 72 inches.

73. If 6 pairs of hose are equal in value to 4 pieces of Hol-

land, and 6 pieces of Holland to 14 yards of satin, and 12 yards of satin to 8 pieces of lace, and 9 pieces of lace to 8 £. 2 s., how many pairs of hose may be bought for 2 £. 16 s.?

Ans. 3 pairs.

74. A man was hired 50 days, on condition that, for every day he worked, he should receive 6 s., and for every day he was idle, he should forfeit 2 s. At the expiration of the time, he received $27.50. How many days did he work, and how many was he idle?

Ans. He labored 40 days, and was idle 10 days.

NOTE.—The currency of the preceding sum is that of New York.

75. A hare starts 40 yards in advance of a greyhound, and is not perceived by him till she has been up 40 seconds. She scuds away at the rate of 10 miles per hour, and the hound pursues after her at the rate of 18 miles per hour. In what time will the hound overtake the hare, and how far will he have run?

Ans. The required time is $60\frac{5}{22}$ sec.; the distance run, 530 yards.

76. If 8 men can build a wall 15 rods long in 10 days, how many men will be required to build 45 rods of wall in 5 days?

Ans. 48 men.

77. A man, when he married, was three times as old as his wife; after they had been married 15 years, his age was only double that of his wife's. How old were they when they married? *Ans.* The man was 45 years, his wife 15 years, old.

78. There is in a pasture a certain number of sheep, cows, and oxen; there are twice as many sheep as cows, and three times as many cows as oxen, and the whole number is 80. How many are there of each kind?

Ans. 8 oxen, 24 cows, and 48 sheep.

79. Sold coffee at 15 cents per pound, and thereby lost 10 per cent. on the first cost. Afterwards sold a quantity of the same for $525, by which I gained 40 per cent. What was the quantity sold, and what the price per pound?

Ans. Quantity, 20 cwt. $32\frac{8}{11}$ lb.; price, .231

80. Two sons, one 11 and the other 16 years of age, received a bequest of $10,000, to be so divided between them, that, the shares being put on interest at 5 per cent., should amount to equal sums, when they became respectively 21 years of age. What were the shares of each?

Ans. The elder received $\$5454\frac{6}{11}$, and the younger, $\$4545\frac{5}{11}$.

81. The fraction $\frac{197257}{120143}$ can be reduced to lower terms. Will the scholar reduce it?

82. There is a house, 60 feet high to the eaves, with a stream of water flowing past it. A line 65 feet long will reach from

the eaves to the nearest bank of the brook, and one 75 feet long, to the farther bank. How wide is the stream?

Ans. 20 ft.

83. Suppose a hall to be 48 feet long, 36 feet wide, and 25 feet high. How far is it from one of the lower corners, diagonally through the room, to the opposite upper corner?

Ans. 65 ft.

84. A, B, and C, wishing to build a house worth \$3000, find that no two of them can furnish money sufficient for the purpose. A and B together lack \$264; A and C lack \$457; and B and C lack \$1191. How many dollars has each?

Ans. A has \$1735; B, \$1001; and C, \$808.

85. A market-woman bought a certain number of eggs at 2 for a cent, and as many more at 3 for a cent. The eggs being placed together in one box, she sold them all at the rate of 5 for 2 cents; and thereby lost 12 cents. How many eggs did she have? *Ans.* 720.

86. The sum of two numbers is 180, and if the greater be divided by the less, the quotient will be 24. What are the numbers? *Ans.* The greater, $172\frac{4}{5}$; the less, $7\frac{1}{5}$.

87. Two drovers met upon the way,
And thus said one — "'Tis true,
If half your flock you give to me,
I'll have just eighty-two."
"Nay, friend," the other soon replied,
"Add but one third to mine,
Of your best sheep, then I shall have
One hundred twenty-nine."
His answer was exactly true;
No scholar will impeach;
Then by your knowledge show to me
How many sheep had each.

Ans. One had 21; the other, 122.

88. How much corn must a farmer carry to mill, so that he shall have a bushel left for grinding after the toll is taken out; the legal toll being 3 quarts per bushel?

Ans. 1 bushel, $3\frac{9}{29}$ qt.

89. Sound moves at the rate of 1142 feet per second. At what distance from me is a cannon, the report of which reaches me in $4\frac{7}{11}$ seconds after I see the flash? *Ans.* $320\frac{108}{121}$ rods.

90. The distance in feet through which a heavy body will fall in the air, in a given time, is very nearly equal to the square of the time in quarters of a second. How far, then, will a bullet drop in $2\frac{1}{2}$ seconds? *Ans.* 100 feet.

91. How long will it take a grape-shot to descend, by the

force of gravity, from the height of Mt. Washington, supposing it to be 6084 feet? *Ans.* $19\frac{1}{2}$ sec.

92. If a certain sum of money be reduced to farthings, and the farthings be increased by 1; the square root of their sum extracted and divided by 8; that result cubed and increased by the ratio 4 to 5 plus 45; the cube root of this sum extracted, and the root diminished by 3; and the remainder trebled and increased by 78, — the result will be one third the number of quarts in a hogshead. What is the sum of money?

Ans. £ 1, 1 s. 3 d. 3qr.

93. How many more rods of fence are required to enclose a square field of 2 acres, than to enclose a similar one containing 1 acre? *Ans.* 20.95 +.

94. A gentleman, having just rails sufficient to make 156 rods of fence, with them wished to enclose a pasture bounded by four straight lines, and having four square corners, so as to contain the greatest possible amount of land. Will the scholar satisfy himself as to what must be its length and breadth? 156

95. An army cut down $\frac{2}{3}$ of the trees in a forest on a certain day; and on each of the two following days, they cut $\frac{2}{3}$ of the trees not cut down the day preceding, when 20 trees only were left standing. How many were there at first? *Ans.* 540.

96. Some newly-married couples met at an inn, and, in their mirth, agreed that each husband should pay a shilling for every wife, and each wife a shilling for every husband. The landlord thus received 16 £. 18 s. The number of couples is required.

Ans. 13.

97. What is the difference between the simple, annual, and compound interest of \$600 for 4 years, at 6 per cent.?

Ans. Between the simple and annual, \$12.96 +; between the annual and compound, \$0.525 +.

APPENDIX.

Prob. 1. To find the greatest common measure of two or more numbers.

Note.—The greatest common measure of two or more numbers, is the *greatest number that will divide them separately without remainders.*

Rule.—*If two numbers only are given, divide the greater of them by the less; and if nothing remain, that divisor is the common measure; but if there be a remainder, divide the preceding divisor by it; and so continue to divide each preceding divisor by the last remainder, till the division is effected without remainder; the last divisor will be the common measure required. When more than two numbers are given, find the common measure of any two of them first, and then of that common measure, and either of the remaining numbers. This process, carried through all the numbers, will give their greatest common measure.*

Ex. 1. What is the greatest common measure of 72 and 108?

OPERATION.

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72)108(1
    72
    --
    36)72(2
       72
       --
       00
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36 is, therefore, the common measure required. Proof, $108 \div 36 = 3$, and no remainder; $72 \div 36 = 2$, and no remainder.

2. What is the greatest common measure of 27 and 99? *Ans.* 9.

3. What is the greatest common measure of 25, 45, and 90? *Ans.* 5.

4. What is the greatest common measure of 16, 32, 48, and 96? *Ans.* 16.

Prob. 2. To determine how many different positions any given number of objects may assume with regard to each other.

Rule.—*Represent the number of objects by the figures* 1, 2, 3, 4, 5, *&c., making the numbers of figures equal to the number of objects. The product of these figures will determine the number of changes.*

Ex. 1. How many changes may be made by the first 3 letters of the alphabet?

Operation: $1 \times 2 \times 3 = 6$, *Ans.* These changes are as follows:—1, a, b, c; 2, a, c, b; 3, b, a, c; 4, b, c, a; 5, c, b, a; 6, c, a, b,

2. How many changes may be made by the first 6 letters of the alphabet? *Ans.* 720.

3. At a certain boarding-house there are 12 boarders. How many different positions may they occupy at the table? *Ans.* 479001600.

4. Five men engaged board, at a tavern, for as many days as the landlord might be able to seat them in different positions. How long did they remain? *Ans.* 120 days.

Prob. 3. To find the area of a square.

Rule.—*Multiply the length of one of the sides by itself; or, square its linear measure.* (See Fig. 2, Square Root.)

Ex. 1. There is a room, just 8 feet square. What is the area?

Operation: $8 \times 8 = 64$ square feet, *Ans.*

2. What is the area of a floor 19 feet square? *Ans.* 361 square feet.

3. What is the area, in square inches, of a board 3 feet square? *Ans.* 1296 square inches.

Prob. 4. To find the area of a parallelogram.

Rule.—*Multiply the length of the parallelogram by its breadth.* (See Fig. 6, Square Root.)

Ex. 1. How many square feet are there in a floor 16 feet long and 12 broad? $16 \times 12 = 192$ square feet, *Ans.*

Note 1.—If the dimensions of a field, in the form of a square or parallelogram, are given in rods, the area is reduced to acres by dividing the square rods by 160.

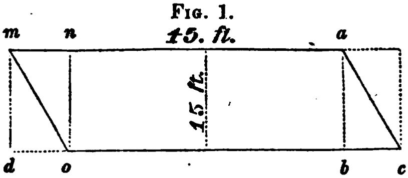
2. How many acres are there in a piece of land 80 rods in length and 40 in breadth?

$80 \times 40 = 3200$, and $3200 \div 160 = 20$ acres, *Ans.*

3. What is the number of acres in a piece of land 63 rods long and 49 rods wide? *Ans.* $19\frac{47}{160}$.

Note 2.—If the parallelogram be not right-angled, the length must be multiplied into the perpendicular distance between the sides. (See Fig. 1.)

Fig. 1.

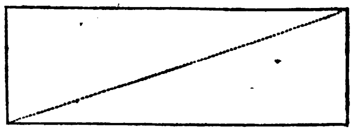


The triangle a, b, c, is obviously equal in area to the triangle m, n, o, and m, n, o, to d, m, o, and their form is similar; consequently, the two triangles are of equal area. If, therefore, the triangle a, b, c, be applied to the triangle m, n, o, so that the point c shall fall upon the point o, and the point a upon the point m, the point b will obviously fall upon the point d, and thus complete the right-angled parallelogram a, b, d, m, the area of which equals the area of the oblique-angled parallelogram a, c, o, m, and is found by multiplying the length, d, b, into the breadth, d, m. The area of the oblique-angled parallelogram is therefore found by multiplying its length by the perpendicular distance between its parallel sides. Hence, $45 \times 15 = 675$, the area of the parallelogram.

PROB. 5. To find the area of triangles.

The area of any triangle is just equal to one half the area of a square or parallelogram of the same height. This is obvious, with regard to right-angled triangles, and true of all others. The diagonal of a square or parallelogram, divides it into two equal right-angled triangles; and since the area of the whole figure is found by multiplying the length into the breadth, the area of its half, that is, of either of the triangles into which it is divided by the diagonal, is found by multiplying the same length or base into one half the breadth.

FIG. 2.



We have, then, the following rule: —

RULE. — *If the triangle be right-angled, multiply its base into half its perpendicular height. But if it be not right-angled, drop a perpendicular from one of the angles to the opposite side, or base; then multiply the base, or side, upon which the perpendicular falls, by one half the perpendicular.*

EX. 1. What is the area of a triangular piece of land, whose base is 40 rods, and whose perpendicular is 30 rods?

$30 \div 2 = 15$, and $40 \times 15 = 600$ square rods, *Ans.*

NOTE. — The same result is obtained by multiplying the perpendicular by one half the base. Thus, $40 \div 2 = 20$, and $30 \times 20 = 600$.

2. There is a room, 26 feet long, and 18 feet high. If a line be drawn from one of the upper corners to its opposite lower corner, the side of the room will be divided into two equal triangles. What is the area of each triangle, and also of the side of the room?

Ans. Each triangle contains 234 square feet; and the side, 468.

3. How many acres are there in a triangular piece of land, whose base is 42 rods, and whose perpendicular height is 36 rods?

Ans. $4\frac{29}{40}$ acres.

PROB. 6. Given one leg of a triangle, to find another leg and a hypotenuse, in *whole* numbers, that shall form a right-angled triangle.

RULE. — *Square the given leg, and resolve the square into two factors, each of which shall be an* EVEN *or an* ODD *number; then will half the sum of the two factors be the hypotenuse, and half the difference, the other leg.*

EX. 1. Suppose 8 feet to be taken as the base of a right-angled triangle, what must be the length of the perpendicular and hypotenuse, in whole numbers? *Ans.* Hypot., 10 feet, and perpend., 6 feet.

2. If 15 feet be taken as the base of a right-angled triangle, what perpendicular and hypotenuse can be found, in whole numbers?

Ans. Hypot., 25 feet; perpend., 20 feet.

Prob. 7. To find the sides of a right-angled triangle, in whole numbers.

Rule.—*Take any odd square, greater than unity;—then shall its root be one leg, and its half, rejecting the remainder of* 1, *the other leg.*

Ex. 1. It is required to find the sides of a right-angled triangle, in whole numbers, the square of whose base is 9 inches.

Ans. Base, 3 inches; perpend., 4 inches; and hypot., 5 inches.

2. Find the sides of a right-angled triangle, in whole numbers, whose base shall be 7 feet. *Ans.* Perpend., 24 feet; hypot., 25 feet.

Prob. 8. Given the *sum* of the hypotenuse and perpendicular, and also the *base* of a right-angled triangle, to find the hypotenuse and perpendicular.

Rule.—*Divide the difference of the squares of the two numbers by twice the greater number.*

Ex. 1. At what point from the ground must a tree, 80 feet high, be broken off, so that its top shall strike the ground 20 feet from its roots, the part broken off, at the same time, resting upon the stump?

Ans. $37\frac{1}{2}$ feet.

2. A liberty-pole, 100 feet high, was broken off, in a storm, so that its top struck 18 feet from the base, the two parts at the same time remaining attached to each other. What was the length of each part?

Ans. Part broken off, 51 feet, 7.44 inches; the other part, 48 feet, 4.56 inches.

Prob. 9. Given *one* leg and the *difference* between the hypotenuse and the other leg, to find that other leg.

Rule.—*From the square of the given leg subtract the square of the difference, and divide the remainder by twice the difference.*

Ex. 1. If the base of a triangle be 6 feet, and the difference between the perpendicular and hypotenuse be 2 feet, what is the perpendicular and hypotenuse? *Ans.* Perpend., 8 feet; hypot., 10 feet.

2. If the length of a field, in the form of a right-angled parallelogram, be 80 rods, and the difference between the diagonal and width be 40 rods, what is the area of the field? *Ans.* 30 acres.

Prob. 10. Given the perpendicular height of each of two objects, and the distance between them, to find the *position* and *length* of a line that shall reach to the top of each.

Rule.—*To the square of the distance between the objects, add the square of* either *height, and from the sum subtract the square of the other height, and divide the remainder by twice the distance between the objects. The quotient will be the distance of the* point *required from the object, the square of whose height was subtracted.*

Ex. 1. Suppose a street to be 30 feet wide, and to have a house

on one side, the eaves of which are 42 feet above the level of the street; and directly opposite to this, another house, the eaves of which are 36 feet above the same level. It is required to find the length of a ladder, and where it must stand, so that, when turned in either direction, it shall just reach the eaves of each house.

Ans. It must stand 22 feet $9\frac{3}{5}$ inches from the lower house, and be 42 feet $7\frac{1}{5}$ inches in length.

2. Two boys, standing between two trees, one of which was 48, and the other 60, feet in height, found that the same length of line would enable them to throw a kite to the top of each. Now, supposing the distance between the trees to have been 50 feet, how far did they stand from the lowest tree, and what length of line was required?

Ans. $37\frac{2}{2}\frac{4}{5}$ feet from the tree; and the length of line was $61\frac{1}{5}$ feet, nearly.

Prob. 11. Given the difference between the diagonal and side of a square, to find the side and diagonal.

Rule.—*Multiply the difference by 2.4142, and the product will be the side. Then multiply the square of the side by 2, and extract the square root of the product for the diagonal.*

Ex. 1. What is the length of the side and diagonal of a square, the diagonal of which is 7 feet longer than the side?

Ans. The side is 16.9 feet, nearly; and the diagonal, 23.9 feet.

2. What is the area of a square field, the diagonal of which measures 10 rods more than the side? *Ans.* 3 acres, 2 roods, 22.6 + rods.

CIRCULAR AND OTHER GEOMETRICAL FIGURES.

Prob. 12. Given the diameter of a circle, to find its circumference.

The diameter of a circle is to its circumference, as 7 to 22, or as 1 to 3.141592. Therefore,

Rule.—*Multiply the diameter of a circle by 3.141592, and the product will be the circumference; or, multiply the diameter by 22, and divide the product by 7.*

Ex. 1. What is the circumference of a wheel, whose diameter is 8 feet?

$8 \times 22 \div 7 = 25.14 +$ feet, *Ans.* Or, $8 \times 3.141592 = 25.132736$, nearly the same as before.

2. What is the circumference of a circle, whose diameter is 6 feet?

Ans. 18.85 feet.

3. What is the circumference of a circle whose diameter is 42 feet?

Ans. 132 feet.

Prob. 13. Given the circumference of a circle, to find the diameter.

Rule.—*Multiply the circumference by 7, and divide the product by 22;*

or (what will produce nearly the same result) *divide the circumference by* 3.141592.

Ex. 1. If the circumference of a wheel be 26 feet, what is the diameter? $26 \times 7 \div 22 = 8.27 +$ feet, *Ans.*

2. What is the diameter of a wheel whose circumference is 50 feet? *Ans.* 15.9 + feet.

Prob. 14. To find the area of a circle.

Rule.—*Multiply the square of the* DIAMETER *by the decimal* .7854, *or by the fraction* $\frac{11}{14}$; *or, multiply the square of the* CIRCUMFERENCE *by the decimal* .07958.

Ex. 1. If the diameter of a circle be 6 feet, what is its area? *Ans.* 28.285 + square feet.

2. If the diameter of a circle be 20 feet, what is its area? *Ans.* 314.285 + square feet.

3. What is the area of a circle, 66 feet in circumference? *Ans.* 346.65 + square feet.

Prob. 15. Given the area of a circle, to find its diameter.

Rule.—*Divide the area by the decimal* .7854, *and extract the square root of the quotient; or, divide by* 11, *and multiply by* 14, *and extract the square root.*

Ex. 1. If the area of a circle be 16 square feet, what is its diameter? *Ans.* 4.513 + feet, or 4 feet 6 inches +.

2. What is the diameter of a circular piece of land containing just one acre? *Ans.* 14.27 rods.

Prob. 16. Given the area of a circle, to find its circumference.

Rule.—*Divide the area by the decimal* .07958, *and extract the square root of the quotient.*

Ex. 1. What is the circumference of a circle, the area of which is 16 square feet? *Ans.* 14.18 feet, nearly.

2. What is the circumference of a circular piece of land containing just one acre? *Ans.* 44 rods, 13 feet, 8 inches.

Prob. 17. To find the superficial area of a globe.

Note.—The superficial area of a globe is four times as great as that of a circle of the same diameter; therefore,

Rule.—*Find the area of a circle of the same diameter, and multiply it by* 4.

Ex. 1. What is the superficial area of a globe 16 inches in diameter? *Ans.* 804.57 + square inches.

2. If the diameter of the earth were just 8000 miles, how many square miles of surface would it contain? *Ans.* 201142857$\frac{1}{7}$ square miles.

Prob. 18. To find the solid content of a globe.

Rule.—*Multiply the cube of the diameter by the decimal .5236, or by the fraction $\frac{11}{21}$.*

Ex. 1. If the diameter of a solid sphere be 12 inches, how many cubic inches does it contain? *Ans.* $905\frac{1}{7}$.

2. If the diameter of the earth be 8000 miles, how many cubic miles does it contain? *Ans.* $268190476190\frac{10}{21}$.

3. What is the capacity, in wine gallons, of a hollow globe, of which the internal diameter is 28 inches? *Ans.* $49\frac{7}{9}$ gallons.

Prob. 19. Given the diameter of a sphere, to ascertain how large a cube may be cut from it, or inscribed in it.

Rule.—*Divide the diameter of the sphere by 1.73205; or, multiply it either by the fraction $\frac{15}{26}$ or $\frac{56}{97}$. The result will be the linear measure of the inscribed cube.*

Ex. 1. Suppose the diameter of a sphere to be 12 inches, what is the linear measure of the greatest cube that can be cut from it? *Ans.* 6.928 inches.

2. If the diameter of a globe be 26 inches, what is the capacity, in wine gallons, of a cubical vessel, whose linear inside measure is equal to the linear measure of the greatest cube that can be cut from the globe? *Ans.* $14\frac{4}{7}$ gallons.

3. Suppose a globe to be 3 feet in diameter, what is the linear measure of the greatest cube that can be cut from it, and what is the solid content of that cube? also, how many cubic feet would be cut off in reducing the sphere to a cube?

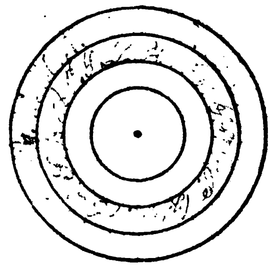
Ans. Linear measure, $20\frac{10}{13}$ inches; content, 5 cubic feet, 319. inches; cut off, 8 cubic feet, 1655 inches.

Prob. 20. To find the solid content of the shell of a concave sphere.

Rule.—*Multiply the difference of the cubes of the internal and external diameters by the decimal .5236, or the fraction $\frac{11}{21}$.*

Ex. 1. Suppose the internal diameter of a shell to be 7 inches, and its thickness $1\frac{1}{2}$ inches, what is its solidity? *Ans.* $344\frac{1}{7}$ cubic inches.

Prob. 21. To divide a circle into any number of concentric circles, each containing the same area.



Note.—Concentric circles are such as have a common center, as may be seen by the adjoining figure.

Rule.—*Find the area of the whole circle, and divide it into the required num-*

ber of equal parts, one of which will be the area of the center circle; from which find its diameter by Prob. 15. Again, take double the area of the center circle, and find its diameter; from which subtract the diameter of the center circle, and divide the remainder by 2. The quotient will be the breadth of the circle second from the center. Take also three times the area of the center circle, and proceed as before, subtracting from the result the diameter of the preceding circle, for the breadth of the third circle; and so on, through the required number of circles.

RULE 2. — *Divide the given diameter by the square root of the number of parts, for the diameter of the center circle, which multiply successively by the square roots of the numbers 2, 3, 4, 5, &c., for the other diameters.*

NOTE. — The square root of 2 = 1.41421; of 3 = 1.73205; of 5 = 2.23606.

Or, the sums under this problem may be solved, with still greater facility, by the adjoining table, as follows: Multiply the diameter of the given circle, by each of the fractions standing opposite the required number of circles, or by *either* of them, when two are given. The results will be the diameters of the required concentric circles. Then, to find the breadth of each circle, subtract the diameter of each *less* circle from that of the next greater, and divide the remainders by 2.

No. Cir.	MULTIPLIERS. 1st.	2d.	3d.	4th.
2	$\frac{12}{17}$ or $\frac{70}{99}$			
3	$\frac{15}{26}$ or $\frac{56}{97}$	$\frac{49}{60}$		
4	$\frac{1}{2}$	$\frac{12}{17}$ or $\frac{70}{99}$	$\frac{84}{97}$	
5	$\frac{17}{38}$	$\frac{31}{49}$	$\frac{55}{71}$	$\frac{17}{19}$

NOTE 2. — *Solid spheres* may be divided in a similar manner, by finding what fraction of the whole each inner sphere is, and multiplying the diameter of the sphere by the cube root of that fraction, and then proceeding as directed in dividing the circle. The cube root of $\frac{1}{4}$, = .25, is very nearly .63; that of $\frac{1}{2}$, = .5, is .7936 +, and of $\frac{3}{4}$, = .75, is .9085 +.

Ex. 1. Suppose the diameter of a circle to be 4 feet; it is required to divide it into four concentric circles of equal area.

Ans. The diameter of the center circle is 2 feet; the breadth of the second from the center, 4.968 inches; of the third, 3.816 inches; of the fourth, 3.216 inches.

2. Three men, A, B, and C, purchased a grindstone, 3 feet in diameter, and agreed that each should grind off one third of it; A grinding first; B, second; and C, third. Allowing the axle to have been equal to a circle 3 inches in diameter, what portion of the stone ought each to grind off?

Ans. A, 3.279 inches; B, 4.257 inches; C, 8.964 inches.

3. A nobleman presented to four jewellers a solid globe of silver, 8 inches in diameter, which he proposed to give them, on condition that they would divide it into four equal parts, by taking each part uniformly from the whole surface. What was the thickness of each portion?

Ans. The first, .366 thousandths; the second, .4596 ten thousandths; the third, .6544 ten thousandths of an inch; and the fourth or center portion, 5.04 inches.

PROB. 22. Given the distance over which a carriage has been drawn, to determine the space through which a given point in the tire of the wheel has passed.

NOTE.—The point in the tire, as the wheel revolves, describes a *cycloidal* arc, or a somewhat *flattened circle.* The following rule for finding such distances, is correct, provided the given wheel performs an exact number of revolutions or half-revolutions; otherwise it is nearly so.

RULE.—*Multiply the distance by* 14, *and divide by* 11; *or, multiply by* 4, *and divide by* 3.14159.

Ex. 1. Suppose a carriage to be drawn forward 4 miles, how far will a nail-head in the tire of a wheel move? *Ans.* 5 miles, 29 rods +.

2. Suppose the wheels of a locomotive engine to make an exact number of revolutions, in traveling 6 miles; how far has a given point in the circumference of either wheel traveled?

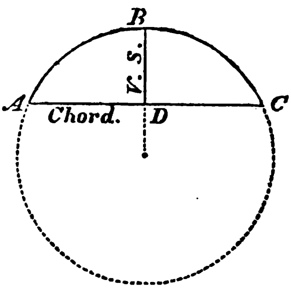
Ans. 7 miles, 203 rods, 10 feet, 6 inches.

NOTE.—Sums the reverse of the preceding, may be performed by reversing the process.

3. Suppose a carriage to be drawn forward, till a point in the tire of either wheel has passed through the distance of 3 miles; how far has the carriage been drawn?

Ans. 2 miles, 114 rods, 4 feet, 8$\frac{4}{7}$ inches.

PROB. 23. Given the chord and versed sine of an arc of a circle, to find its center.



DEFINITION.—An *arc* of a circle is any part of a circle cut off by a straight line. The line cutting off the arc is called the *chord;* and the line standing upon the center point of the chord, and extending perpendicularly to meet the arc, is called the *versed sine.* Thus, the curve line A, B, C, in the adjoining figure, is an arc of a circle; the line A, C, is the chord; and the line B, D, is the versed sine.

RULE.—*Divide the sum of the squares of the half-chord and the versed sine by twice the versed sine. The quotient will be the radius or semidiameter of the circle, which, being set off from the point B,* (*see Fig.*) *in the direction of the versed sine, or line B, D, will mark the center of the circle.*

Ex. 1. Suppose the chord of an arc to be 12 inches, and the versed sine, 4 inches; what is the semidiameter of the circle?

Ans. 6½ inches.

2. Suppose the chord, cutting off a portion of a circular piece of land, to be 18 rods, and the versed sine to be 3 rods; what is the diameter of the whole plot, and what is its area?

Ans. Diameter, 30 rods; area, 4 acres, 1 rood, 27 rods.

PROB. 24. To find the solid content of a cylinder.

A cylinder is a round body, of uniform diameter, and of any assignable length. Each end of a cylinder is, therefore, a circle.

RULE. — *Find the area of one end of the cylinder, and multiply that area by its length.*

Ex. 1. What is the solid content of a cylinder, 6 feet in length, and 3 feet in diameter?

The area of the end, as found by Prob. 14, is 7.06; and $7.06 \times 6 = 42.36$.

2. What is the solid content of a log, the uniform diameter of which is 2 feet, and the length, 30 feet? *Ans.* 94.247 solid feet.

3. What fraction of a cord is there in a log 10 feet long, and 4 feet in diameter? *Ans.* $\frac{63}{64}$, nearly.

PROB. 25. In a stick of timber, of uniform thickness, to find what length will make a solid foot.

RULE. — *Find the area of one end, in inches, and divide* 1728 *by it; the quotient will be the required length, in inches.*

Ex. 1. If a stick of timber be 16 inches square, what length will be required to make a solid foot?

$16 \times 16 = 256$, the area of one end, and $1728 \div 256 = 6\frac{3}{4}$ inches, *Ans.*

2. How much length of plank, 3 inches thick, and 24 inches wide, will be required, to make a solid foot? *Ans.* 2 feet.

PROB. 26. To find the solid content of cones and pyramids.

A *pyramid* or *cone* is a solid whose base is a plain surface, and which gradually and uniformly tapers till it comes to a point. It may be either round, square, or triangular.

The solid content of such figures is $\frac{1}{3}$ as much as the content of a cylinder of the same base and height; therefore,

RULE. — *Multiply the area of the base by one third of the perpendicular height.*

Ex. 1. What is the solid content of a cone, 60 feet high, the base of which is 8 feet in diameter? *Ans.* 1005.3 + cubic feet.

2. What is the solid content of a pyramid, whose base is 4 feet square, and whose height is 15 feet? *Ans.* 80 solid feet.

3. How many cubic inches in a cone 18 inches high, and 12 inches in diameter at the base? *Ans.* 678.58

4. There is a pyramid whose base is 30 feet in diameter, and the height of it is 90 feet. What is its solid content?

Ans. 21205.8 + solid feet.

PROB. 27. To find the solidity of the frustum of a cone or pyramid.

A *frustum* is that part of a cone or pyramid that remains after a part has been cut off from the apex or top.

RULE. — *Find the diameter of each end of the cone or pyramid, and multiply them together; then to the product add one third of the square of the difference of the diameters, and multiply the sum by the decimal* .785398; *the product will be the mean area between the two bases; therefore, multiply this mean area by the length of the frustum.*

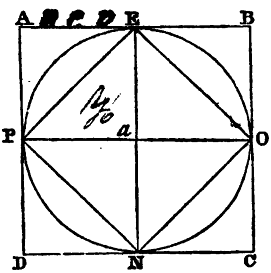
Ex. 1. What is the solid content of a frustum of a cone, 30 feet in length, whose base measures 4 feet in diameter, and whose top measures 2 feet in diameter?

$4 \times 2 = 8$. The difference of diameters is $4 - 2 = 2$, and $2^2 = 4$, and $4 \div 3 = 1\frac{1}{3}$. Therefore, $8 + 1\frac{1}{3} \times .785398 = 7.33$, nearly the mean diameter; and $7.33 \times 30 = 219.9$, the required content.

2. What is the content of a stick of timber, of which the larger end measures 2 feet in diameter, and the smaller end, 1 foot, the length being 40 feet? *Ans.* 73.3, nearly.

PROB. 28. Given the length and diameter of a log or stick of round timber, to determine the solid content of the greatest square timber that can be obtained from it.

The area of the greatest square that can be described upon the *end* of the given log, must first be determined; that is, the area of a square inscribed in a circle, the diameter of which equals the diameter of the log. How, then, is the area of an inscribed square, that is, of a square, the corners of which all terminate in the circumference of a circle, to be found, the diameter of that circle being given?



If the scholar will examine the adjoining figure, he will see that the square A, B, C, D, which is circumscribed about the circle, is equal to the square of the diameter of the circle, since the diameter, E, N, equals the side of A, D; and A, D, squared, gives the area of the square, A, B, C, D; also, that the inscribed square, P, E, O, N, is just one half of the circumscribed square, since each of the triangles, E, *a*, O; O, *a*, N; N, *a*, P, and P, *a*, E, into which the inscribed square is divided, is precisely one half of each of the four squares, A, P, *a*, E; E, *a*, O, B; O, *a*, N, C, and N, *a*, P, D, into which the circumscribed square, A, B, C, D, is divided. If, therefore, a fourth part of the inscribed square equals *one half* of a fourth part of the circumscribed square, the whole inscribed square equals one half of the circumscribed square. Hence,

RULE. — *Square the diameter of the small end of the log, and multiply one half of this square by the length of the log; the product will be the content, in solid feet, if the length and diameter are both given in feet; but if one of them be expressed in inches, divide this product by 144; and if both, by 1728, and the quotient will express the solid content in feet.*

Ex. 1. How many solid feet of square timoer may be sawn from a log 2 feet in diameter, and 20 feet in length?

$2^2 = 4$, and $4 \div 2 = 2$; then, $2 \times 20 = 40$ solid feet, *Ans.*

2. How many solid feet of square timber may be obtained from a log 10 inches in diameter, and 15 feet long? *Ans.* $5\frac{5}{24}$ solid feet.

3. How many square feet of timber are there in a log 43 feet long, allowing the diameter to be 1 foot 6 inches at the small end? and how many solid feet of slab would be taken off in squaring it, allowing the diameter at the large end to be 2 feet?

Ans. 48.37 + solid feet of timber, and 55.76 solid feet of slab.

Prob. 29. To find the capacity of casks in gallons, &c.

Rule.—*Take the length of the cask, between the heads, and also its interior bung and head diameter, in inches. Multiply the difference between the head and bung diameter by .62, when the staves are of ordinary curvature; but by .66, if the curvature be more than ordinary, and by .58, if they be curved less than usual, and add the product to the head diameter; the sum will be the mean diameter. Multiply the square of the mean diameter by the length, and this product by 34, for wine, and by 28, for beer measure, and from the product cut off four decimals. The number obtained will be the gallons and the decimal of a gallon the cask will contain.*

Ex. 1. How many gallons of wine are there in a cask whose bung diameter is 34 inches, head diameter, 30 inches, and whose length is 60 inches?

Operation: $34 - 30 = 4$, the difference between the head and bung diameter; then, $4 \times .62 = 2.48$, and $2.48 + 30 = 32.48$, mean diameter, and $32.48 \times 32.48 = 1054.9504$, square of mean diameter. $1054.9504 \times 60 = 63297.0240 \times 34 = 215.20988160$, or nearly 215.2 gallons, *Ans.*

2. Suppose the bung diameter of a cask of wine to be 24 inches, and the head diameter to be 20 inches, and the length, 40 inches; how many gallons will it contain? *Ans.* $68\frac{1}{4}$ gallons, nearly.

Prob. 30. To find the tonnage of double and single-decked vessels.

The following is the rule established by Congress:—

Rule.—"*If the vessel be double-decked, take the length thereof, from the fore part of the main stern to the after part of the stern-post, above the upper deck; the breadth thereof, at the broadest part, above the main wales, half of which breadth shall be accounted the depth of such vessel; and then deduct from the length three fifths of the breadth. Multiply the remainder by the breadth, and the product by the depth, and divide this last product by 95; the quotient whereof shall be deemed the true contents or tonnage of such ship or vessel. But if such ship or vessel*

be single-decked, take the length and breadth, as above directed, deduct from said length three fifths of the breadth, and take the depth from the under side of the deck plank to the ceiling in the hold; then multiply and divide as aforesaid, and the quotient shall be deemed the tonnage."

NOTE. — Carpenters multiply together the length of the keel, the breadth at the main-beam, and the depth of the hold, each dimension being taken in feet; and divide the product by 95, for the tonnage of single-decked vessels. But for double-decked vessels, they take half the breadth at the beam for the depth of the hold, and then work as before.

Ex. 1. Suppose the length of a double-decked vessel to be 80 feet, and its breadth to be 30 feet; what is its tonnage?

Operation: $30 \div 2 = 15$, the depth of the vessel; $\frac{3}{5}$ of $30 = 18$, and $80 - 18 = 62$; and $62 \times 30 = 1860$; then, $1860 \times 15 = 27900$; and, lastly, $27900 \div 95 = 293\frac{13}{19}$ tons, *Ans.*

2. Required the tonnage of a double-decked vessel, whose length is 95 feet, and whose breadth is 35 feet. *Ans.* $477\frac{2}{19}$.

3. What is the tonnage of a single-decked vessel, which is 75 feet in length, 30 feet in breadth, and 10 feet in depth? *Ans.* 180 tons.

NOTE. — The tonnage of a ship of war is found by dividing the continued product of the length, breadth, and depth by 100.

4. What is the tonnage of a sloop of war, whose length is 96 feet, breadth 32 feet, and depth 15 feet? *Ans.* 460.8 tons.

PROB. 31. The difference of longitude between two places being given, to find the difference in time.

As the circumference of the earth is divided into 360 degrees, and as the sun appears to pass round the earth in 24 hours, it is evident that $360 \div 24$, $= 15$ degrees of the earth's circumference, must pass under the sun every hour, and consequently 1 degree every 4 minutes, and 1′ every 4 seconds. We have, then, the following general rule: —

RULE. — *If the given degrees be more than* 15, *divide them by* 15; *the quotient will be the hours. But if the degrees be less than* 15, *multiply them by* 4, *and the product will be minutes. Multiply also the given minutes of motion by* 4, *and the product will be seconds of time.*

Ex. 1. Warsaw is situated about 17° east longitude from Brussels. What is the difference in time?

$17 \div 15 = 1\frac{2}{15}$ hours, or 1 hour 8 minutes.

It is obvious that the sun will rise first at the most eastern of any two places. Hence, when the time of one place is given, and it is required to find the time of some other place, *the difference of time must be added to the given time, when that place is situated to the east, and subtracted, when it is situated to the west of the place at which the time is given.*

2. Turin is situated 8° east from London, and Moscow 38° east from the same place. What is the hour at Moscow, when it is 9 o'clock at Turin? *Ans.* 11 o'clock.

3. When it is 9 o'clock at Philadelphia, what time is it at Council Bluff, 21° west of Philadelphia? *Ans.* 36 minutes past 7 o'clock.

4. What is the difference of time between Washington and Harmony, a missionary station west of Missouri, the difference of longitude being 18°? *Ans.* 1 hour, 12 minutes.

PROB. 32. The difference in time between any two places given, to find the difference of longitude.

This is the reverse of the preceding; therefore,

RULE.—*Multiply the hours by* 15, *and the products will be degrees. If minutes and seconds of time be given, divide them by* 4; *the quotient of the minutes will be degrees, and that of the seconds will be minutes of motion.*

Ex. 1. If the difference of time between two places be 3 hours and 32 minutes, what is the difference of longitude?

$3 \times 15 = 45°$, and $32 \div 4 = 8°$, and $45° + 8° = 53°$, *Ans.*

2. What is the difference of longitude between two places, if the time of one place be 5 hours 15 minutes in advance of the time of the other place? *Ans.* 78° 45′.

PROB. 33. Given the sum and difference of two numbers, to find the numbers.

RULE.—*Having placed the smaller of the two numbers under the greater, add and subtract, and divide each result by* 2. *The quotients will be the numbers.*

NOTE.—When the sum and difference of the *squares* of two numbers are given, the *squares* may be found by the preceding rule, the *roots* of which will be the numbers. The same is true of any and *all powers* of numbers.

Ex. 1. Two boys, together, had 164 cents; but the elder had 22 cents more than the younger. How many had each?

Ans. The elder, 93 cents; the younger, 71 cents.

2. Suppose a tree, 80 feet high, to be broken off, so that the top part shall be 18 feet longer than the lower part; what would be the length of each part? *Ans.* The top part, 49 feet; the lower part, 31 feet.

PROB. 34. Given either the sum or difference of two numbers, and the quotient of the greater divided by the less, to find the numbers.

RULE.—*Divide the sum, or difference, of the two numbers, by the quotient increased by* 1, *if their* SUM *be given, and diminished by* 1, *if the* DIFFERENCE *be given, and the quotient in each case will be the smaller number, which, multiplied by the given quotient, will be the greater number.*

Ex. 1. Two farmers, together, had 90 sheep, of whom one had 4 times as many as the other. How many had each?

Ans. One had 72; the other, 18.

2. Two neighbors sold their butter at the same market, one of whom, who sold 720 pounds more than the other, sold 5 lb. as often as the other sold 1 lb. How many pounds did each sell?

Ans. One, 180 lbs.; the other, 900 lb.

PROB. 35. Given the product and quotient of two numbers, to find the numbers.

RULE.—*Multiply together the product and quotient. The square root of* THEIR *product will be the greater number, by which divide the given product for the smaller number.*

Ex. 1. Suppose a field, in the form of a right-angled parallelogram, to contain 2 acres and 4 square rods, and that its length is four times its breadth; what are its length and breadth?

Ans. Length, 36 rods; breadth, 9 rods.

NOTE.—It will be observed that the area of the field, reduced to *square rods*, is the product of its length and breadth.

2. Suppose the area of a triangular field to contain $1\frac{3}{5}$ acres, and that one leg is twice as long as the other; what is the length of the legs? *Ans.* 32 and 16 rods.

PROB. 36. Given the *product* and either the sum or difference of two numbers, to find the numbers.

RULE.—*Multiply the product by 4, and square the other given number; then, if the* SUM *be given, subtract one result from the other; but if the* DIFFERENCE, *add them, and extract the square root, for the difference, if the* SUM *be given; but for the* SUM, *if the difference be given. Then, having the sum and difference, proceed by Prob.* 33.

Ex. 1. Suppose the area of a right-angled parallelogram to be $13\frac{1}{2}$ acres, and that its length exceeds its breadth by 24 rods; what are its length and breadth? *Ans.* Length, 60 rods; breadth, 36 rods.

2. I wish to lay out a piece of land, in the form of a right-angled parallelogram, to be enclosed by 136 rods of fence, and to contain 6 acres. The length and breadth are required.

Ans. Length, 48 rods; breadth, 20 rods.

PROB. 37. Given the difference of the *squares* of two numbers, and either their sum or difference, to find the numbers.

RULE.—*Divide the difference of the squares by the* DIFFERENCE *of the numbers, for the sum; and by the* SUM *of the numbers, for the difference; then, having the sum and difference of the numbers, proceed by Prob.* 33.

Ex. 1. The difference in the area of two square fields is 112 square rods, and the sum of their sides is 28 rods. What is the length of the side of each? *Ans.* 16 and 12 rods.

2. The difference of the area of two square fields is 180 square rods, and the difference in the measure of their sides is 6 rods. What is the lineal measure of each?

Ans. Of the larger, 18 rods; the smaller, 12 rods.

3. If the base of a triangle be 48 rods, and the hypotenuse be 24 rods longer than the perpendicular, what is the length of each?

Ans. Hypotenuse, 60 rods; perpendicular, 36 rods.

PROB. 38. Given the sum of the squares of two numbers, and either their sum or difference, to find the numbers.

RULE.—*From twice the sum of the squares subtract the square of the other given quantity, and extract the square root for the difference, if the sum be given; but for the sum, if the difference be given; then proceed by Prob.* 33.

Ex. 1. The diagonal of a parallelogram is 50 feet, and the sum of all its sides is 124 feet. What are its length and breadth?

Ans. Length, 48 feet; breadth, 14 feet.

2. The diagonal of a parallelogram is 35, and its length exceeds its breadth by 7 rods. What are its length and breadth?

Ans. Length, 28 rods; breadth, 21 rods.

PROB. 39. Given the sum of the squares of two numbers, and their product, to find the numbers.

RULE.—*To and from the* SUM *of the squares, add and subtract twice their product, and extract the square root of each result, for the* SUM *and* DIFFERENCE *of the numbers; then proceed by Prob.* 33.

Ex. 1. What are the length and breadth of a piece of land, lying in the form of a right-angled parallelogram, the area of which is 30 acres, and its diagonal 100 rods? *Ans.* Length, 80 rods; breadth, 60 rods.

2. Required the base and perpendicular of a right-angled triangle, of which the area is 5 acres, 1 rood, and 24 rods, and the hypotenuse 60 rods. *Ans.* Base, 48 rods; perpendicular, 36 rods.

PROB. 40. Given the product of two numbers, and the difference of their squares, to find the numbers.

RULE.—*To the square of the difference add four times the square of the product, and the square root of the result will be the sum of the squares; then proceed according to the note following Prob.* 33.

Ex. 1. The length of a parallelogram is 84 rods; but, were its length equal to its diagonal, it would, with the same breadth, contain 19 acres, 3 roods, 25 rods. What is its diagonal, and what its breadth?

Ans. Diagonal, 91 rods; breadth, 35 rods.

NOTE.—The inquisitive student will be able to apply the preceding rules to a great variety of useful problems, not *apparently* included in the above; as follows:—

Hannah.

 Laura.

To find the numbers from,

1. The sum of the numbers, and the sum of their square roots,	see Prob. 38.
2. The sum of their squares, and the sum of their fourth powers,	" Prob. 38.
3. The sum of their cubes, and the sum of their sixth powers,	" Prob. 38.
4. The sum of their cubes, and the sum of their ninth powers,	" Prob. 41.

Prob. 41. Given the sum of two numbers, and the sum of their cubes, to find the numbers.

Rule.—*From the cube of their sum subtract the sum of their cubes, and divide the remainder by three times their sum. The result will be the product of the two numbers; then proceed according to Prob.* 36.

Ex. 1. The sum of two numbers is 7, and the sum of their cubes, 91. What are the numbers? *Ans.* 3 and 4.

2. Suppose there are two cubical excavations, whose united capacity is equal to 480249 wine gallons, and whose united depth is 57 feet 9 inches; what is the depth of each?

Ans. The less, 19 feet, 3 inches; the greater, 38 feet, 6 inches.

Prob. 42. Given the *difference* of two numbers, and the difference of their cubes, to find the numbers.

Rule.—*From the difference of their cubes subtract the cube of their difference, and divide the remainder by three times their difference. The result will be the product of the numbers; whence proceed by Prob.* 36.

Ex. 1. A man, having a cubical bin, found that, by enlarging each side an inch, it would hold $26\frac{295}{231}$ gallons more than before. What was the measure of its side? *Ans.* 45 inches.

2. If a solid globe of glass, whose diameter is 8 inehes, be blown at the furnace into a hollow globe, till its shell is but $\frac{1}{5}$ of an inch in thickness, what would be its diameter, and how many wine gallons would it contain?

Ans. Diameter, 20.655 inches; contents, 18.33 gallons.

Prob. 43. Given the sum of every *two* of three, or the sum of every *three* of four numbers, to find the numbers.

Rule.—*Add together all the given quantities, and divide their sum by the number of quantities, less* 1. *The quotient will be the sum of all the quantities required; from which subtract the sum of any given group, and you will obtain the number* NOT *contained in that group.*

Ex. 1. A and B, together, have \$534; A and C, \$517; and B and C, \$417. How many dollars has each?

Ans. A, \$317; B, \$217; and C, \$200

2. A, B, and C, made a purchase together, for $1000. On payment, they, however, found that no two of them had money sufficient to cancel the debt. If A and B advanced all they had, $112 would remain unpaid; if A and C advanced all of theirs, $44 would remain unpaid; and, if B and C advanced what they had, $214 would be unpaid. How many dollars had each? *Ans.* A, $529; B, $359; and C, $427.

Prob. 44. Given the *product* of every *two* of three, or of every *three* of four numbers, to find the numbers.

Rule. — *Multiply the given quantities together, and extract the square root of their product. The result will be the* product *of all the quantities; which, divided by the given product of any group, will give the number* not *contained in that group.*

Note. — The rule is the same when the product of every 3 of 4 numbers is given, except that the *cube root* must be extracted. Also, if the product of every 4 of 5 numbers be given, the *fourth* root must be extracted, &c.

Ex. 1. A captain of a Liverpool packet took out from New York a certain number of men, women, and children. For each man he asked as many dollars as there were women, and thus received $1241; — for each woman he asked as many dollars as there were children, and thus received $3942; — for each child he asked as many dollars as there were men, and thus received $918. How many men, women, and children, were there? *Ans.* 17 men, 73 women, and 54 children.

2. A merchant sold a certain quantity of satinet, muslin, and ribbon. For the satinet he received as many cents, by the yard, as there were yards of muslin, the value of which was $17.64; for the muslin he received as many cents, by the yard, as there were yards of ribbon, the value of which was $15.12; and for the ribbon he received as many cents, by the yard, as there were yards of satinet, the value of which was $3.78. How many yards of each kind did he sell?

Ans. 21 yards of satinet, 84 yards of muslin, and 18 yards of ribbon.

SUMS TO BE SOLVED BY THE PRECEDING RULES.

Ex. 1. A gentleman buys 5 acres, 1 rood, 1 rod of land, which he wishes to lay out in two squares, so that the side of one shall be a rod longer than the side of the other. What must be the linear measure of each? *Ans.* The larger, 21 rods; the smaller, 20 rods.

2. There is a square public green, surrounded by a gravel walk, of the uniform breadth of 2 rods, and containing just 1 acre. What is the side of the square? *Ans.* 18 rods.

3. How much more land will 100 rods of fence enclose, in the form of a circle, than in the form of a square? *Ans.* 1 acre, 10.8 rods.

4. What is the greatest possible quantity of land, that can be enclosed by 137 rods of fence? *Ans.* 9 acres, 1 rood, 13.6+ rods.

5. How many rods less will it take to fence in an acre, if it be laid out in the form of a circle, than in the form of a square?

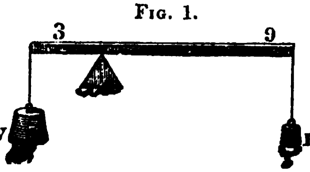
Ans. $5\frac{3}{4}$ + rods.

MECHANICAL POWERS.

When one body is made to communicate motion to another, the body communicating the motion is called *a power*, and the one to which the motion is communicated is called *a weight*. The various instruments employed to increase the effect of a given power, are called *Mechanical Powers.*

The mechanical powers are six in number, viz., the *Lever*, the *Pulley*, the *Wheel and Axle*, the *Inclined Plane*, the *Screw*, and the *Wedge.*

FIG. 1.



The lever is a bar, movable about a fixed point. It is represented at Fig. 1. P represents the power, and W the weight, and 3 and 9 the comparative length of the arms of the lever.

When the power and weight are such that the products obtained by multiplying each into the length of the arm to which it is attached, are equal, they will remain in equilibrium, that is, they will balance each other.

In Fig. 1, the arm to which the power is applied is three times as long as that to which the weight is applied; consequently, the power is required to be only one third as heavy as the weight, in order that they reciprocally balance each other. *We hence perceive that the power is to the weight, as the arm to which the weight is applied, is to that to which the power is applied.*

PROB. 45. Given the two arms of a lever, and the power, to find what weight that power will balance.

RULE.—*Multiply the length of the arm to which the power is applied, by the power, and divide the product by the other arm.*

Ex. 1. There is a lever, 16 feet in length, divided by the fulcrum or point of support into two arms, one of which is 12 feet, and the other 4 feet, in length. What weight on the short arm will balance 20 lb. suspended at the end of the long arm?

$20 \times 12 = 240$, and $240 \div 4 = 60$ lb., *Ans.*

2. The arms of a lever are, the one 30 feet, and the other 3 feet in length. What weight will a power of 160 lb., at the extremity of the long arm, balance at the extremity of the short arm? *Ans.* 1600 lb.

3. How many lb. will a power of 9 lb., placed 15 feet from the fulcrum of a lever, support at the extremity of the other arm, 2 feet in length? *Ans.* $67\frac{1}{2}$ lb.

PROB. 46. Given the arms of the lever and the weight, to find the power.

RULE.—*Multiply the weight by the length of the arm to which it is suspended, and divide the product by the other arm. The quotient will be the power required.*

Ex. 1. A weight of 80 lb. is attached to an arm of a lever 3 feet in length. What power must be applied to the other arm, 27 feet in length, to balance it? $80 \times 3 = 240$, and $240 \div 27 = 8\frac{8}{9}$ lb., *Ans.*

2. A weight of 20 tons is suspended to an arm of a lever 6 inches in length. What weight, at the extremity of the other arm, 40 feet in length, will balance the same? *Ans.* 5 cwt.

PROB. 47. Given the power, the weight, and the length of the lever, to find the fulcrum.

RULE. — *Add together the numbers representing the power and weight, and say, As their sum is to the number representing the power, so is the whole lever to the length of the arm to which the* WEIGHT *is to be applied; which, subtracted from the whole lever, will give the other arm.*

Ex. 1. Where must be the fulcrum, in a lever 16 feet in length, so that a power of 20 lb. shall balance a weight of 60 lb.?

$20 + 60 = 80$. Therefore, $80 : 60 :: 16 : 12$, *Ans.*, or length of the arm to which the power is to be applied; and $16 - 12 = 4$, length of the other arm.

2. If the power be 72 lb., the weight 1728 lb, and the lever 36 feet, where must be the fulcrum, in order that they shall balance each other?

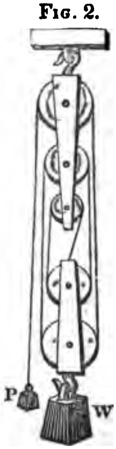
Ans. $1\frac{11}{25}$ feet from the weight.

3. If the length of a lever be 10 feet, the power 170 lb., and the weight to be balanced 1530 lb., where must be the fulcrum?

Ans. 9 feet from the power.

PULLEY.

FIG. 2.



A pulley consists of a wheel, movable about its axis, and so arranged as to be put in motion by means of a cord passing over it. The movable pulley is the only one by which a gain in power is effected. A double movable pulley is represented at Fig. 2. By such a pulley an equilibrium is produced, when the power is to the weight as 1 to the number of ropes sustaining the weight. If a single movable pulley be employed, the weight is sustained by two ropes; and the power will be to the weight as 1 to 2. If two movable pulleys be employed, the weight is sustained by four ropes, and the power will be to the weight as 1 to 4, &c.; that is, each movable pulley is sustained by two ropes, and consequently doubles the effect of the power employed.

PROB. 48. Given the number of movable pulleys and the power, to find the *weight*.

RULE. — *Multiply the power applied by twice the number of movable pulleys.*

Ex. 1. If, in a system of pulleys, there be three movable pulleys, what weight will a power of 72 lb. balance? $72 \times 6 = 432$ lb., *Ans.*

2. What weight will a power of 15 lb. balance by a system of 6 movable pulleys? *Ans.* 180 lb.

3. By the aid of a system of 12 movable pulleys, how many pounds will a man sustain who is capable of applying a power of 150 lb.? *Ans.* 3600.

Prob. 49. Given the number of movable pulleys, and the weight to be balanced, to find the required *power.*

Rule.—*Divide the weight by twice the number of movable pulleys.*

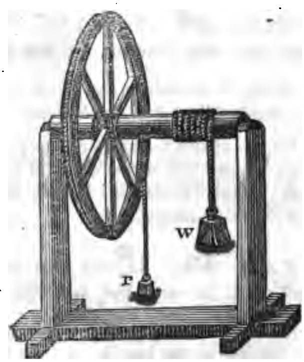
Ex. 1. How many pounds would be required by the aid of two movable pulleys, to sustain 800 lb.? *Ans.* 200 lb.

2. By the aid of 10 movable pulleys, how many lb. would be required to balance 2000 lb.? *Ans.* 100 lb.

3. What weight would balance 10 lb. of silver by the aid of a system of 100 pulleys? *Ans.* 12 pwt.

WHEEL AND AXLE.

Fig. 3.



The wheel and axle form a kind of perpetual lever, the long arm of which is the *semidiameter of the wheel*, and the short arm, the *semidiameter of the axle.* Consequently, the power of the wheel and axle is increased either *by making the wheel larger*, while the axle remains unaltered, or by *making the axle smaller*, while the wheel remains the same. The power must always be applied to the wheel, and the weight to the axle, when an increase of action is to be gained.

In the use of the wheel and axle, an equilibrium is produced, when the power is to the weight as the *semidiameter of the axle is to the semidiameter of the wheel.*

Prob. 50. Given the diameter of the wheel, the diameter of the axle, and the power, to find the weight.

Rule.—*Multiply the diameter of the wheel by the power applied, and divide the product by the diameter of the axle.*

Ex. 1. If the diameter of the axle be 6 inches, and that of the wheel 6 feet, what weight, attached to the axle, will 16 lb., attached to the wheel, balance? *Ans.* 192 lb.

2. If the diameter of a wheel be 20 feet, and that of the axle 2 feet,

and a power of 400 lb. be applied to the wheel, what weight will be balanced at the axle? *Ans.* 4000 lb.

Prob. 51. Given the diameter of the wheel, the diameter of the axle, and the weight to be balanced, to find the required power.

Rule.—*Multiply the diameter of the axle by the weight, and divide the product by the diameter of the wheel.*

Ex. 1. If the diameter of the axle be 6 inches, and the diameter of the wheel 12 feet, what power will balance a weight of 360 lb.? *Ans.* 15 lb.

2. If the diameter of the axle be 2 feet, and the diameter of the wheel 20 feet, what power will balance a weight of 4000 lb.? *Ans.* 400 lb.

INCLINED PLANE.

An inclined plane is a plane that makes with the ground or floor some *certain angle less than a right angle.* In the application of this instrument, the ratio of the power and weight is always the same as that of the *height and length of the plane.*

Prob. 52. Given the length and height of the plane, and also the power, to determine the weight.

Rule.—*Multiply the power by the length of the plane, and divide the product by its perpendicular height.*

Ex. 1. If the length of a plane be 16 feet, and its height 4 feet, what weight will a power of 32 lb. sustain? *Ans.* 128 lb.

2. What weight will 164 lb. sustain, on a plane 112 feet in length, and 3 feet in height? *Ans.* $6122\frac{2}{3}$ lb.

Prob. 53. Given the length and height of the plane, and also the weight, to find what power will sustain it.

Rule.—*Multiply the weight by the height of the plane, and divide the product by the length.*

Ex. 1. What power will balance 128 lb. on an inclined plane, the length of which is 16 feet, and the height 4 feet? *Ans.* 32 lb.

2. Suppose, on a railroad, there is an inclined plane 150 rods in length, and rising to the perpendicular height of 50 feet; what power will be required to sustain a weight of 84000 lb.? *Ans.* 28000 lb.

WEDGE

The wedge is composed of two inclined planes, whose bases unite and form the base of the wedge. The power applied to the wedge is

to the effect produced at the side of the wedge, *as the thickness of the head to the length of the wedge*, no allowance being made for friction.

PROB. 54. Given the thickness of the head, the length of the side, and the power acting upon the head of a wedge, to determine the force produced on the side.

RULE.—*Multiply the length of the wedge by the power applied, and divide the product by the thickness of the head.*

Ex. 1. If the length of a wedge be 12 inches, the thickness of the head 3 inches, and the force applied 64 lb., what will be the resistance at the side? *Ans.* 256 lb.

2. If the length of a wedge be 32 inches, the thickness of the head 2 inches, and the force applied 1600 lb., what will be the resistance at the side? *Ans.* 25600 lb.

PROB. 55. Given the length of the side, the thickness of the head, and the resistance upon the side of a wedge, to find the force acting upon the head.

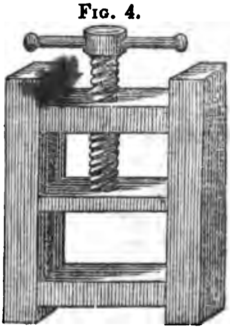
RULE.—*Multiply the resistance at the side by the thickness of the head, and divide the product by the length of the side of the wedge.*

Ex. 1. If the length of the wedge be 32 inches, the thickness of the head 2 inches, and the resistance at the side be 25600 lb., what must be the force upon the head, no allowance being made for friction? *Ans.* 1600 lb.

2. If the resistance at the side of a wedge be 20000 lb., the length of the wedge 20 inches, and the thickness of the head 3 inches, what force is required to be applied to counteract the resistance at the sides? *Ans.* 3000 lb.

THE SCREW.

FIG. 4.



The screw is a cylindrical piece of wood or metal, with a spiral thread running round it with a gradual and uniform inclination. The thread, therefore, forms an *inclined plane* of the same length as the thread itself, and whose height equals the length of the screw.

It is, then, obvious, that the power of the screw depends in part upon the fineness of the threads, since the finer they are, the greater will be the length of the plane, while its height remains the same. The power of the screw also depends in part on the *length of the lever* employed to move it. It is,

therefore, a *compound power*, involving in its action both the principle of the lever and the inclined plane, and is, consequently, more efficient than either of the other mechanical powers. In the action of this instrument, an equilibrium between the power and weight is produced, when the *power is to the weight as the distance between two contiguous threads of the screw is to the circumference of the circle described by the power in one revolution.*

PROB. 56. Given the distance between the threads of the screw, the length of the lever, and the power applied, to find the weight.

RULE.—*Multiply the circumference of the circle described by one revolution of the lever by the power applied, and divide the product by the distance between the threads of the screw.*

NOTE.—The lever is the semidiameter of the circle it describes; the circumference of the circle may, therefore, be found by Prob. 12.

Ex. 1. If the threads of a screw be 2 inches apart, the lever 40 inches in length, and a power of 60 lb. be applied to the end of the lever, what weight will be required to produce an equilibrium?

Ans. 7542 lb. 13 oz. 11$\frac{3}{7}$ drams.

2. If the distance between the threads of a screw be 1 inch, the length of the lever 16 inches, and the power applied to the end of the lever be 40 lb, what weight will produce an equilibrium?

Ans. 4022.8 + lb.

PROB. 57. Given the weight, the length of the lever, and the distance between the threads of the screw, to find the power required to produce an equilibrium.

RULE.—*Multiply the given weight by the distance between the threads of the screw, and divide the product by the circumference of the circle described by one revolution of the lever.*

Ex. 1. How many pounds, applied to the end of a lever 16 inches long, will balance 4022.8 lb. upon a screw whose threads are one inch apart? *Ans.* 40 lb.

2. How many pounds, applied to the end of a lever 30 inches long, will produce an equilibrium with 4000 lb. attached to a screw whose threads are 2 inches apart? *Ans.* 42$\frac{1}{2}$ lb., nearly.

STEAM POWER.

The power of the steam engine is usually estimated by comparing it with that of the horse. There are, however, various circumstances, which tend to render the power of the horse an indefinite standard;

such as the natural strength of the animal, — the training it may have received to enable it to exert its strength to the best advantage, — its position while exerting its strength, — the length of time the exertion is to be continued, — and the velocity of motion required. Before the term "horse power" could be employed to communicate any uniform idea of force exerted, it was, therefore, necessary to determine, by experiment, what was the medium power of the horse, making due allowance for varying circumstances.

Since, however, the power required to move any given body in a horizontal direction is dependent on various circumstances besides its actual weight, and as a diminished force only is required to continue a body in motion, when motion in that direction is once communicated, a medium power could be obtained only by repeated experiments on the power of the horse, in raising weights in a perpendicular direction, like raising a stone from a well, while the animal was made to act in a horizontal direction through the agency of a single pulley. Experiments were, accordingly, instituted, the result of which has been to establish the fact, that a one horse power is a power competent to raise 2250 lb. 1 mile, in a perpendicular direction, in 6 hours of time. This fact being thus established, the term "horse power" has acquired a distinctness of meaning, which renders it a convenient standard by which to estimate the power of machinery, especially when propelled by steam.

The weight given, viz., 2250 lb., is a little more than a ton; an amount entirely beyond the power of any horse to move as above stated. The velocity of 1 mile in 6 hours of time is, on the other hand, far less than the natural velocity of the horse. How are we, then, to understand the estimation of horse power here presented? It will be remembered, that, in all machinery by which *a gain of power* is effected, the power applied to put the machinery in motion always moves through a greater space than the body moved by the same machinery; or, in other words, "what is gained in power, is lost in speed." This fact presents us with a full explanation of the matter. A horse, with a moderate burden, will easily move 3 miles per hour, and in 6 hours, $6 \times 3 = 18$ miles. Now, suppose the horse to act through machinery, so constructed, that, while he moves at the rate of 18 miles in 6 hours, the weight attached to and acted upon by the same machinery, moves only 1 mile; how great a power would the horse be required to exert to raise 2250 lb. that distance? Evidently, a power equal to $\frac{1}{18}$ of 2250 lb., or 125 lb., a weight entirely within the power of a horse of ordinary strength to raise at the velocity proposed. This reduces the statement of a single horse power to a form easy to be comprehended; viz., it is a power competent to raise 125 lb. 18 miles in 6 hours, or 3 miles per hour.

Prob. 58. To determine the horse power required to raise a given weight any specified distance per hour.

Rule. — *Multiply the weight, reduced to pounds, by the distance through which it is to be moved, and make the product a dividend. Then, having reduced* 3 *miles to the same denomination with the given distance, multiply it by* 125, *and make the product a divisor. Divide, and the quotient will express the required horse power.*

Ex. 1. What amount of horse power will be required to raise 14 T. 14 cwt. 2 qr. 16 lb. 60 feet per hour?

Operation: 14 T., 14 cwt., 2 qr., 16 lb., $=33000$ lb.; and $33000 \times 60=1980000$, the dividend. 3 miles $=15840$ feet; and $15840 \times 125=1980000$, the divisor; then, $1980000 \div 1980000=1$, *Ans.*

NOTE. — If the time given be less than 1 hour, the required power must be *proportionally greater*. Had the time, in the preceding sum, been 20 minutes, instead of one hour, that is, $\frac{1}{3}$ of the time given, *three times as much power would have been required. Hence, the power required is found by multiplying* 1 *by the direct ratio of* 125 *and the given weight, and by the inverse ratio of* 1 *hour and the given time.*

2. What amount of horse power is necessary to raise 60 tons 3 feet per minute? *Ans.* 12.2 + horse power.

PROB. 59. Given the horse power, the distance, and the time, to determine the weight that may be raised.

RULE. — *Multiply* 125 *by the given horse power; the product will be the weight that power will raise* 3 *miles per hour. Multiply this by* 3 *miles reduced to the same denomination as the given distance, and divide the product by that distance, and the number obtained will determine the pounds that may be raised the given distance in one hour. Then, lastly, multiply by the given time in minutes, and divide the product by* 60; *the quotient will be the weight required.*

Ex. 1. What weight will a 5 horse power raise 120 feet in 20 minutes of time?

Solution: $125 \times 5=625$; then, since 3 miles equal 15840 feet, $625 \times 15840=9900000$; and $9900000 \div 120=82500$; and $82500 \times 20 \div 60=27500$ lb., *Ans.*

2. What weight will an 8 horse power raise 360 feet in 90 minutes of time? *Ans.* 29 T. 9 cwt. 1 qr. 4 lb.

PROBLEMS IN INTEREST.

PROB. 60. Time, rate per cent., and amount given, to find the principal.

RULE. — *Divide the given amount by the amount of* \$1 *or* 1£. *for the given rate and time.*

Ex. 1. What sum of money will amount to \$360, at 6 per cent., in 3 years and 4 months?

The amount of \$1 for 3 years and 4 months, is \$1.20; and \$360 ÷ \$1.20 = \$300, *Ans.*

2. What sum of money will, in 4 years, 6 months, at 6 per cent. per annum, amount to $1016? *Ans.* $800.

Prob. 61. Time, rate per cent., and interest given, to find the principal.

Rule.—*Divide the given interest by the interest of one dollar for the given time and rate per cent.*

Ex. 1. If a man's annual interest be $1200, what is his capital, the rate per cent. being 6? *Ans.* $20000.

2. How much money on interest, at 6 per cent., will gain $36 in 1 year and 8 months? *Ans.* $360.

Prob. 62. Given the principal, interest, and time, to find the rate per cent.

Rule.—*Divide the given interest by the interest of the given principal, at one per cent., for the given time.*

Ex. 1. A man, having $4000 on interest, at the expiration of one year received $240 interest on the same. What rate per cent. did he receive? *Ans.* 6 per cent.

2. If $200 be paid as interest on $2000, for 2 years and 6 months, what is the rate per cent.? *Ans.* 4 per cent.

Prob. 63. Given the principal, rate per cent., and interest, to find the time.

Rule.—*Divide the given interest by the interest of the principal for one year; the quotient will be the time.*

Ex. 1. Paid $108 interest on a note of $600. How long had the note been on interest, the rate per cent. being 6. *Ans.* 3 years.

2. Paid $400 interest on a note of $800, the rate per cent. being 4. How long had the note been on interest?

Ans. 12 years and 6 months

ANNUITIES.

An *annuity* is a sum of money payable annually for any given number of years, or forever.

When any sum of money, payable annually, has remained unpaid for any number of years, the amount due *is the sum of all the annuities which are in arrears, together with the interest on each annuity for the time it has remained unpaid.*

Prob. 64. Given the annuity, the rate per cent., and the time of arrears, to find the amount due.

Rule.—*Find the amount of each annuity for the time it has remained unpaid, at the given rate per cent.; the sum of the amounts thus obtained will be the sum required.*

Ex. 1. A man, who was in the receipt of an annual pension of $200, was required to forego the payment four years in succession. What was due at the expiration of that time, the rate per cent. being 6?

The annuity of the fourth year would receive no interest, as it was not due till the year expired, and consequently amounted to only $200. The annuity of the third year was entitled to one year's interest, and, therefore, amounted to $212. The annuity of the second year was entitled to two years' interest, and amounted to $224. The annuity of the first year was entitled to three years' interest, and amounted to $236. Therefore, 200 + 212 + 224 + 236 = $872, *Ans.*

2. What is the value of an annuity of $600, which has remained unpaid for 8 years, interest at 6 per cent. being allowed?

Ans. $5808.

Prob. 65. Given the annuity and rate per cent., to find its present worth for any number of years.

Rule.—*Divide each annuity by the amount of $1 or 1£. for the time before it becomes due; the quotient will be the present worth of the several annuities, and their sum will be the amount required.*

Ex. 1. What is the present worth of an annuity of $600, for three years to come, 6 per cent. being allowed for present payment?

The present worth of the first year is $600 ÷ $1.06 = 566.037; of the second year, $600 ÷ $1.12 = $535.714; and of the third year, $600 ÷ $1.18 = $508.474. Then, $566.037 + $535.714 + $508.474 = $1610.225, *Ans.*

2. What is the present worth of an annuity of 30£., for 5 years to come, at 4 per cent.? *Ans.* 134£. 5 s. 5 d. +.

ANNUITIES AT COMPOUND INTEREST.

The amount of an annuity at compound interest, is obtained by *computing the compound interest of the several payments, and then finding their sum.* Operations of this nature may obviously be solved by geometrical progression, by making $1 the first term of a geometrical series, and the amount of $1 for one year, at the given rate per cent., the ratio. The number of terms will always be the same as the number of years. (See Geometrical Progression.)

Ex. 1. What will an annuity of $60 per annum amount to, in 4 years, at 6 per cent.?

$1 is the first term, $1.06 the ratio. Therefore,

$$\frac{1.06^4 - 1}{.06} \times 60 = \$262.476 +, \textit{Ans.}$$

We have, then, the following general principle:—*Raise the ratio* (which is always found by adding the per cent. to $1) *to a power equal to the number of years; from this subtract* 1; *then divide the remainder by the ratio, less* 1, (that is, by the decimal part of the ratio only,) *and multiply the quotient by the annuity.*

2. What is the amount of an annuity of $400, which has remained unpaid for 20 years, at 6 per cent., compound interest?

Ans. $14714.23 +.

Or, the answers to such sums may be found by *multiplying the amount of one dollar for the given number of years and rate per cent., as found in the following table, by the annuity:*—

TABLE,

Showing the Amount of $1, at Five or Six per Cent. Compound Interest, for any Number of Years between One and Forty.

Yr.	Five per Cent.	Year.	Six per Cent.	Year.	Five per Cent.	Year.	Six per Cent.
1	1.000000	1	1.000000	21	35.719252	21	39.992727
2	2.050000	2	2.060000	22	38.505214	22	43.392290
3	3.152500	3	3.183600	23	41.430475	23	46.995828
4	4.310125	4	4.374616	24	44.501999	24	50.815577
5	5.525631	5	5.637093	25	47.727099	25	54.864512
6	6.801913	6	6.975319	26	51.113454	26	59.156383
7	8.142008	7	8.393838	27	54.669126	27	63.705766
8	9.549109	8	9.897468	28	58.402583	28	68.528112
9	11.026564	9	11.491316	29	62.322712	29	73.639798
10	12.577893	10	13.180795	30	66.438847	30	79.058186
11	14.206787	11	14.971643	31	70.760790	31	84.801677
12	15.917127	12	16.869941	32	75.298829	32	90.889778
13	17.712983	13	18.882138	33	80.063771	33	97.343165
14	19.598632	14	21.015066	34	85.066959	34	104.183755
15	21.578564	15	23.275970	35	90.220307	35	111.434780
16	23.657492	16	25.672528	36	95.836323	36	119.120867
17	25.840366	17	28.212880	37	101.628139	37	127.268119
18	28.132385	18	30.905653	38	107.709546	38	135.904206
19	30.539004	19	33.759992	39	114.095023	39	145.058458
20	33.065954	20	36.785591	40	120.799774	40	154.761966

3. What is the amount of an annuity of $150, which has remained unpaid for 12 years, compound interest, 6 per cent.?

Ans. $2530.49 +.

The amount of $1 for 12 years, in the above table, is $16.869941, and $16.869941 × 150 = $2530.49 +, *Ans.*

4. What is the worth of an annual salary of $1000, which has remained unpaid for 15 years, compound interest, at 6 per cent.? *Ans.* $23275.97.

5. What is the value of an annuity of $150, to continue 30 years, at 5 per cent. compound interest? *Ans.* $9965.827+.

TABLE,

Showing the Present Worth of an Annuity of $1, at Five or Six per Cent., for any Number of Years between One and Forty.

Yr.	Five per Cent	Year.	Six per Cent.	Year	Five per Cent.	Year.	Six per Cent.
1	0.952381	1	0.943396	21	12.821153	21	11.764077
2	1.859410	2	1.833393	22	13.163003	22	12.041582
3	2.723248	3	2.673012	23	13.488574	23	12.303379
4	3.545950	4	3.465106	24	13.798642	24	12.550358
5	4.329477	5	4.212364	25	14.093945	25	12.783356
6	5.075692	6	4.917324	26	14.375185	26	13.003166
7	5.786373	7	5.582381	27	14.643034	27	13.210534
8	6.463213	8	6.209794	28	14.898127	28	13.406164
9	7.107822	9	6.801692	29	15.141074	29	13.590721
10	7.721735	10	7.360087	30	15.372451	30	13.764831
11	8.306414	11	7.886875	31	15.592810	31	13.929086
12	8.863252	12	8.388844	32	15.802677	32	14.084043
13	9.393573	13	8.852683	33	16.002549	33	14.230230
14	9.898641	14	9.294984	34	16.192904	34	14.368141
15	10.379658	15	9.712249	35	16.374194	35	14.498246
16	10.837770	16	10.105895	36	16.546852	36	14.620987
17	11.274066	17	10.477260	37	16.711287	37	14.736780
18	11.689587	18	10.827603	38	16.867893	38	14.846019
19	12.085321	19	11.158116	39	17.017041	39	14.949075
20	12.462216	20	11.469921	40	17.159086	40	15.046297

To find the value of an annuity, by the preceding table, *multiply the value of one dollar, as given in the preceding table, for the given number of years, and rate per cent., by the given number of dollars.*

Ex. 1. What sum of money will purchase an annuity of $400, to continue 12 years, at 6 per cent. discount?

The value of $1, for 12 years, is $8.388844; and 8.388844 × 400 = $3355.537+, *Ans.*

2. What is the present value of an annuity of $1200, to continue 16 years, at 6 per cent. discount? *Ans.* $12127.074+.

3. How much must be paid for an annuity of $75, to continue 30 years, at 6 per cent. discount? *Ans.* $1032.352+.

ASSESSMENT OF TAXES.

A tax is a sum of money collected from the citizens of a state, county, or town, for defraying the expenses necessarily incurred in the administration of justice, and in works of common utility.

The sum each man is required to pay, depends, for the most part, on the amount of his property. To this the poll tax forms the only exception; which is a tax required of every male inhabitant of a state, who has attained the age of twenty-one years, *independently of the property he may possess.*

In levying a tax upon any community, first take a complete list of all the property of the town on which the tax is to be laid, and also of the number of polls to be taxed. Next, determine the amount of the poll taxes, by multiplying the number of polls by the tax on each. This being determined, subtract it from the whole sum to be raised, and the remainder will be the sum to be raised on the property of the town, both real and personal. Then, to ascertain the percentage, divide the tax to be raised by the whole amount of taxable property, and the quotient will be the tax on one dollar. To find what tax any individual pays, multiply his inventory by this quotient, or tax per dollar.

Ex. 1. What will be A's tax, if he own real estate to the amount of \$1340, and personal property to the amount of \$874, if the town in which he lives, the inventory of which is \$32265, raise a tax of \$1129.95, there being 270 polls, which are taxed 60 cents each, and for two of which he pays?

270 polls $\times$ 60 $=$ \$162.00, amount of the poll taxes. Therefore, \$1129.95 $-$ \$162.00 $=$ \$967.95, the sum to be levied on the property. Hence, \$967.95 $\div$ \$32265 $=$ 3 cents, the tax on one dollar. Then, \$1340 $\times$ .03 $=$ \$40.20, and \$874 $\times$ .03 $=$ \$26.22, and two polls, at 60 cents each, $=$ \$1.20. Therefore, \$40.20 $+$ \$26.22 $+$ \$1.20 $=$ 67.62, *Ans.*

After finding what the tax is per dollar, a table may be formed like the following, which will expedite the operations: —

TABLE.

Tax	on	\$1	is	.03	Tax	on	\$10	is	.30	Tax	on	\$100	is	3.00
"	"	2	"	.06	"	"	20	"	.60	"	"	200	"	6.00
"	"	3	"	.09	"	"	30	"	.90	"	"	300	"	9.00
"	"	4	"	.12	"	"	40	"	1.20	"	"	400	"	12.00
"	"	5	"	.15	"	"	50	"	1.50	"	"	500	"	15.00
"	"	6	"	.18	"	"	60	"	1.80	"	"	600	"	18.00
"	"	7	"	.21	"	"	70	"	2.10	"	"	700	"	21.00
"	"	8	"	.24	"	"	80	"	2.40	"	"	800	"	24.00
"	"	9	"	.27	"	"	90	"	2.70	"	"	900	"	27.00
										"	"	1000	"	30.00

Now, to determine A's tax from this table, $1340 + $874 = $2214.

The tax on	$2000 =	$60.00
" " "	200 =	6.00
" " "	10 =	.30
" " "	4 =	.12
Two polls, at 60 cts. =		1.20
		$67.62, *Ans.*, as before.

2. From the preceding table, calculate B's tax, whose real estate is valued at $5620, and his personal property at $1162, and who pays 3 poll taxes, each 75 cents. *Ans.* $205.71.

3. Form a table, and calculate the tax of A, whose property is valued at $1500; of B, whose property is $2000; of C, whose property is $1200; of D, whose property is $3000; and of E, whose property is estimated at $3500; supposing them to live in a town whose inventory is $1000000, and on which a tax of $4280 is to be levied, the number of polls being 400, and paying each 70 cents.

Ans. A's tax will be $6; B's, $8; C's, $4.80; D's, $12; and E's, $14.

4. A tax of $3000 is to be raised on a certain town, the inventory of which is $60000; the number of polls, 250, taxed each 75 cents. What is the amount of A's tax, whose property is valued at $2500, and who pays for two polls? *Ans.* 118.68\frac{3}{4}$.

TABLE

OF

STANDARD WEIGHT OF GOLD AND SILVER COIN.

	WEIGHT. pwt.	gr.
American Coin.		
Eagle, (half and quarter in proportion,)	10	16
Silver Dollar, (parts in proportion,)	17	6
English Currency		
One grain British gold is worth 2 d. English currency, = .037$\frac{1}{27}$, Federal Money; consequently, per pwt., it is worth .888$\frac{8}{9}$, and per oz. \$17.777$\frac{7}{9}$. But \$17.777$\frac{7}{9}$ ÷ 4 = \$4.444 +, the Federal value of 1 £. sterling; hence, every oz. of British gold = 4 £. sterling.		
English Guinea, (half in proportion,)	5	8½
Heavy Sovereign, ...	5	2½
Seven-Shilling Piece, . . .	1	19
French Coin.		
Double Louis, coined before 1786, ...	10	11
Double Louis, since 1786,	9	20
Double Napoleon, or 40 francs, (single in prop.,)	8	7
Same as the new Louis Guinea, ...	5	
Spanish Coin.		
Doubloon, (shares in proportion,)	17	8¼
Doubloon, Mexican or Colombian, ...	17	9
Coronilla, or Vintern, . . .	1	3
Portugal.		
Dobraon, (Brazilian, the same,)	34	12
Dobra, ...	18	6
Johannes, (Brazilian, same,)	18	
Moidore, (half in prop.,) .	6	22
Sixteen-Testoon Piece, 1600 Rees, ...	2	6
Old Crusade, 400 Rees, . .		15
New Crusade, 480 Rees, .		16¼
Milree, coined since 1755,		19¾
Frankfort and Hamburg.		
Ducat, (Hungarian and Russian, same,)	2	5¾
Geneva.		
Pistole, old, ...	4	7¼
Pistole, new, ...	3	15¾
Sequin of Genoa, ...	2	5¾
Austrian Dominions.		
Souverein, ...	3	14
Double Ducat, ...	4	12
Russia.		
Ducat of 1796, ...	2	6
Gold Ruble, 1756, ...	1	0½
Gold Ruble, 1799, ...		18¾
Gold Polten, 1777, ...		9
Imperial, 1801, (half in proportion,)	7	17¼
Half Imperial, 1818, ...	4	3½
Holland.		
Ryder, ...	6	9
Double Ryder, . .	12	21
Ducat, ...	2	5¾
Ten-Guilder Piece, (half in proportion,)	4	8
Prussia.		
Ducat, ...	2	5¾
Frederick, double, ...	8	14
Frederick, single, ...	4	7
Turkey.		
Sequin fonducli of Constantinople, ...	2	5¾

	WEIGHT. pwt.	gr.
Sequin fonducli,	2	5
Yeermeeblekblek,	3	1¾
Half Misseir, 1818,		18¼
Milan.		
Sequin,	2	5¾
Doppia, or Pistole,	4	1½
Forty-Livre Piece, 1808,	8	8
Naples.		
Six-Ducat Piece, 1783,	5	16
Two do., or Sequin, 1762,	1	20¼
Three do., or Oncetta, 1818,	2	10¼
Netherlands.		
Gold Lion, or Fourteen-Florin Piece,	5	7¾
Ten-Florin Piece, 1820,	4	7¾
Piedmont.		
Pistole, since 1785, (half in proportion,)	5	20
Sequin, (half in prop.)	2	5
Carlino, since 1785, (half in proportion,)	29	6
Marengo, or 20 francs,	4	3¼
Saxony.		
Ducat,	2	5¾
Augustus,	4	6½
Sicily.		
Double Ounce, (half in proportion,)	5	17
Denmark.		
Ducat, Current,	2	
Ducat, Specie,	2	5¾
Christian d'Or,	4	7
East India.		
Rupee, Bombay, 1818,	7	11
Rupee, Madras, 1818,	7	12
Pagoda, Star,	2	4¾

	WEIGHT. pwt.	gr.
Hanover.		
Double George d'Or, (single in proportion,)	8	13
Ducat,	2	5¼
Gold Florin,	2	2
Malta.		
Double Louis, (half in proportion,)	10	16
Demi-Louis,	2	16
Parma.		
Quadruple Pistole,	18	9
Pistole, or Doppia,	4	14
Maria Theresa,	4	3½
Rome.		
Sequin, coined since 1760,	2	4¼
Scudo of Republic,	17	0¼
Sweden.		
Ducat,	2	5
Switzerland.		
Pistole of the Helvetic Republic, 1800,	4	21½
Treves.		
Ducat,	2	5¾
Tuscany.		
Zechino, or Sequin,	2	5¾
Ruspone of the Kingdom of Etruria,	6	17¼
Venice.		
Zechino, or Sequin,	2	6
Wirtemberg.		
Carolin,	6	3½
Ducat,	2	5
Zurich.		
Ducat, (double in proportion,)	2	5¾

What will $8\frac{2}{4}$ pounds of codfish cost at
8 cents and $\frac{2}{4}$ a pound. Edmund —

If $\frac{1}{3}$ of 6 be 8 what will $\frac{1}{4}$ of 20 be. Henry [illegible]
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